

Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points)

Score.....

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = \ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c . Please read and answer the following questions carefully and completely.

Score.....

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1 . Then, explain the relationship between the level of consumption and the risk free rate in this economy.

$$\begin{aligned} U(C) &= \ln(C) & U'(C_0) &= R_f \delta E[U'(C_1)] \\ &= C^{-1} & C_0^{-1} &= R_f \delta E[C_1^{-1}] \\ & & \frac{1}{R_f} &= \delta E\left[\frac{C_1}{C_0}\right]^{-1} \\ & & R_f &= \frac{1}{\delta} \left(\frac{C_1}{C_0}\right) \end{aligned}$$

The level of consumption in period 0 increases
 The risk free rate decrease
 The level of consumption in period 1 increases
 The risk free rate also increase

Score.....

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$\epsilon = \frac{R_f}{\frac{C_1}{C_0}} \times \frac{\partial \frac{C_1}{C_0}}{\partial R} = \frac{\partial \ln(C_1/C_0)}{\partial \ln(R_f)}$$

$$R_f = \frac{1}{\delta} \left(\frac{C_1}{C_0} \right)$$

$$\frac{\partial R_f}{\partial \ln(C_1/C_0)} = \frac{1}{\delta}$$

$$\partial \ln(R_f) = \ln\left(\frac{1}{\delta} \left(\frac{C_1}{C_0}\right)\right)$$

$$= -\ln(\delta) + \ln\left(\frac{C_1}{C_0}\right)$$

$$= 1 \frac{R_f}{C_1/C_0}$$

$$\epsilon = \frac{\partial \ln(C_1/C_0)}{\partial \ln(R_f)}$$

$$\epsilon = \frac{1}{1}$$

$$\epsilon = 1$$

if the interest rate increase
the individual's consumption level remain the same
since $\epsilon = 1$ the IE and SE offset each other

Score.....

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$$P_i = E \left[\frac{\delta U'(C_1)}{U'(C_0)} X_i \right] \quad E(C_1 X_i^N)$$

Let $CPI = \text{consumer index}$

$$P_i^N = E \left[\frac{1}{I_0} \frac{U'(C_1)}{U'(C_0)} X_i^N \right]$$

$$= E \left[\frac{1}{I_0} \frac{1}{C_1} X_i^N \right]$$

$$= E \left[\frac{1}{I_0} \left(\frac{C_0}{C_1} \right) X_i^N \right]$$

Pricing kernel is SDF for nominal return

Score.....

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

$$U(C) = \ln(C) \quad m_1, \delta(C_1/C_0) = \delta e^{\ln(C_1/C_0)}$$

$$\frac{U(C)}{E[m_1]} : \sqrt{\quad}$$