assignment_5_achariyaporn.R

10.10

2021-04-26

```
cat(rep("\n",50))

library(quantmod)

## Loading required package: xts

## Loading required package: zoo

## Warning: package 'zoo' was built under R version 4.0.4

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: TTR

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

library(fBasics)

## Loading required package: timeDate

## Loading required package: timeSeries

##
## Attaching package: 'timeSeries'

## The following object is masked from 'package:zoo':
##
##     time<-

##
## Attaching package: 'fBasics'

## The following object is masked from 'package:TTR':
##
##     volatility

library(sn)
```

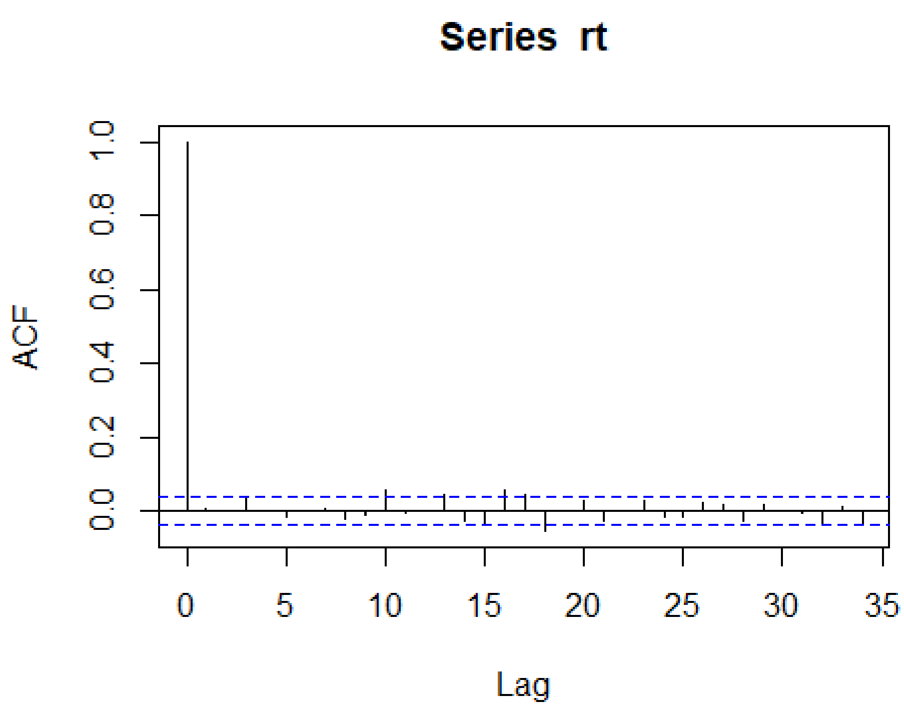
```
## Warning: package 'sn' was built under R version 4.0.5

## Loading required package: stats4

##
## Attaching package: 'sn'

## The following objects are masked from 'package:fBasics':
##
##     tr, vech

## The following object is masked from 'package:stats':
##
##     sd

library(PerformanceAnalytics)

##
## Attaching package: 'PerformanceAnalytics'

## The following objects are masked from 'package:timeDate':
##
##     kurtosis, skewness

## The following object is masked from 'package:graphics':
##
##     legend

library(car)

## Loading required package: carData

##
## Attaching package: 'car'

## The following object is masked from 'package:fBasics':
##
##     densityPlot

library(tseries)
library(forecast)

## Warning: package 'forecast' was built under R version 4.0.4

library(fGarch)

## Warning: package 'fGarch' was built under R version 4.0.5

require(quantmod)
```

```r
#question 1
getSymbols("CAT",from="2006-01-03", to="2017-04-13")
```

```
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
```

```
## [1] "CAT"
```

```r
rt=diff(log(as.numeric(CAT[,6])))



#1a Are there any serial correlations in the log return series rt? Why?
acf(rt)
```



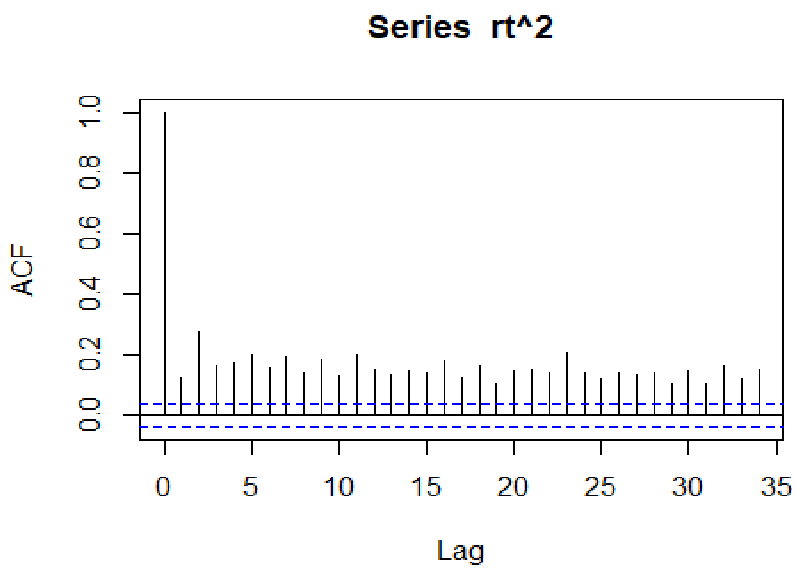

```r
Box.test(rt, lag = 10,type = 'Ljung')
```

```
##
##  Box-Ljung test
##
```

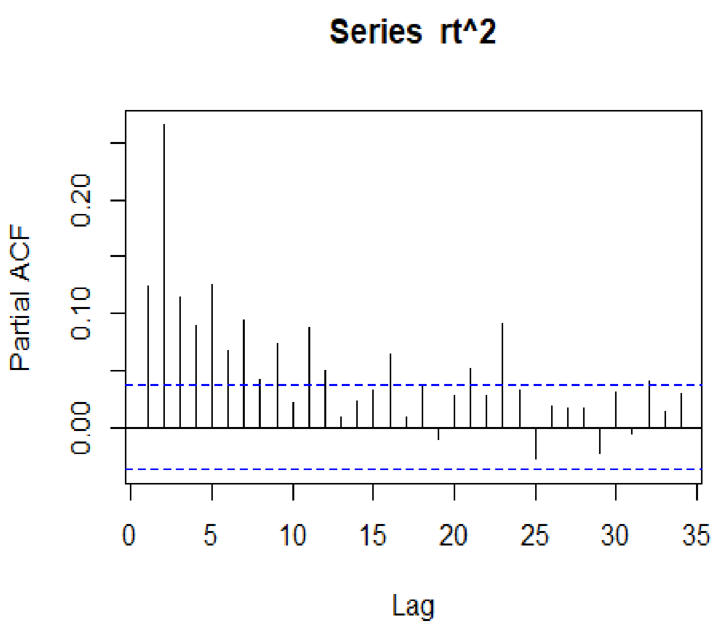
```
## data:  rt
## X-squared = 16.291, df = 10, p-value = 0.09159
```

Ans: There is no serial correlation among the log return series as you can see from the box test that we do not reject H0 (p-value is greater than alpha).

```
#1b Are there any ARCH effects in the log return series rt? Why?
acf(rt^2)
```




```
pacf(rt^2)
```



```r
Box.test(rt^2,lag=10,type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  rt^2
## X-squared = 917.21, df = 10, p-value < 2.2e-16
```

Ans: Since the result from the box test of rt^2 showing that the p-value is less than 2.2e-16 which is definitely less than 0.05 (alpha), therefore, we reject H0, there is ARCH effect in rt with 95% confidence level.

```r
#1c Fit a Gaussian ARMA(1,0)-GARCH(1,1) model to the rt series. Perform model checking, including showing the normal QQ-plot of the standardized residuals. Is model adequate? Write down the fitted model.
m1=garchFit(formula = ~arma(1,0)+garch(1,1),data = rt,trace = F)
```

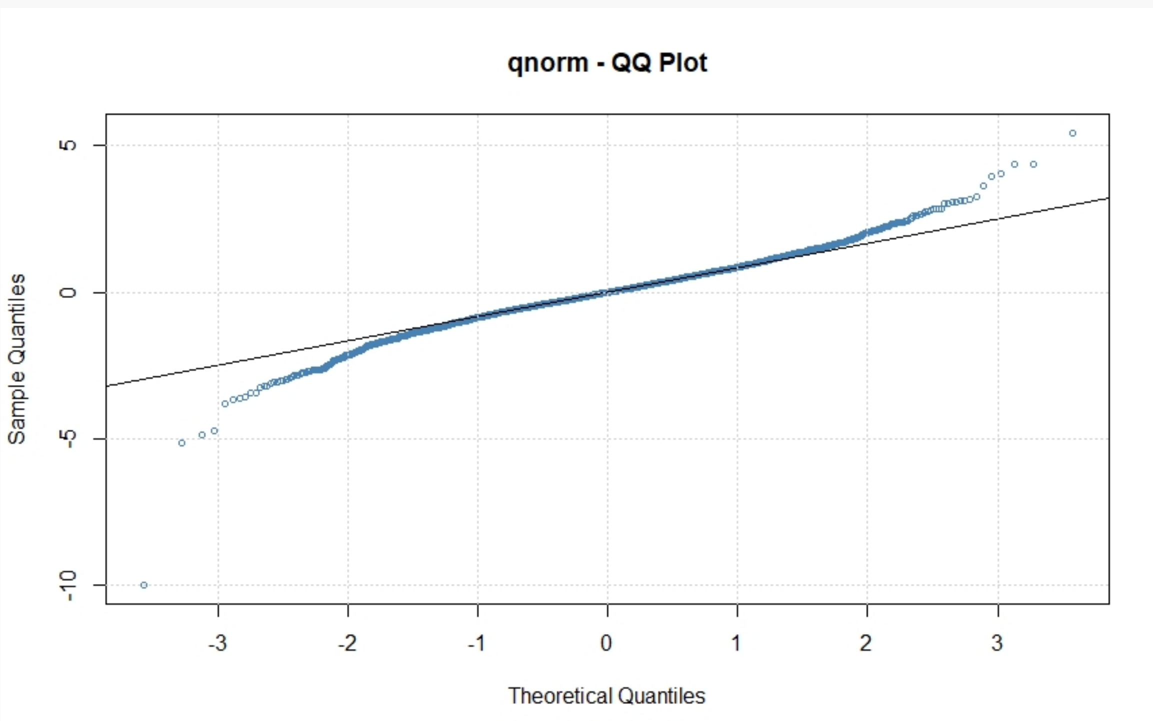
```
## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```r
summary(m1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x0000000020680c98>
##  [data = rt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu          ar1        omega       alpha1        beta1
## 4.8298e-04  1.6866e-02  4.4779e-06  4.9720e-02  9.3866e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.830e-04   3.075e-04    1.571 0.116297
## ar1    1.687e-02   2.006e-02    0.841 0.400353
## omega  4.478e-06   1.278e-06    3.503 0.000461 ***
## alpha1 4.972e-02   8.191e-03    6.070 1.28e-09 ***
```

```
## beta1  9.387e-01   1.031e-02   91.048  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7378.56    normalized:  2.599916
##
## Description:
##  Mon Apr 26 15:06:53 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3298.441  0
##  Shapiro-Wilk Test  R    W      0.9663735 0
##  Ljung-Box Test     R    Q(10)  12.37554  0.2607088
##  Ljung-Box Test     R    Q(15)  14.79514  0.4662719
##  Ljung-Box Test     R    Q(20)  19.20107  0.5087928
##  Ljung-Box Test     R^2  Q(10)  0.980939  0.9998424
##  Ljung-Box Test     R^2  Q(15)  3.682825  0.9986048
##  Ljung-Box Test     R^2  Q(20)  6.9285    0.996913
##  LM Arch Test       R    TR^2   2.723165  0.9972029
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.196308 -5.185823 -5.196314 -5.192526
```

plot(m1)



Ans: Fitted model

Mean equation -> rt (hat) = 4.830e-04+(1.687e-02)(ϕ1 hat)

(3.075e-04) (2.006e-02)

Variance equation -> σt^2 = 4.478e-06+(4.972e-02)(a t-1 ^2)

(1.278e-06) (8.191e-03)

#1d Build a GARCH(1,1) model with standardized Student-1 innovations for the rt series. Perform model checking, including the QQ-plot. Is model adequate? Why?

```
m2=garchFit(~garch(1,1),data = rt,cond.dist = "std",trace = F)
```

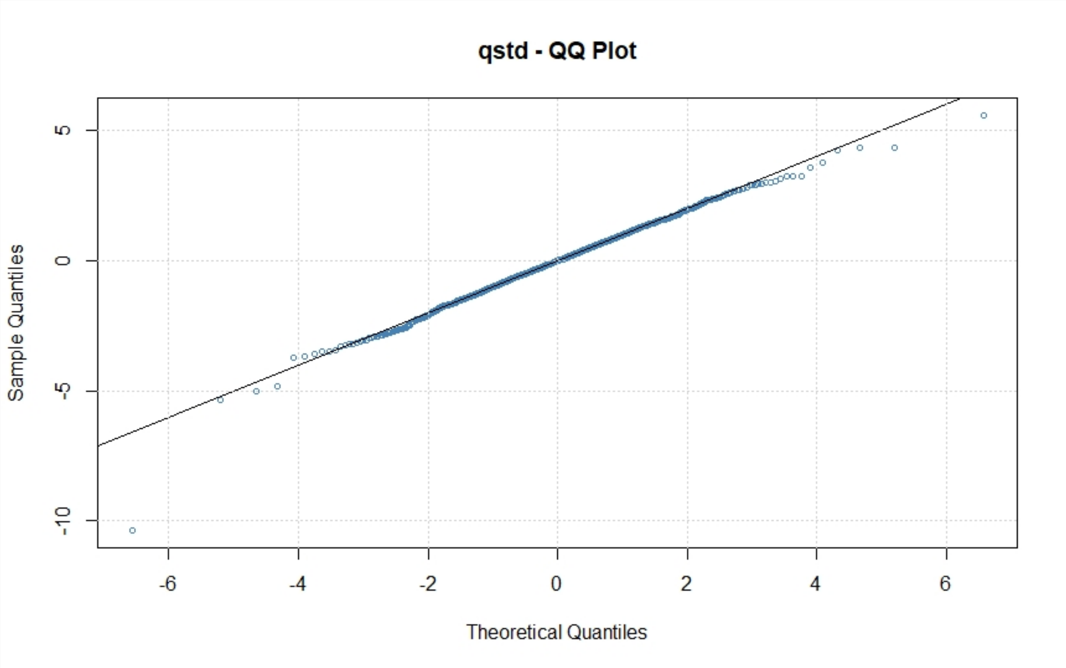
```
## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001ef4e758>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 5.9780e-04  4.2035e-06  7.2376e-02  9.2033e-01  5.0958e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.978e-04   2.702e-04    2.212  0.02695 *
## omega  4.203e-06   1.571e-06    2.675  0.00747 **
## alpha1 7.238e-02   1.374e-02    5.267 1.39e-07 ***
```

```
## beta1  9.203e-01   1.472e-02   62.503  < 2e-16 ***
## shape  5.096e+00   4.825e-01   10.561  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7507.106    normalized:  2.64521
##
## Description:
##  Mon Apr 26 15:06:54 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  4056.502  0
##  Shapiro-Wilk Test  R    W      0.9639091 0
##  Ljung-Box Test     R    Q(10)  14.77968  0.140303
##  Ljung-Box Test     R    Q(15)  16.74279  0.3344718
##  Ljung-Box Test     R    Q(20)  20.39783  0.433304
##  Ljung-Box Test     R^2  Q(10)  2.953085  0.9825066
##  Ljung-Box Test     R^2  Q(15)  5.482428  0.9871938
##  Ljung-Box Test     R^2  Q(20)  9.458146  0.9769677
##  LM Arch Test       R    TR^2   4.273688  0.977976
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.286897 -5.276412 -5.286903 -5.283115
```

plot(m1)



Ans: Since the Ljung-box test for both mean and volatility equations, the results of the test is that all value that obtained are greater that 0.05 (alpha) meaning that the model is adequate as we do not reject H0.

```
#1e write down the fitted model.
```

Ans: Fitted model

Mean equation: rt (hat) = 5.978e-04

(2.702e-04)

Variance equation:

σt^2 = 4.203e-06 + (7.238e-02)(at-1 ^2) + (9.203e-01)(σt-1^2)

(1.571e-06) (1.374e-02) (1.472e-02)

```
#1f Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted GARCH(1,1) model with standardized Student-t innovations.
predict(m2,5)
```

Ans:

```
##   meanForecast  meanError standardDeviation
## 1 0.0005977987 0.01515760        0.01515760
## 2 0.0005977987 0.01524072        0.01524072
## 3 0.0005977987 0.01532280        0.01532280
## 4 0.0005977987 0.01540384        0.01540384
## 5 0.0005977987 0.01548387        0.01548387
```

```
#1g Compute the 95% 1-step to 5-step interval predictions of the log return series using standardized student-t innovations.
```

Ans: meanforecast-(1.96)(meanerror) to meanforecast+(1.96)(meanerror)

1 step: -0.02911 to 0.03031

2 step: -0.02927 to 0.0305

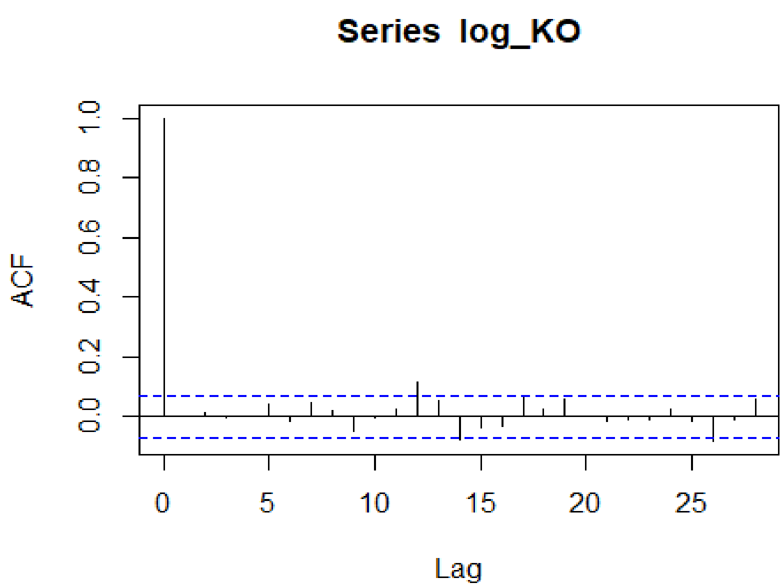
3 step: -0.02943 to 0.03063

4 step: -0.02959 to 0.0308

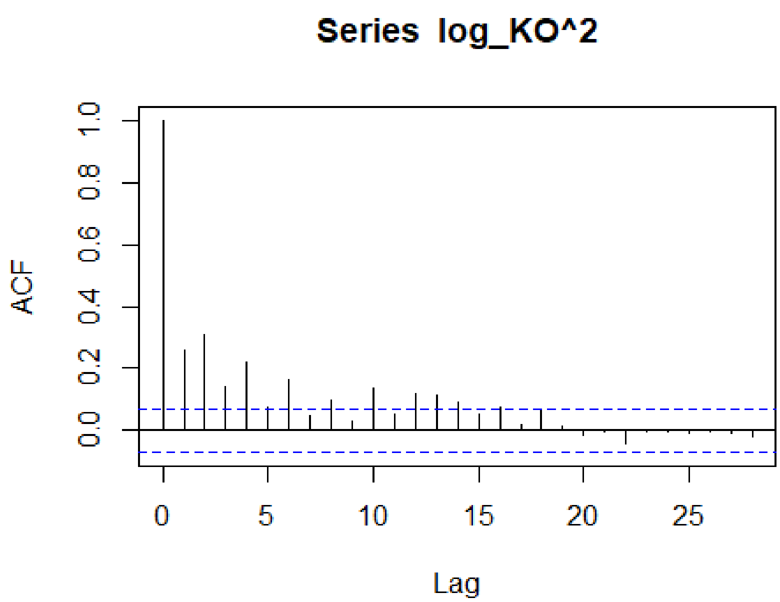
5 step: -0.02975 to 0.0309

```
#question 2
KO= read.table(file = "m-kovw-5116.txt",header = TRUE)
log_KO=log(1+KO[,3])
```

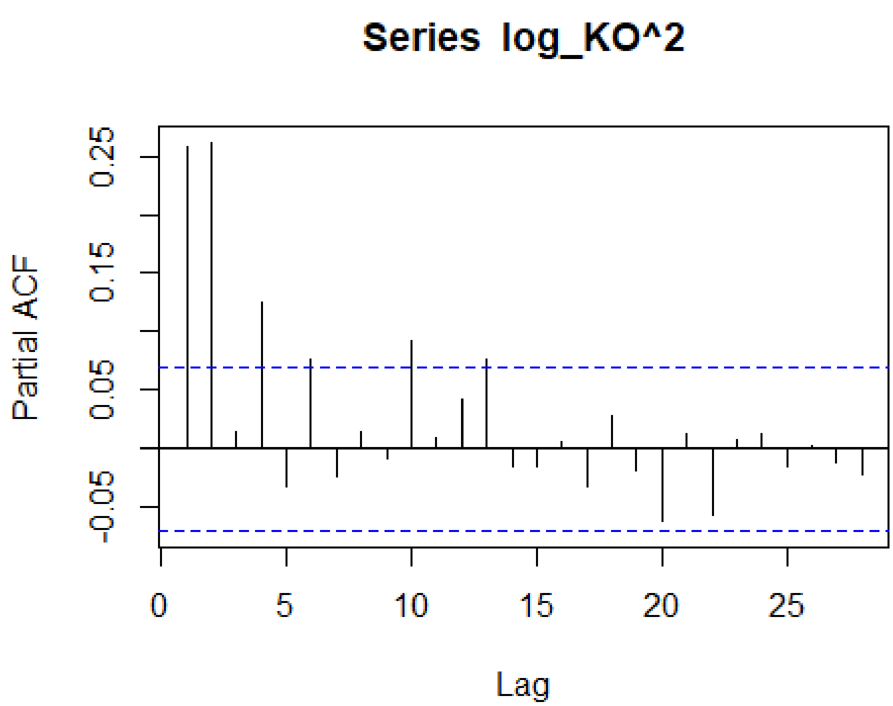
```
#2a Is the expected value of KO log return zero? Why? Is there any serial cor
relation in the log returns? Why? Is there any ARCH effect in the log returns
? Why?
t.test(log_KO)

##
##  One Sample t-test
##
## data:  log_KO
## t = 4.9853, df = 779, p-value = 7.628e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.00625636 0.01438347
## sample estimates:
##  mean of x
## 0.01031992

acf(log_KO)
```




```
acf(log_KO^2)
```




```
pacf(log_KO^2)
```




```
Box.test(log_KO,lag = 10,type = 'Ljung')

##
##  Box-Ljung test
##
```

```
## data:  log_KO
## X-squared = 5.9201, df = 10, p-value = 0.8219
```

```
Box.test(log_KO^2,lag = 10,type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  log_KO^2
## X-squared = 228.23, df = 10, p-value < 2.2e-16
```

Ans: The expected return of KO is not equal to zero as the p-value that we obtain from t-test is less that 0.05 (alpha), therefore, we reject H0, or the mean is not equal to zero at 95% confidence level. For the serial correlation, the result box test of KO is p-value is greater than 0.05 (alpha),so we do not reject H0, therefore, there is no serial correlation. And there is ARCH effect in KO^2 as the value obtain from box test is 2.2e-16 which is less than 0.05 (alpha) so we reject H0, there is ARCH effect in this return series.

```
#2b build a AR(1)-GARCH(1,1) model with Gaussian innovations for the log retu
rn series. Perform model checking and write down the fitted model.
m3=garchFit(formula = ~arma(1,0)+garch(1,1),data = log_KO,trace = F )
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = log_KO,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001d5a15c0>
##  [data = log_KO]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124544  -0.02633742   0.00018112   0.09535029   0.84861593
##
## Std. Errors:
##  based on Hessian
##
```

```
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.125e-02   1.897e-03    5.929 3.05e-09 ***
## ar1    -2.634e-02   3.881e-02   -0.679  0.49740
## omega   1.811e-04   5.852e-05    3.095  0.00197 **
## alpha1  9.535e-02   1.915e-02    4.978 6.42e-07 ***
## beta1   8.486e-01   2.766e-02   30.675  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.664    normalized:  1.500852
##
## Description:
##  Mon Apr 26 15:06:54 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  92.91946  0
##  Shapiro-Wilk Test  R    W      0.9857081 6.655604e-07
##  Ljung-Box Test     R    Q(10)  9.306169  0.5033144
##  Ljung-Box Test     R    Q(15)  22.9901   0.0843502
##  Ljung-Box Test     R    Q(20)  27.44814  0.1231201
##  Ljung-Box Test     R^2  Q(10)  12.63377  0.2448749
##  Ljung-Box Test     R^2  Q(15)  13.62088  0.5544545
##  Ljung-Box Test     R^2  Q(20)  15.19817  0.7649584
##  LM Arch Test       R    TR^2   10.65102  0.5590389
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.988883 -2.959016 -2.988965 -2.977396
```

Ans: Fitted model

Mean equation: rt(hat)= 1.125e-02 + (-2.634e-02)(ϕ1 hat)

(1.897e-03) (3.881e-02)

Variance equation: σt^2 = 1.811e-04 + (9.535e-02)(at-1^2) + (8.486e-01)(σt-1^2)

(5.852e-05) (1.915e-02) (2.766e-02)

```
#2c fit a AR(1)-GARCH(1,1) model with standardized Student-t innovations to t
he log return series. Perform model checking and write down the fitted model.
m4=garchFit(formular = ~arma(1,0)+garch(1,1),data = log_KO,cond.dist = "std",
trace = F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m4)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(data = log_KO, cond.dist = "std", trace = F, formular = ~arma(1,
##     0) + garch(1, 1))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x00000000201c60c8>
##  [data = log_KO]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
## Description:
##  Mon Apr 26 15:06:54 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
```

```
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
##  Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
##  LM Arch Test       R    TR^2   10.62989  0.560875
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

Ans: Fitted model

Mean equation: rt(hat)= 1.125e-02

(1.757e-03)

Variance equation: σt^2 = 1.753e-04 + (9.644e-02)(at-1^2) + (8.501e-01)(σt-1^2)

(6.627e-05) (2.337e-02)   (3.277-02)

```
#2d build a GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.
m5=garchFit(formula = ~garch(1,1),data = log_KO,trace = F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m5)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = log_KO, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001d984a08>
##  [data = log_KO]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu       omega      alpha1       beta1
## 0.01098417  0.00018497  0.09479925  0.84780406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
```

```
## mu     1.098e-02   1.846e-03    5.950 2.68e-09 ***
## omega  1.850e-04   5.899e-05    3.135  0.00172 **
## alpha1 9.480e-02   1.912e-02    4.958 7.11e-07 ***
## beta1  8.478e-01   2.787e-02   30.416  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.393    normalized:  1.500504
##
## Description:
##  Mon Apr 26 15:06:54 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.07163  0
##  Shapiro-Wilk Test  R    W      0.9856773 6.481596e-07
##  Ljung-Box Test     R    Q(10)  8.125181  0.6166108
##  Ljung-Box Test     R    Q(15)  21.27199  0.128362
##  Ljung-Box Test     R    Q(20)  25.62765  0.1784646
##  Ljung-Box Test     R^2  Q(10)  12.90586  0.228983
##  Ljung-Box Test     R^2  Q(15)  13.87463  0.5350581
##  Ljung-Box Test     R^2  Q(20)  15.35522  0.755734
##  LM Arch Test       R    TR^2   10.96004  0.532346
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.990752 -2.966858 -2.990804 -2.981562
```

Ans: Fitted model

Mean equation: rt(hat)= 1.098e-02

(1.846e-03)

Variance equation: σt^2 = 1.850e-04 + (9.480e-02)(at-1^2) + (8.478e-01)(σt-1^2)

(5.899e-05) (1.912e-02) (2.787e-02)

```
#2e fit a GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.
m6=garchFit(formula = ~garch(1,1),data = log_KO,cond.dist = "std",trace = F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m6)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = log_KO, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001facf2b8>
##  [data = log_KO]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
## Description:
##  Mon Apr 26 15:06:54 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
##  Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
```

```
##  LM Arch Test       R    TR^2   10.62989  0.560875
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

Ans: Fitted model

Mean equation: rt(hat)= 1.105e-02

(1.757e-03)

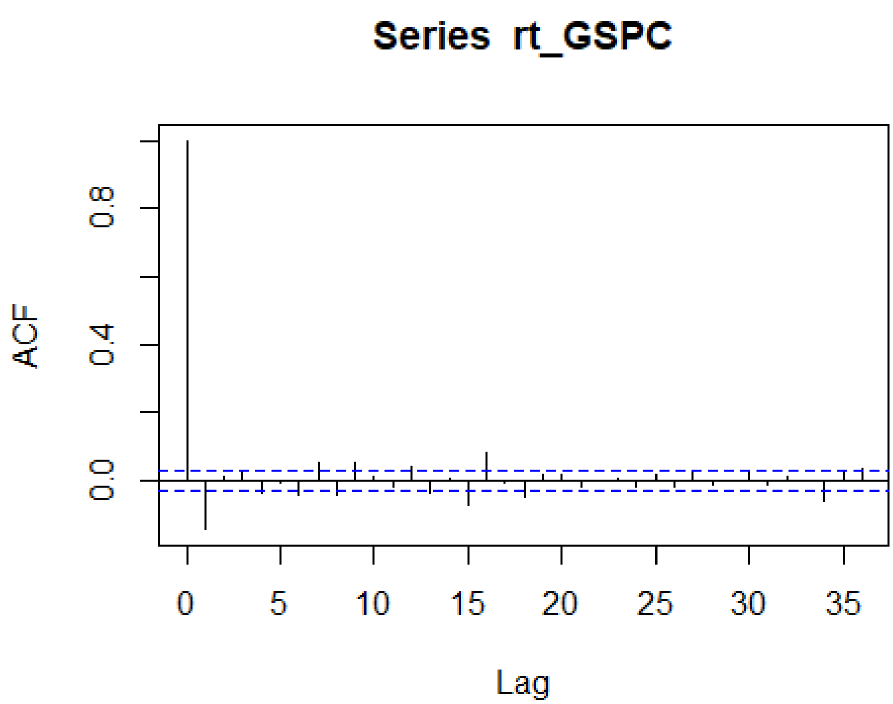
Variance equation: σt^2 = 1.753e-04 + (9.633e-02)(at-1^2) + (8.501e-01)(σt-1^2)

(6.627e-05) (2.337e-02)   (3.277e-02)

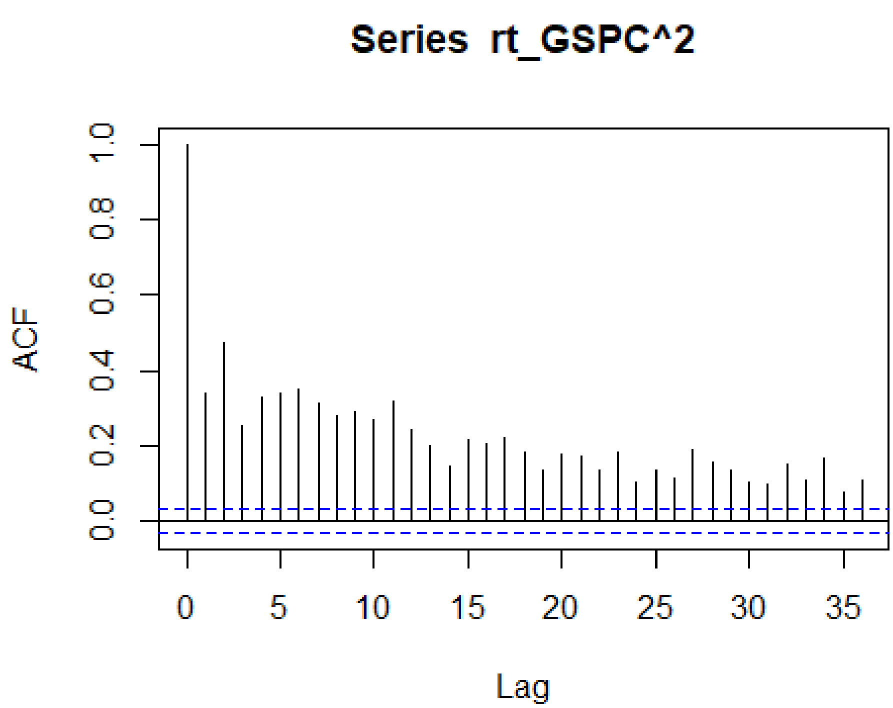
```
#2f compare the model (b)-(e)which model you select.
```

Ans: I would choose model m6 (GARCH(1,1) with standardized student-t innovations) as it is no need for us to add AR(1) with GARCH(1,1) because the coefficient in front of AR(1) is insignificant. Therefore, pure GARCH would work and it is better to use the model with student-t as you can see that the value of AIC is lower when we use GARCH(1,1) with standardized student-t rather than GARCH(1,1) with Gaussian innovations.

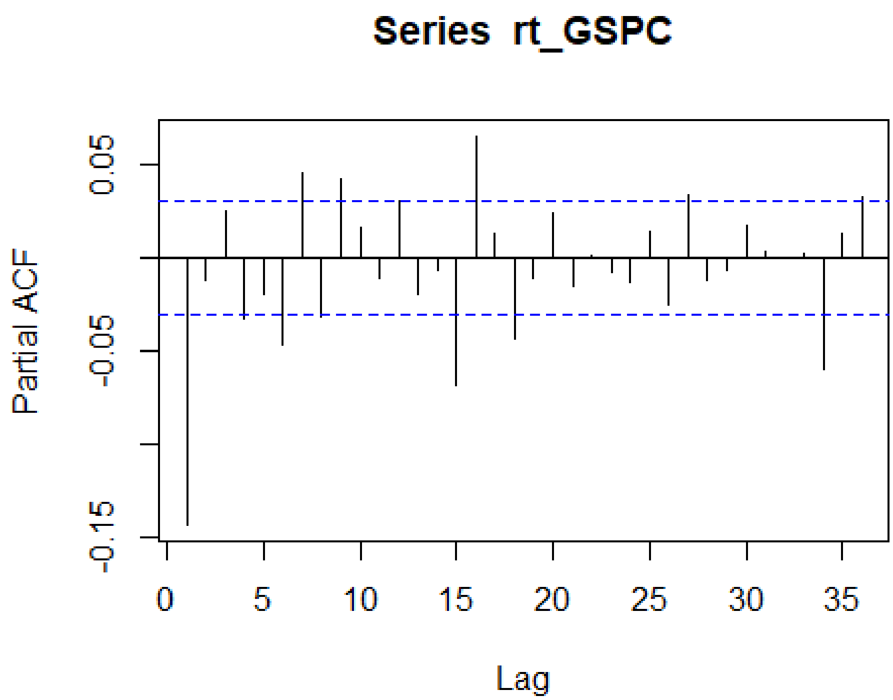
```
#question 3
require(quantmod)
getSymbols("^GSPC",from="2005-01-02",to="2021-03-31")

## [1] "^GSPC"

rt_GSPC=diff(log(as.numeric(GSPC[,6])))

#3a Is the expected value of rt zero? Why? Are there any serial correlations
in rt? Why?
t.test(rt_GSPC)

##
##  One Sample t-test
##
## data:  rt_GSPC
## t = 1.4961, df = 4086, p-value = 0.1347
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -9.051939e-05  6.737464e-04
## sample estimates:
##    mean of x
## 0.0002916135

acf(rt_GSPC)
```




```
acf(rt_GSPC^2)
```



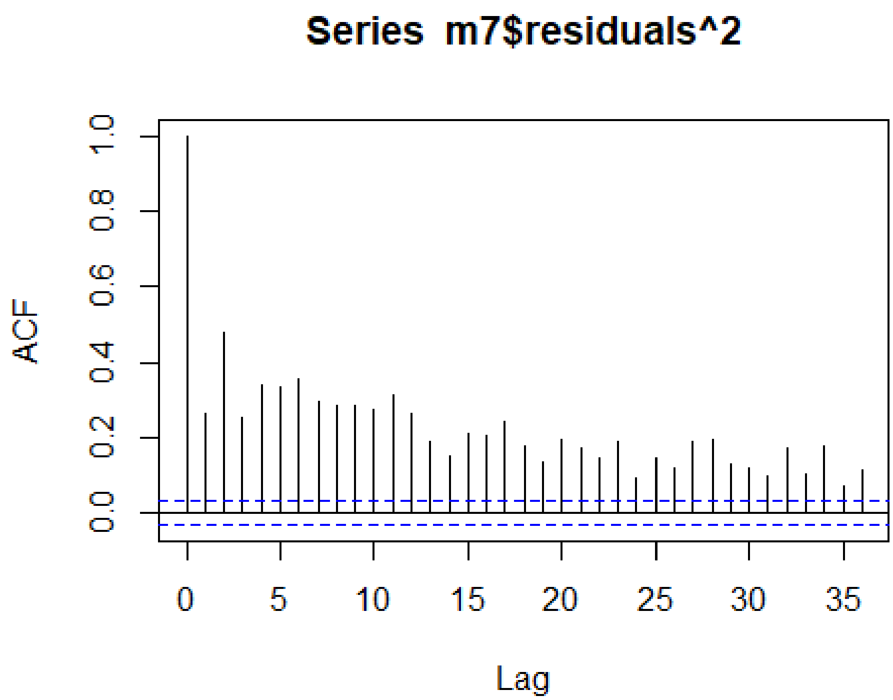

```
pacf(rt_GSPC)
```




```
Box.test(rt_GSPC,lag = 10,type = 'Ljung')

##
##  Box-Ljung test
##
## data:  rt_GSPC
## X-squared = 131.85, df = 10, p-value < 2.2e-16
```

Ans: The expected return of S&P500 is equal to zero as the p-value that we obtain from the t-test is greater than 0.05 (alpha) so we reject H0 or the mean of the return of S&P500 is equal to zero at 95% confidence interval. There is serial correlation among the return as the p-value from box test is less than 0.05 (alpha), so we reject H0.

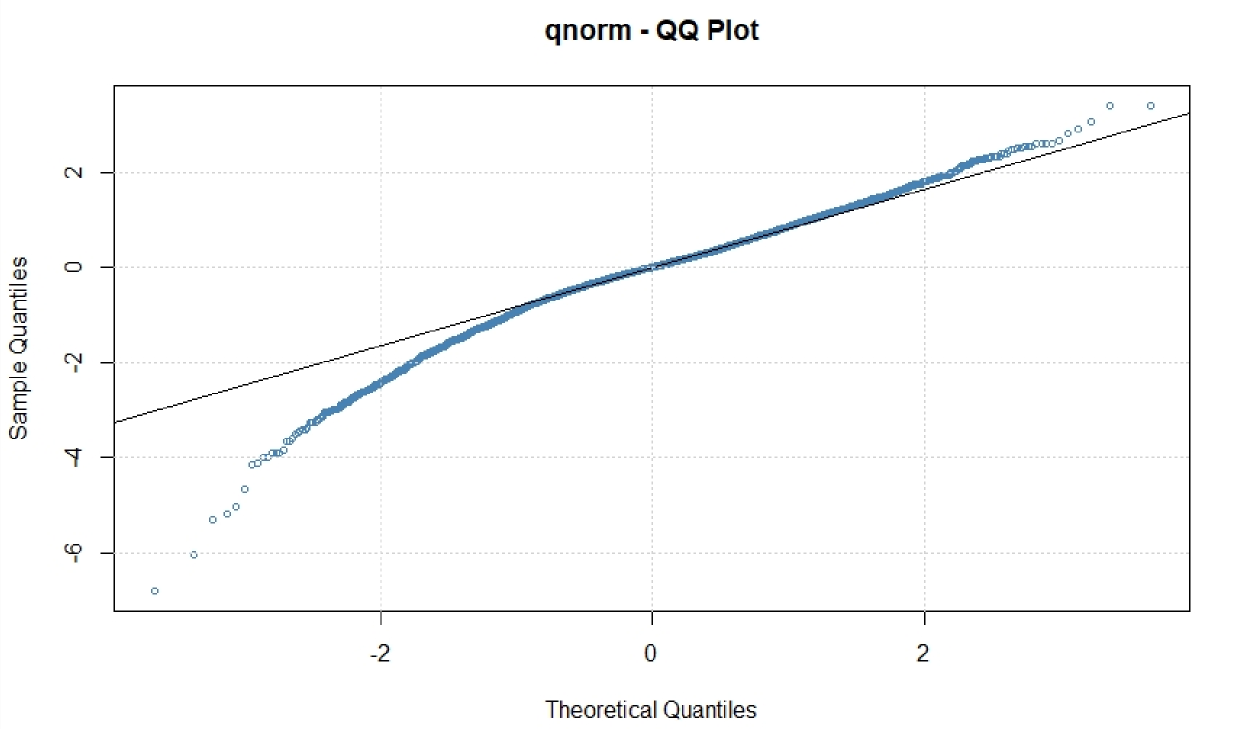
```
#3b Fit a Gaussian ARMA-GARCH model to the rt series. Obtain the normal QQ-pl
ot of the standardized residuals, and write down the fitted model. Is the mod
el adequate? Why?
m7=arima(rt_GSPC,order = c(4,0,0))
acf(m7$residuals^2)
```




```
Box.test(m7$residuals^2,lag = 10,type = 'Ljung')

##
##  Box-Ljung test
##
## data:  m7$residuals^2
## X-squared = 4255.3, df = 10, p-value < 2.2e-16

m8=garchFit(~arma(1,0)+garch(1,1),data = rt_GSPC,trace = F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m8)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt_GSPC,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001532f3d8>
##  [data = rt_GSPC]
```

```
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  7.4132e-04  -7.5408e-02   2.7169e-06   1.4182e-01   8.3740e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      7.413e-04   1.178e-04    6.292 3.14e-10 ***
## ar1    -7.541e-02   1.730e-02   -4.358 1.31e-05 ***
## omega   2.717e-06   3.504e-07    7.753 8.88e-15 ***
## alpha1  1.418e-01   1.202e-02   11.800  < 2e-16 ***
## beta1   8.374e-01   1.196e-02   70.032  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  13428.16    normalized:  3.285579
##
## Description:
##  Mon Apr 26 15:06:55 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1128.595  0
##  Shapiro-Wilk Test  R    W      0.9719221 0
##  Ljung-Box Test     R    Q(10)  17.65631  0.06104542
##  Ljung-Box Test     R    Q(15)  25.98986  0.03812936
##  Ljung-Box Test     R    Q(20)  31.68964  0.04671967
##  Ljung-Box Test     R^2  Q(10)  15.98919  0.09994225
##  Ljung-Box Test     R^2  Q(15)  18.25271  0.2496128
##  Ljung-Box Test     R^2  Q(20)  19.6233   0.4817047
##  LM Arch Test       R    TR^2   16.7807   0.1580347
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.568712 -6.560985 -6.568715 -6.565976
```

```
plot(m8)
```



Ans: Fitted model

Mean equation: rt(hat)= 7.413e-04 + (-7.541e-02)(ϕ1 hat)

(1.178e-04) (1.730e-02)

Variance equation: σt^2 = 2.717e-06 + (1.418e-01)(at-1^2) + (8.374e-01)(σt-1^2)

(3.504e-07) (1.202e-02) (1.196e-02)

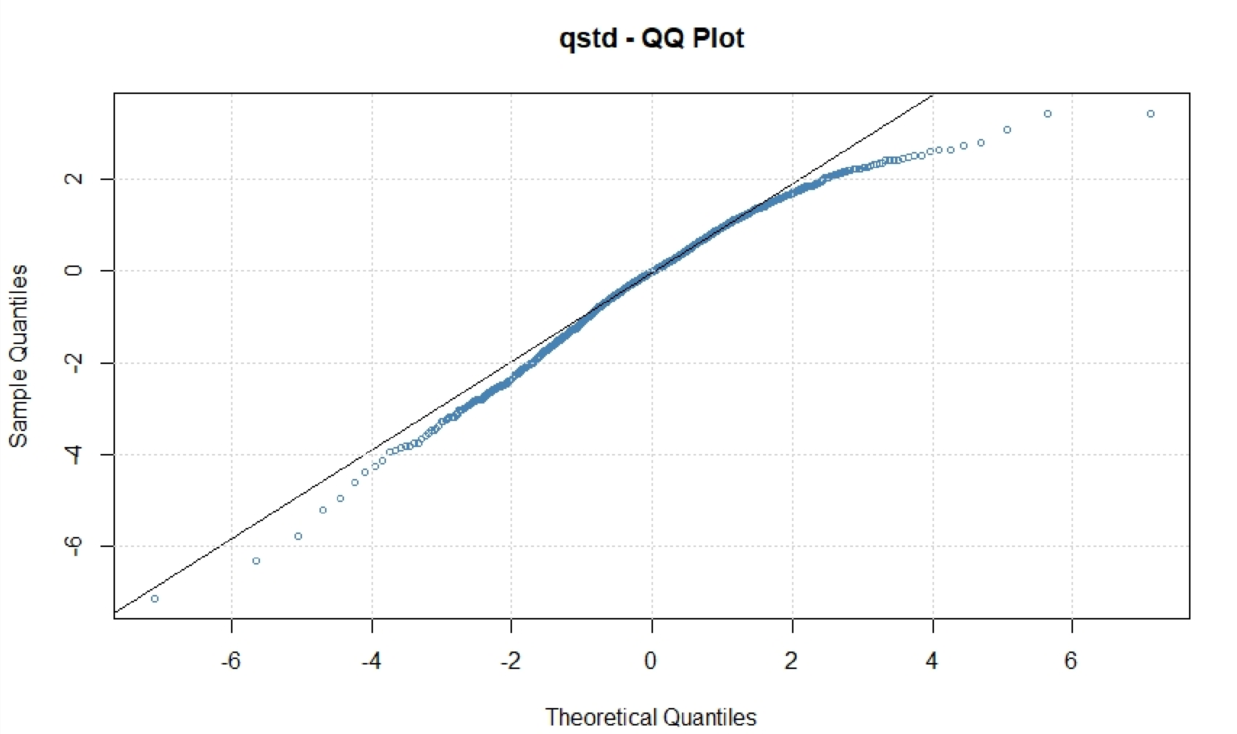
The model is adequate except for the mean equation for 15 and 20 lags.

```
#3c Build a ARMA-GARCH model with Student-t innovations for the log return se
ries. Perform model checking and write down the fitted model.
m9=garchFit(~arma(1,0)+garch(1,1),data = rt_GSPC,cond.dist = "std",trace = F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m9)

##
## Title:
##  GARCH Modelling
##
```

```
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt_GSPC,
##     cond.dist = "std", trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001d554688>
##  [data = rt_GSPC]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1        sha
pe
##  9.1359e-04  -7.0189e-02   1.6949e-06   1.4129e-01   8.5658e-01   5.0787e+
00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      9.136e-04   1.061e-04    8.612  < 2e-16 ***
## ar1    -7.019e-02   1.570e-02   -4.469 7.85e-06 ***
## omega   1.695e-06   3.625e-07    4.676 2.93e-06 ***
## alpha1  1.413e-01   1.460e-02    9.675  < 2e-16 ***
## beta1   8.566e-01   1.286e-02   66.618  < 2e-16 ***
## shape   5.079e+00   4.283e-01   11.857  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  13563.45    normalized:  3.318682
##
## Description:
##  Mon Apr 26 15:06:56 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1426.546  0
##  Shapiro-Wilk Test  R    W      0.9697692 0
##  Ljung-Box Test     R    Q(10)  17.75545  0.05923161
##  Ljung-Box Test     R    Q(15)  26.09726  0.03701271
##  Ljung-Box Test     R    Q(20)  31.5407   0.04844507
##  Ljung-Box Test     R^2  Q(10)  13.10432  0.2178975
##  Ljung-Box Test     R^2  Q(15)  17.76262  0.275351
##  Ljung-Box Test     R^2  Q(20)  20.49609  0.4273063
##  LM Arch Test       R    TR^2   14.77776  0.2538157
```

```
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.634429 -6.625157 -6.634433 -6.631146
```

plot(m9)



Ans: Fitted model

Mean equation: rt(hat)= 9.136e-04 + (-7.019e-02)(ϕ1 hat)

(1.061e-04) (1.570e-02)

Variance equation: σt^2 = 1.695e-06 + (1.413e-01)(at-1^2) + (8.566-01)(σt-1^2)

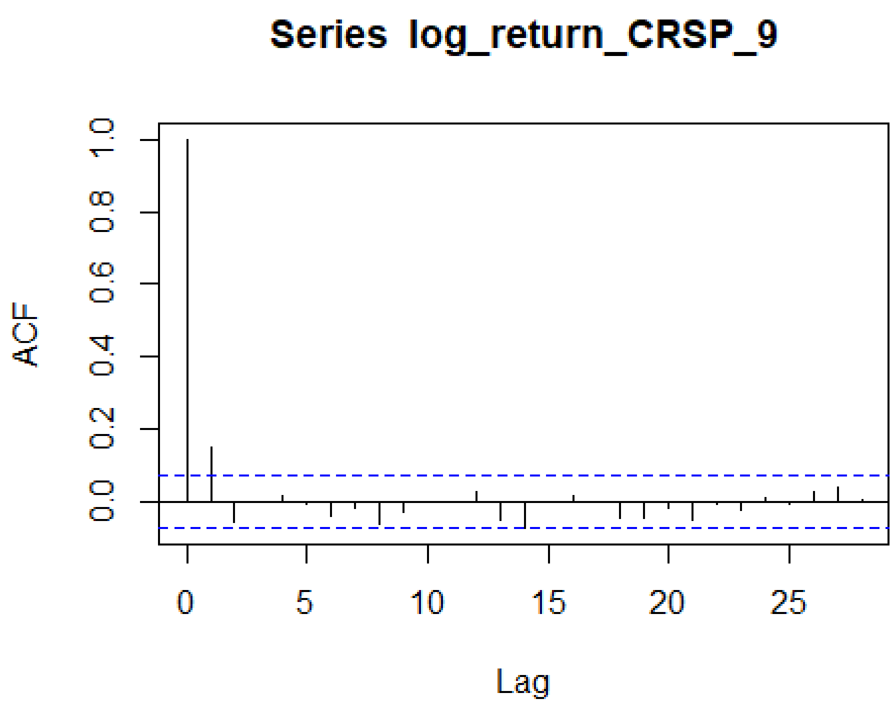
(3.625e-07) (1.460e-02) (1.286e-02)

```
#3d Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted ARMA-GARCH model with Student-t innovations.
predict(m9,5)
```

Ans:

```
##   meanForecast   meanError standardDeviation
## 1 0.0011355886 0.009054724       0.009054724
## 2 0.0008338889 0.009160372       0.009138299
## 3 0.0008550647 0.009243329       0.009220941
## 4 0.0008535784 0.009325270       0.009302675
## 5 0.0008536828 0.009406325       0.009383526
```

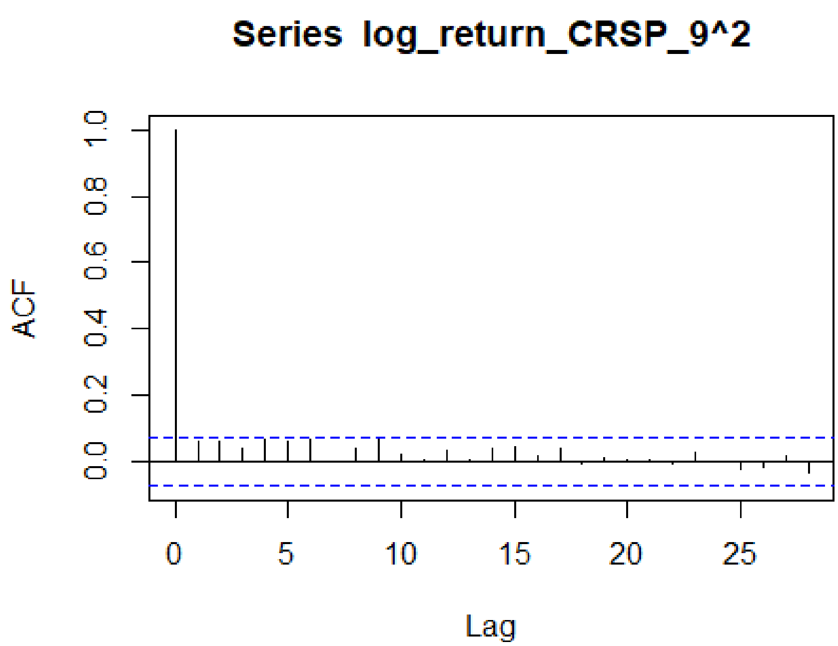
```r
#question 4
CRSP=read.table(file = "m-deciles.txt",header = TRUE)
log_return_CRSP_9=log(1+CRSP[,10])

#4a Is the expected value of the CRSP decile 9 portfolio log return zero? Why
? Is there any serial correlation in the log returns? Why? If necessary, find
an ARMA model to remove the serial correlations.
t.test(log_return_CRSP_9)

##
##  One Sample t-test
##
## data:  log_return_CRSP_9
## t = 5.1808, df = 719, p-value = 2.873e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.005946562 0.013203545
## sample estimates:
##   mean of x
## 0.009575054

acf(log_return_CRSP_9)
```



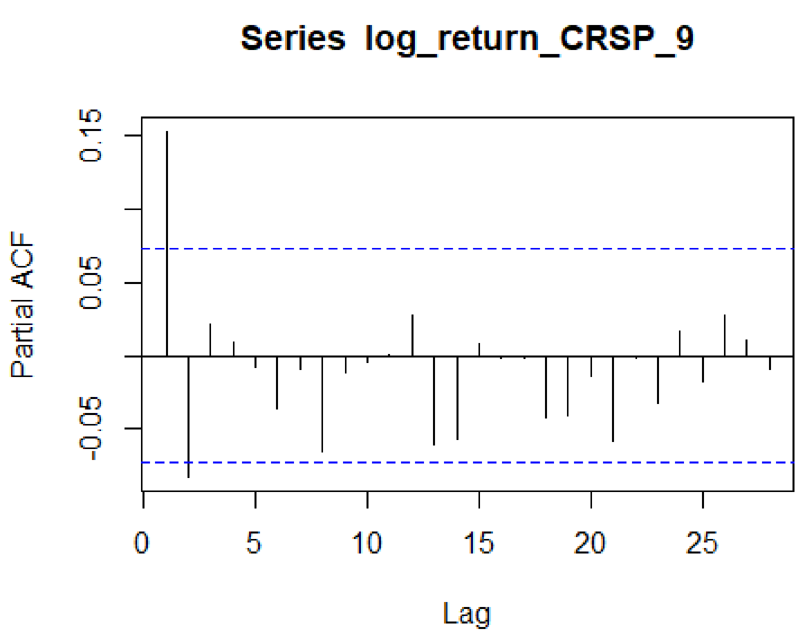

```r
Box.test(log_return_CRSP_9,lag = 10,type = 'Ljung')

##
##  Box-Ljung test
```

```
##
## data:  log_return_CRSP_9
## X-squared = 24.257, df = 10, p-value = 0.006946
```

```
acf(log_return_CRSP_9^2)
```



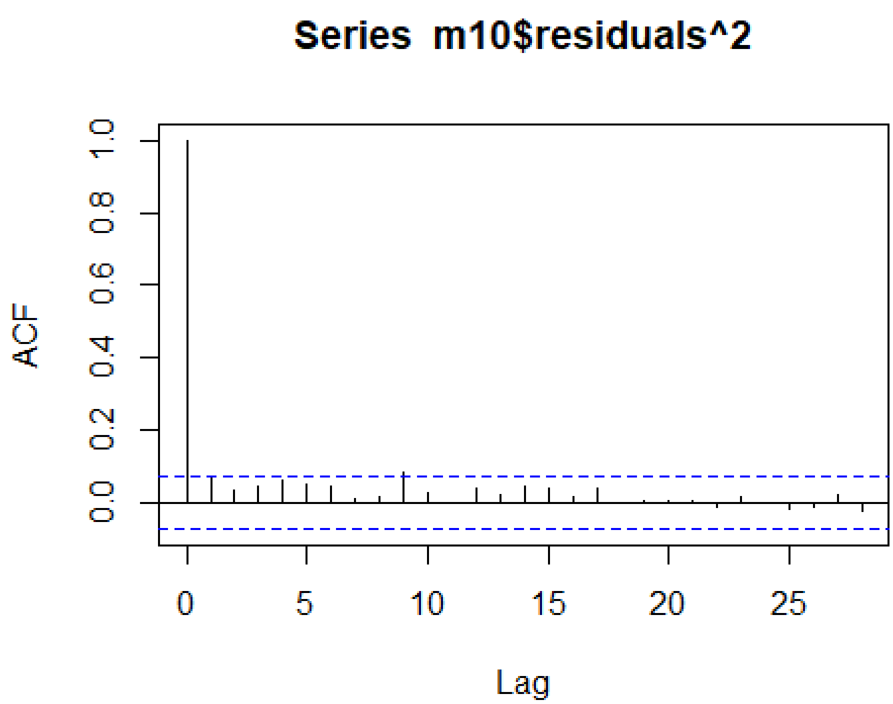

```
pacf(log_return_CRSP_9)
```



```
m10=arima(log_return_CRSP_9,order = c(2,0,0))
summary(m10)
```

```
##
## Call:
## arima(x = log_return_CRSP_9, order = c(2, 0, 0))
##
## Coefficients:
##          ar1      ar2  intercept
##       0.1649  -0.0822     0.0096
## s.e.  0.0372   0.0372     0.0020
##
## sigma^2 estimated as 0.002383:  log likelihood = 1152.58,  aic = -2297.15
##
## Training set error measures:
##                         ME       RMSE       MAE       MPE     MAPE      MASE
## Training set -3.62389e-06 0.04881268 0.0366968 -144.3403 492.917 0.7514755
##                     ACF1
## Training set 0.00181124
```

```
acf(m10$residuals^2)
```




```
Box.test(m10$residuals,lag = 10,type = 'Ljung')
```

```
##
##  Box-Ljung test
##
```

```
## data:  m10$residuals
## X-squared = 4.6209, df = 10, p-value = 0.915
```

```
Box.test(m10$residuals^2,lag = 10,type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  m10$residuals^2
## X-squared = 17.082, df = 10, p-value = 0.07258
```

Ans: The expected value of the CRSP is not equal to zero as the p-value we obtain from t-tes t is less than 0.05 (alpha), so we reject H0 at 95% confidence interval. There is serial correl ation among the return as when we did bow test for the log return of CRSP, the p-value that we get is less than 0.05 (alpha) so we reject H0 at 95% confidence level. The ARMA(2,0) mo del removes the serial correlations among the return as when we did the box test for model 10 which is the ARMA(2,0) model for CRSP stock, the p-value from box test is greater than 0.05 (alpha) meaning that there is no serial correlations among the return at 95% confiden ce level.

```
#4b Is there any ARCH effect in the log returns? Why?
Box.test(log_return_CRSP_9^2,lag = 10,type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  log_return_CRSP_9^2
## X-squared = 19.824, df = 10, p-value = 0.03096
```

Ans: There is ARCH effect as the p-value from box test is less than 0.05 (alpha) se we reject H0 at 95% confidence level.

```
#4c build a AR(1)-ARCH(1) model with Gaussian innovations for the log return
series. Perform model checking and write down the fitted model.
m11=garchFit(formula = ~arma(1,0)+garch(1,0),data = log_return_CRSP_9,trace =
F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m11)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = log_return_CRSP_9,
```

```
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x0000000020e79580>
##  [data = log_return_CRSP_9]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##      mu      ar1    omega   alpha1
## 0.01053  0.14707  0.00200  0.18152
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0105300   0.0019275    5.463 4.68e-08 ***
## ar1    0.1470670   0.0424301    3.466 0.000528 ***
## omega  0.0020000   0.0001482   13.493  < 2e-16 ***
## alpha1 0.1815182   0.0651142    2.788 0.005309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1158.628    normalized:  1.609206
##
## Description:
##  Mon Apr 26 15:06:56 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  647.6357  0
##  Shapiro-Wilk Test  R    W      0.962804  1.502752e-12
##  Ljung-Box Test     R    Q(10)  8.582995  0.572082
##  Ljung-Box Test     R    Q(15)  12.95811  0.6055337
##  Ljung-Box Test     R    Q(20)  15.77362  0.7305655
##  Ljung-Box Test     R^2  Q(10)  8.153476  0.6138486
##  Ljung-Box Test     R^2  Q(15)  12.29206  0.6568012
##  Ljung-Box Test     R^2  Q(20)  13.57787  0.851238
##  LM Arch Test       R    TR^2   8.24593   0.7656286
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.207301 -3.181861 -3.207363 -3.197480
```

Ans: Fitted model

Mean equation: rt(hat)= 0.01053+(0.14707)(ϕ1 hat)

(0.00193) (0.04243)

Variance equation: σt^2 = 0.00200 + (0.18152)(at-1^2)

(0.00015) (0.06511)

```
#4d fit a AR(1)-ARCH(1) model with standardized Student-t innovations to the
log return series. Perform model checking and write down the fitted model.
m12=garchFit(formula = ~arma(1,0)+garch(1,0),data = log_return_CRSP_9,cond.di
st = "std",trace = F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m12)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = log_return_CRSP_9,
##     cond.dist = "std", trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x000000001cb863b8>
##  [data = log_return_CRSP_9]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu        ar1      omega     alpha1      shape
## 0.0116189  0.1076982  0.0019203  0.1900830  6.4225253
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0116189   0.0017710    6.561 5.35e-11 ***
## ar1    0.1076982   0.0403169    2.671  0.00756 **
## omega  0.0019203   0.0001818   10.564  < 2e-16 ***
## alpha1 0.1900830   0.0713692    2.663  0.00774 **
## shape  6.4225253   1.3115905    4.897 9.74e-07 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1187.516    normalized:  1.649328
##
## Description:
##  Mon Apr 26 15:06:56 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  680.4834  0
##  Shapiro-Wilk Test  R    W      0.9612945 7.49198e-13
##  Ljung-Box Test     R    Q(10)  9.486455  0.4866411
##  Ljung-Box Test     R    Q(15)  13.8214   0.5391158
##  Ljung-Box Test     R    Q(20)  17.0087   0.6524086
##  Ljung-Box Test     R^2  Q(10)  7.567444  0.671006
##  Ljung-Box Test     R^2  Q(15)  11.42176  0.7221637
##  Ljung-Box Test     R^2  Q(20)  12.79422  0.8860373
##  LM Arch Test       R    TR^2   7.723697  0.8063327
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.284768 -3.252967 -3.284863 -3.272491
```

Ans: Fitted model

Mean equation: rt(hat)= 0.01162+(0.10770)(ϕ1 hat)

(0.00177) (0.04032)

Variance equation: σt^2 = 0.00192 + (0.19008)(at-1^2)

(0.00018) (0.07137)

```
#4e build a ARCH(1) model with Gaussian innovations for the log return series
. Perform model checking and write down the fitted model.
m13=garchFit(formula = ~garch(1,0),data = log_return_CRSP_9,trace = F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m13)

##
## Title:
##  GARCH Modelling
##
## Call:
```

```
##  garchFit(formula = ~garch(1, 0), data = log_return_CRSP_9, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x0000000020161c80>
##  [data = log_return_CRSP_9]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##       mu     omega    alpha1
## 0.012346  0.002016  0.194126
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.012346    0.001923    6.421 1.35e-10 ***
## omega   0.002016    0.000145   13.900  < 2e-16 ***
## alpha1  0.194126    0.062209    3.121  0.00181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1152.289    normalized:  1.600401
##
## Description:
##  Mon Apr 26 15:06:56 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8024  0
##  Shapiro-Wilk Test  R    W      0.9570702 1.171061e-13
##  Ljung-Box Test     R    Q(10)  18.72223  0.04393591
##  Ljung-Box Test     R    Q(15)  23.27998  0.07837365
##  Ljung-Box Test     R    Q(20)  27.61004  0.1189566
##  Ljung-Box Test     R^2  Q(10)  7.012055  0.7243064
##  Ljung-Box Test     R^2  Q(15)  10.3604   0.7964784
##  Ljung-Box Test     R^2  Q(20)  11.97825  0.9168225
##  LM Arch Test       R    TR^2   7.047101  0.8544853
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.192468 -3.173388 -3.192503 -3.185102
```

Ans: Fitted model

Mean equation: rt(hat)= 0.012346

(0.001923)

Variance equation: σt^2 = 0.002016 + (0.194126)(at-1^2)

(0.000145) (0.062209)

```
#4f fit a ARCH(1) model with standardized student-t innovations to the log re
turn series. Perform model checking and write down the fitted model.
m14=garchFit(formula = ~garch(1,0),data = log_return_CRSP_9,cond.dist = "std"
,trace = F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m14)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = log_return_CRSP_9, cond.dist = "s
td",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x00000000211f04a0>
##  [data = log_return_CRSP_9]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##       mu     omega    alpha1     shape
## 0.013356  0.001928  0.204163  6.220222
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.013356    0.001685    7.927 2.22e-15 ***
## omega   0.001928    0.000185   10.421  < 2e-16 ***
## alpha1  0.204163    0.072375    2.821  0.00479 **
## shape   6.220223    1.236608    5.030 4.90e-07 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1183.349    normalized:  1.64354
##
## Description:
##  Mon Apr 26 15:06:57 2021 by user: 10.10
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.3878  0
##  Shapiro-Wilk Test  R    W      0.9574271 1.363165e-13
##  Ljung-Box Test     R    Q(10)  18.51406  0.04688705
##  Ljung-Box Test     R    Q(15)  22.99926  0.08415548
##  Ljung-Box Test     R    Q(20)  27.3947   0.1245202
##  Ljung-Box Test     R^2  Q(10)  6.667871  0.7563837
##  Ljung-Box Test     R^2  Q(15)  10.0157   0.8187514
##  Ljung-Box Test     R^2  Q(20)  11.63085  0.928196
##  LM Arch Test       R    TR^2   6.819315  0.8693193
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.275969 -3.250529 -3.276031 -3.266148
```

Ans: Fitted model

Mean equation: rt(hat)= 0.013356

(0.001685)

Variance equation: σt^2 = 0.001928 + (0.204163)(at-1^2)

(0.000185) (0.072375)

```
#4g compare the model (c)-(f) which model you select.
```

Ans: I would select model m12 or AR(1)-ARCH(1) with standardized student-t as it is better to put AR(1) rather pure ARCH model because the coefficient of AR model is significant. And model m12(standardized student-t) is better than m11(Gaussian innovation) as it gives lower AIC.