

Constructing Efficient Portfolios Using MarkowitzII.xls

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Introduction

This document shows how to,

- (1) Compute the expected return and standard deviation of a portfolio of N given risky assets;
- (2) Construct the portfolio that gives the highest rate of return for a given standard deviation;
- (3) Construct the portfolio that has the lowest standard deviation for a given expected return;
- (4) Construct the tangent portfolio that has the the steepest capital allocation line CAL when combined with the risk-free rate of asset;
- (5) Compute the optimal combination of the tangent portfolio and the risk-free asset for a risk averse investor;

using *MarkowitzII.xls* (© 2000, Ravi Jagannathan and Andrew Kaplin)

You will require the following information for using *MarkowitzII.xls*.

- a. Expected returns, standard deviations, and the correlation matrix for the N risky asset returns;
- b. The risk-free rate of return;
- c. The risk aversion coefficient, A, of the investor that relates the certainty equivalent rate of return of a risky asset to its expected return and standard deviation¹:

$$\text{Certainty Equivalent Rate of Return} = (\text{Expected Return}) - .005 \times A \times (\text{Variance of Returns}).$$

You have the expected returns, standard deviations, and a correlation matrix for IBM, COLEX, ATX, HPX, IRWINX, and BORYLAND. The risk-free rate of return is 1.4% and the aversion coefficient is 25.

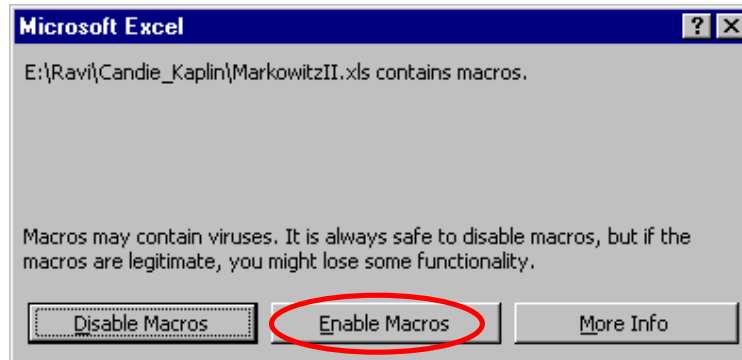
¹ See Bodie, Zvi, Alex Kane and Alan J. Marcus, *Investments* (Homewood, Illinois: Irwin, Fourth Edition 1999)



Section 1: Entering the inputs

1.1. Open *MarkowitzII.xls*

1.2. Enable Macros.



1.3. Input 6 (=N) in the cell C3 for the **Number of Securities** and press the Enter key ↵

1.4. Click the button **Construct Tables**.

No	Name	Fraction	Expected Return	Standard Deviation
1				
2				
3				
4				
5				
6				

Correlations	2	3	4	5
1				
2	1			
3		1		
4			1	
5				1

Results:	
Portfolio's Expected Return	0.0000
Portfolio's Standard Deviation	0.0000

Risk Free Rate	0.0191	Risk Aversion Coefficient	6.00
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Note on Using Excel to Compute Optimal Risky Portfolios

1.5. Input the means, the standard deviations and the correlation matrix as follows.

The screenshot shows the following data in the Excel spreadsheet:

No	Name	Fraction	Expected Return	Standard Deviation
1			0.075	0.139
2			0.109	0.090
3			0.189	0.180
4			0.204	0.452
5			0.223	0.148
6			0.161	0.258

Correlations	2	3	4	5
1	0.00	-0.21	0.13	0.39
2	1.00	-0.05	0.54	-0.09
3		1.00	-0.01	0.45
4			1.00	0.03
5				1.00

Results:	
Portfolio's Expected Return	0.0000
Portfolio's Standard Deviation	0.0000

At the bottom, the Risk Free Rate is 0.0191 and the Risk Aversion Coefficient is 5.00.

1.6. Input the names of the securities in Row B as follows.

1.7. Click the button **Fill in Names** to fill the names for the correlation matrix.

The screenshot shows the following data in the Excel spreadsheet:

No	Name	Fraction	Expected Return	Standard Deviation
1	IBX		0.075	0.139
2	COLEX		0.109	0.090
3	ATX		0.189	0.180
4	HPX		0.204	0.452
5	IRWINK		0.223	0.148
6	BORYLAND		0.161	0.258

Correlations	2	3	4	5
1	0.00	-0.21	0.13	0.39
2	1.00	-0.05	0.54	-0.09
3		1.00	-0.01	0.45
4			1.00	0.03
5				1.00

Results:	
Portfolio's Expected Return	0.0000
Portfolio's Standard Deviation	0.0000

The 'Fill in Names' button is circled in red, and the 'Name' column is highlighted with a yellow background.

Note on Using Excel to Compute Optimal Risky Portfolios

The following window will appear.

The Markowitz Portfolio Selection Model II

Number of securities:

No	Name	Fraction	Expected Return	Standard Deviation
1	IBX		0.075	0.139
2	COLEX		0.109	0.090
3	ATX		0.189	0.180
4	HPX		0.204	0.452
5	IRWINX		0.223	0.148
6	BORYLAND		0.161	0.258

Correlations	2 COLEX	3 ATX	4 HPX	5 IRWINX
1 IBX	0.00	-0.21	0.13	0.39
2 COLEX	1.00	-0.05	0.54	-0.09
3 ATX		1.00	-0.01	0.45
4 HPX			1.00	0.03
5 IRWINX				1.00

Risk Free Rate: Risk Aversion Coefficient:

Results:

Portfolio's Expected Return	0.0000
Portfolio's Standard Deviation	0.0000

1.8. Input **0.014** in the box **Risk Free Rate**.

1.9. Input **25** in the box **Risk Aversion Coefficient** and press the Enter key ↵

Number of securities:

No	Name	Fraction	Expected Return	Standard Deviation
1	IBX		0.075	0.139
2	COLEX		0.109	0.090
3	ATX		0.189	0.180
4	HPX		0.204	0.452
5	IRWINX		0.223	0.148
6	BORYLAND		0.161	0.258

Correlations	2 COLEX	3 ATX	4 HPX	5 IRWINX
1 IBX	0.00	-0.21	0.13	0.39
2 COLEX	1.00	-0.05	0.54	-0.09
3 ATX		1.00	-0.01	0.45
4 HPX			1.00	0.03
5 IRWINX				1.00

Risk Free Rate: Risk Aversion Coefficient:

Slope of CAL: y^* :



Section 2: Calculating the Expected Return and Standard Deviation for an Arbitrary Portfolio

Input the portfolio weights (fractions) of the investment on each of the N risky assets in row C to obtain

- (1) The expected return and standard deviation of a portfolio of N given risky assets.

The screenshot shows an Excel spreadsheet titled 'Markowitz'. The interface is designed for calculating portfolio metrics. Key elements include:

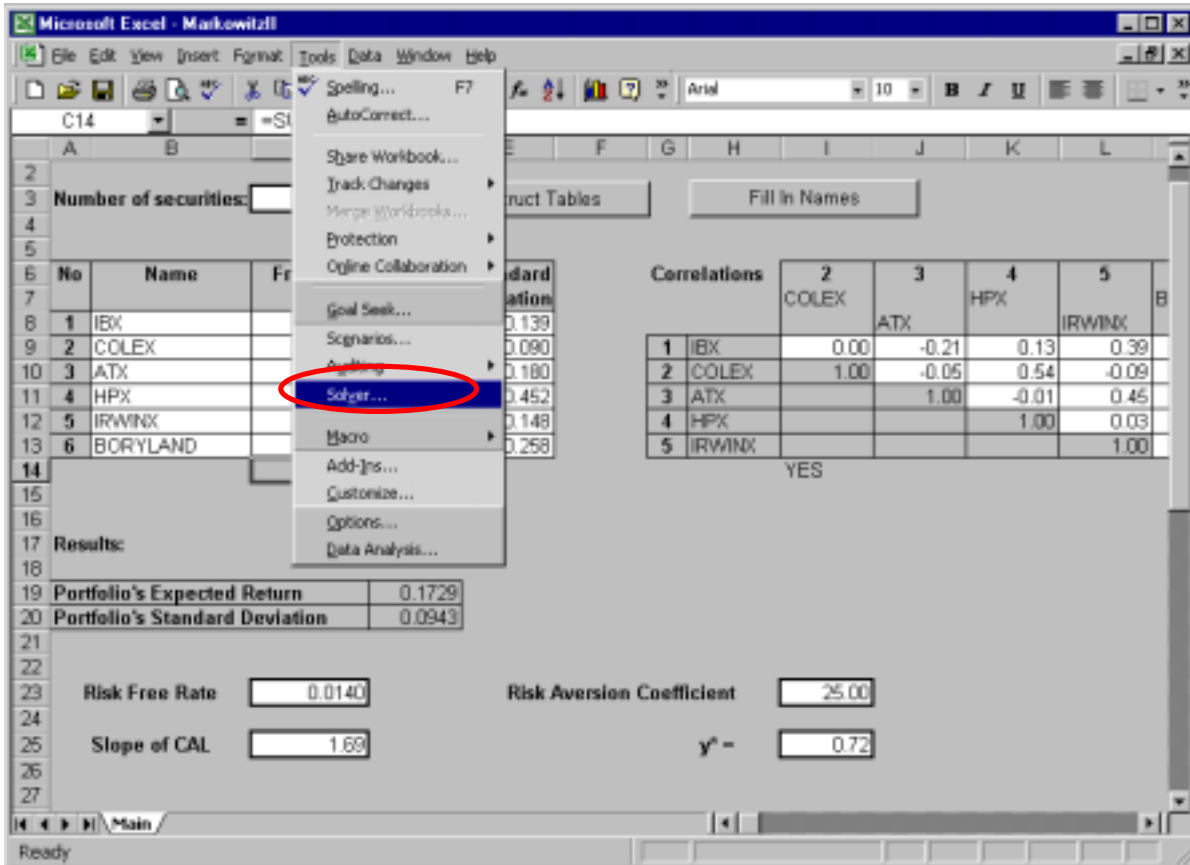
- Input Section:** A table with columns for 'No', 'Name', 'Input' (weights), 'Expected Return', and 'Standard Deviation'. The weights for five assets (IBX, COLEX, ATX, HPX, IRWINX) are entered as 0.1, 0.1, 0.1, 0.1, and 0.3 respectively.
- Correlations Table:** A lower triangular matrix showing correlations between the assets. For example, the correlation between COLEX and ATX is -0.05.
- Results Section:** Labeled 'Output', it displays the 'Portfolio's Expected Return' as 0.1729 and the 'Portfolio's Standard Deviation' as 0.0943.
- Additional Parameters:** Fields for 'Risk Free Rate' (0.0140), 'Risk Aversion Coefficient' (25.00), 'Slope of CAL' (1.69), and y^* (0.72).



Section 3: Calculating Optimal Portfolios

3A. The highest rate of return for a given level of standard deviation

3A.1. Click Tools on the menu bar and choose Solver ...

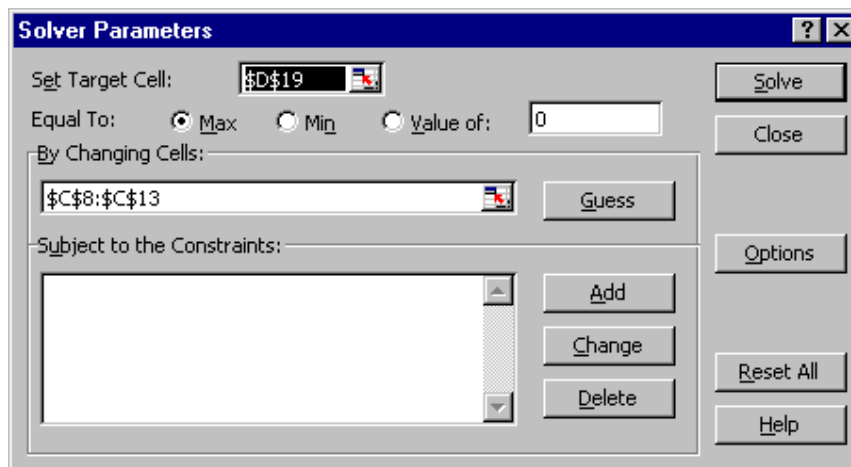


3A.2. In the Solver Parameters window, choose "maximize" to maximize the portfolio's expected return:

Set Target Cell: **\$D\$19** the cell that contains the portfolio's expected return

Equal To: **Max**

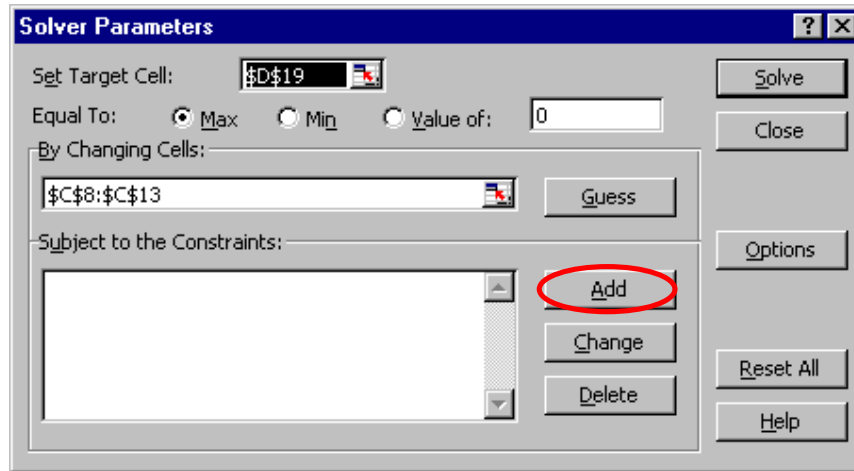
By Changing Cells: **\$C\$8:\$C\$13** the cells that contain the weights for the 6 assets.



Suppose that the maximum risk level you would like to take is 8.5%.

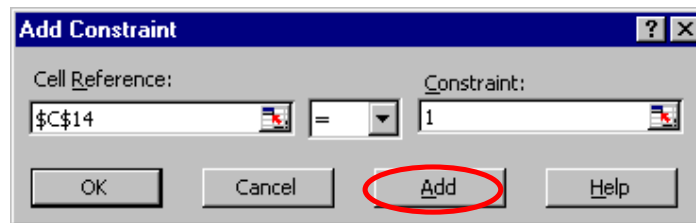
Note on Using Excel to Compute Optimal Risky Portfolios

3A.3. You have to **add the following constraints**.²



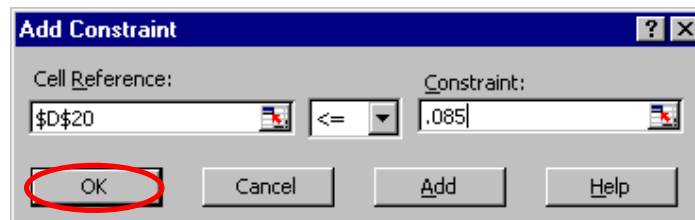
Constraint 1: **\$C\$14** the cell recording the sum of the weights ($=\text{sum}(\$C\$8:\$C\$13)$) equals ($=$) **1**.

Press the button **Add** to add the second constraint.

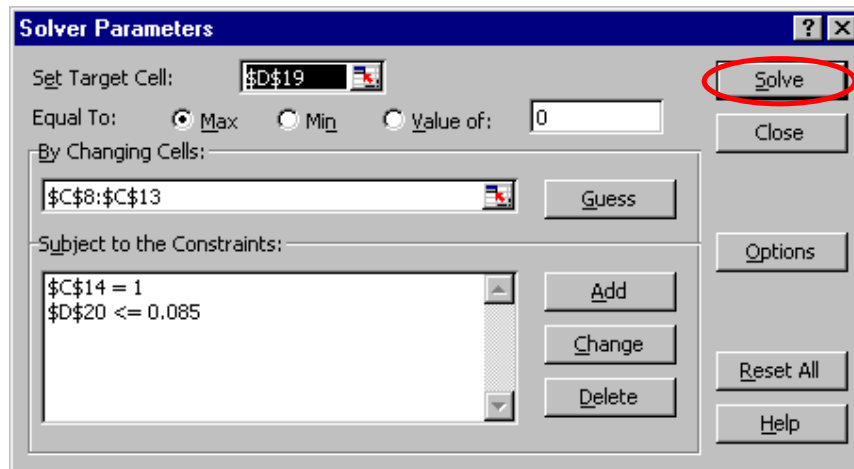


Constraint 2: **\$D\$20** the cell recording the portfolio's standard deviation is **no more than** (\leq) **0.085**.

Press the button **OK** to return to the Solver Parameters window.



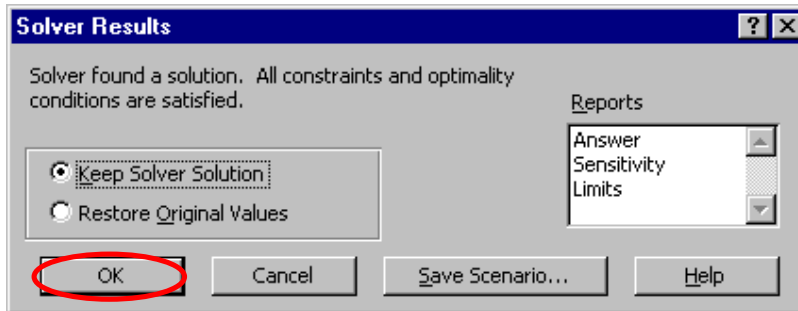
3A.4. The following window will appear. Press the button **Solve** to obtain the desired optimal portfolio.



² You can add other constraints such as “the weight of each asset is at least 5%” and “all the weights are non-negative”.

Note on Using Excel to Compute Optimal Risky Portfolios

3A.5. To keep the results, press the button **OK**.



The optimal portfolio is given in \$C\$8:\$C\$13 and the corresponding expected return in \$D\$19.

No	Name	Fraction	Expected Return	Standard Deviation
1	IBX	-0.23625496	0.075	0.139
2	COLEX	0.46483496	0.109	0.090
3	ATX	6.7989E-05	0.189	0.180
4	HPX	0.00909714	0.204	0.452
5	IRWINX	0.58184917	0.223	0.148
6	BORYLAND	0.18040558	0.161	0.258

Correlations		2	3	4	5
		COLEX	ATX	HPX	IRWINX
1	IBX	0.00	-0.21	0.13	0.39
2	COLEX	1.00	-0.05	0.54	-0.09
3	ATX		1.00	-0.01	0.45
4	HPX			1.00	0.03
5	IRWINX				1.00

Results:	
Portfolio's Expected Return	0.1936
Portfolio's Standard Deviation	0.0950

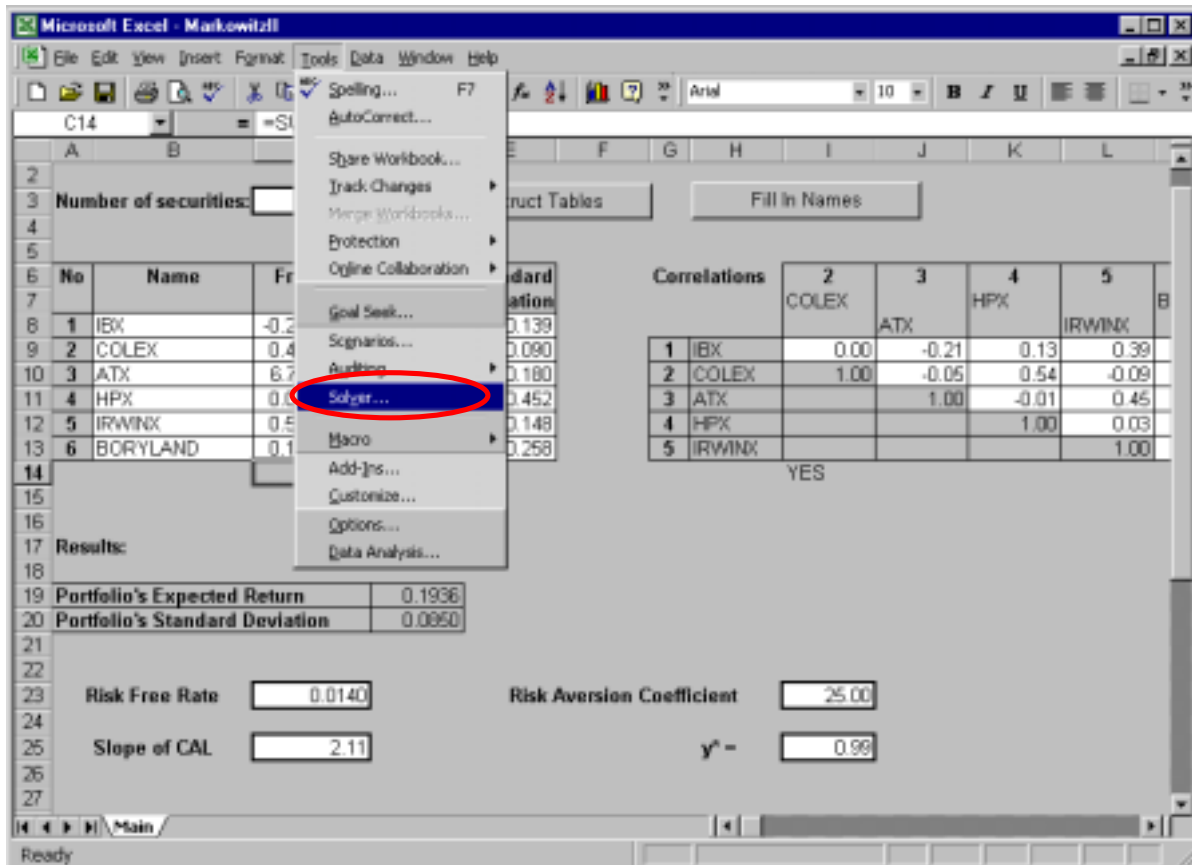
Risk Free Rate	0.0140	Risk Aversion Coefficient	25.00
Slope of CAL	2.11	y*	0.98

Note that the weight for ATX is practically zero, $6.7989E-05 = .000067989$.

Note on Using Excel to Compute Optimal Risky Portfolios

3B. The lowest standard deviation for a given rate of expected return

3B.1. Click Tools on the menu bar and choose Solver ...

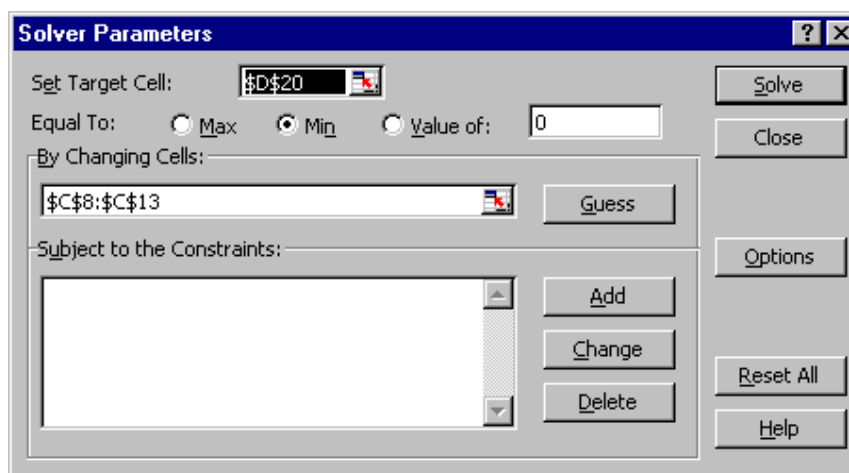


3B.2. In the Solver Parameters window, you ask the solver to minimize the portfolio's standard deviation:

Set Target Cell: **\$D\$20** the cell recording the portfolio's expected return

Equal To: **Min**

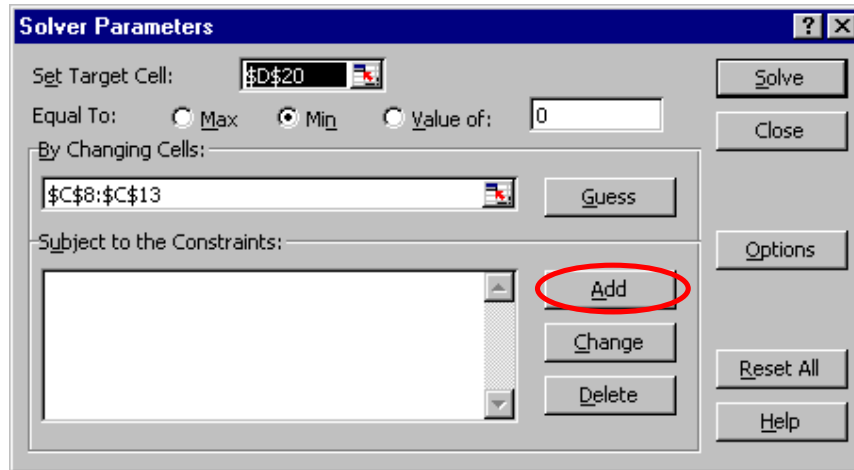
By Changing Cells: **\$C\$8:\$C\$13** the cells recording the weights or the fractions of the 6 assets.



Note on Using Excel to Compute Optimal Risky Portfolios

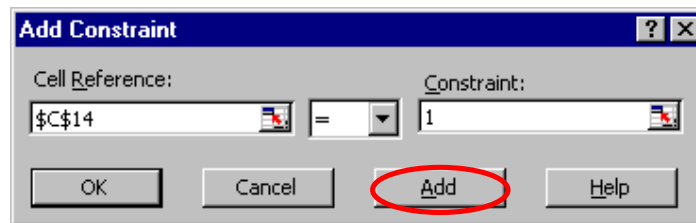
Suppose that you want the portfolio's expected return to be at least 20%.

3B.3. You have to tell the Solver this by **adding constraints (do not forget to delete constraints that may be there already if you are not going to need them).**



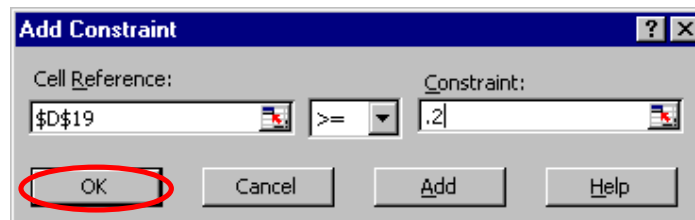
Constraint 1: **\$C\$14** the cell recording the sum of the weights ($=\text{sum}(\$C\$8:\$C\$13)$) equals ($=$) **1**.

Press the button **Add** to add the second constraint.

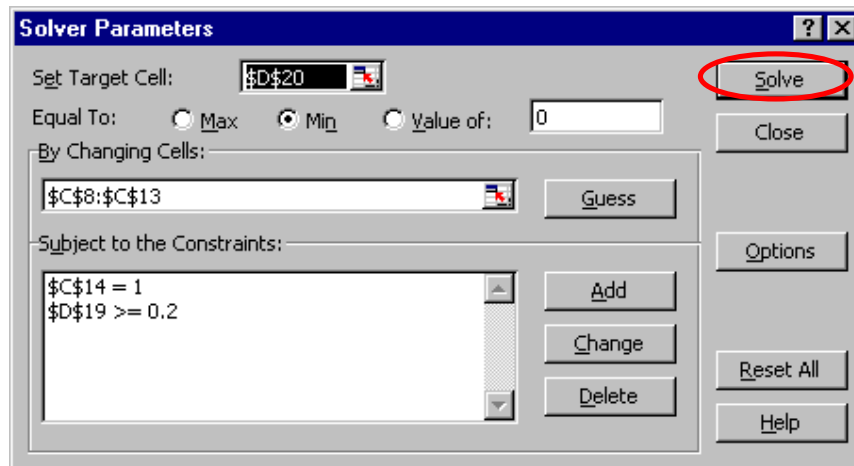


Constraint 2: **\$D\$19** the cell recording the portfolio's expected return is **at least** (\Rightarrow) **0.2**.

Press the button **OK** to return to the Solver Parameters window.

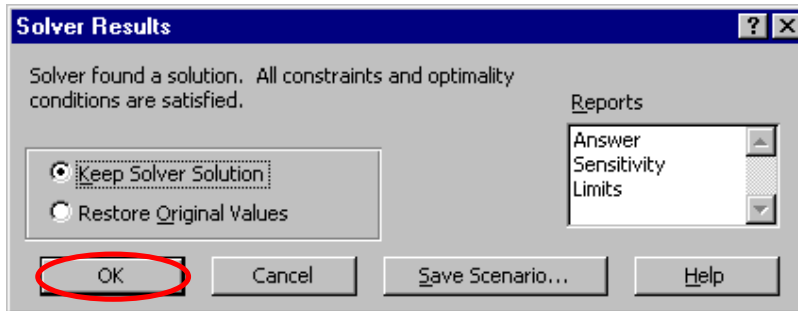


3B.4. The following window will appear. Press the button **Solve** to obtain the desired optimal portfolio.



Note on Using Excel to Compute Optimal Risky Portfolios

3B.5. To keep the results, press the button **OK**.



The optimal portfolio is given in \$C\$8:\$C\$13 and the corresponding standard deviation in \$D\$20.

No	Name	Fraction	Expected Return	Standard Deviation
1	IBX	-0.27230933	0.075	0.139
2	COLEX	0.45557031	0.109	0.090
3	ATX	-0.00994773	0.189	0.180
4	HPX	0.01375516	0.204	0.452
5	IRWINX	0.62835673	0.223	0.148
6	BORYLAND	0.18452486	0.161	0.258

Correlations		2	3	4	5
		COLEX	ATX	HPX	IRWINX
1	IBX	0.00	-0.21	0.13	0.39
2	COLEX	1.00	-0.05	0.54	-0.09
3	ATX		1.00	-0.01	0.45
4	HPX			1.00	0.03
5	IRWINX				1.00

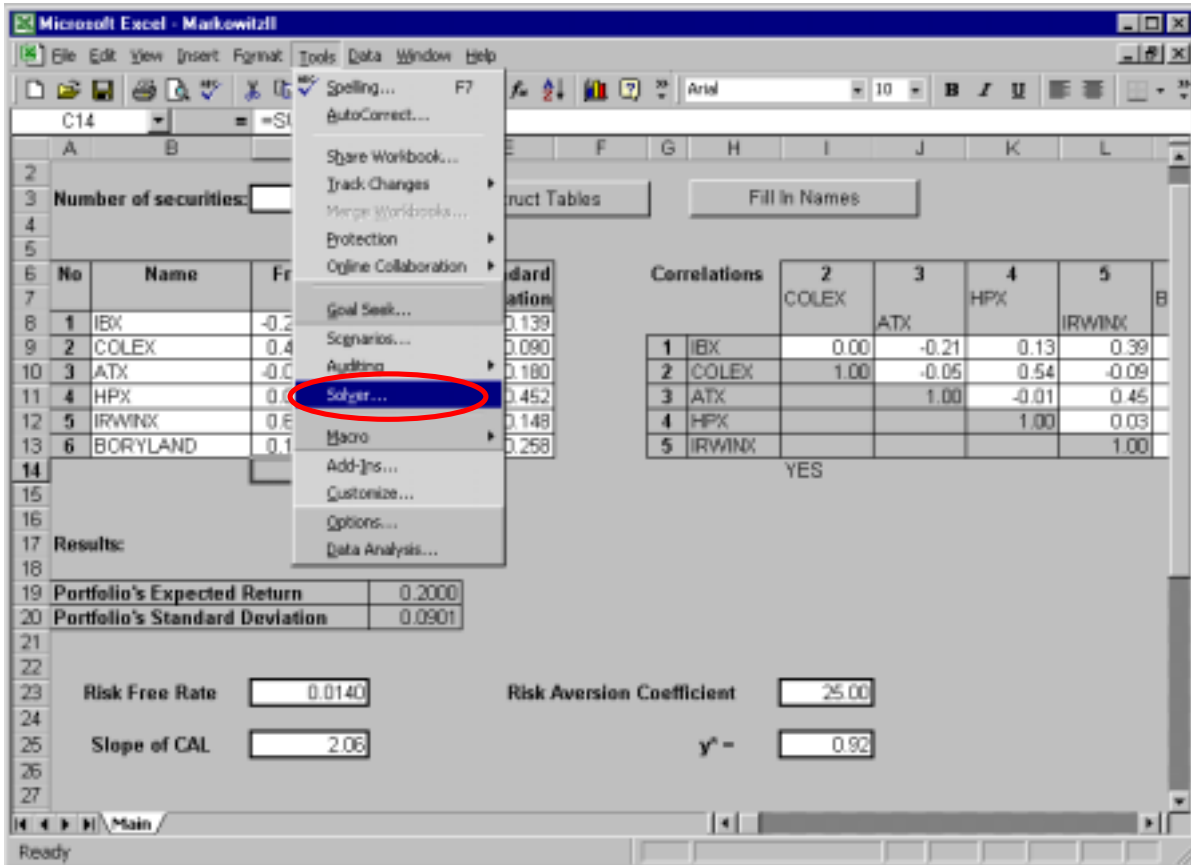
Results:

Portfolio's Expected Return	0.2000
Portfolio's Standard Deviation	0.0901

Risk Free Rate: 0.0140 Risk Aversion Coefficient: 25.00
 Slope of CAL: 2.06 y^* : 0.92

3C. The highest reward-to-variability portfolio & the optimal portfolio for a given level of risk aversion.

3C.1. Click **T**ools on the menu bar and choose **Solver ...**

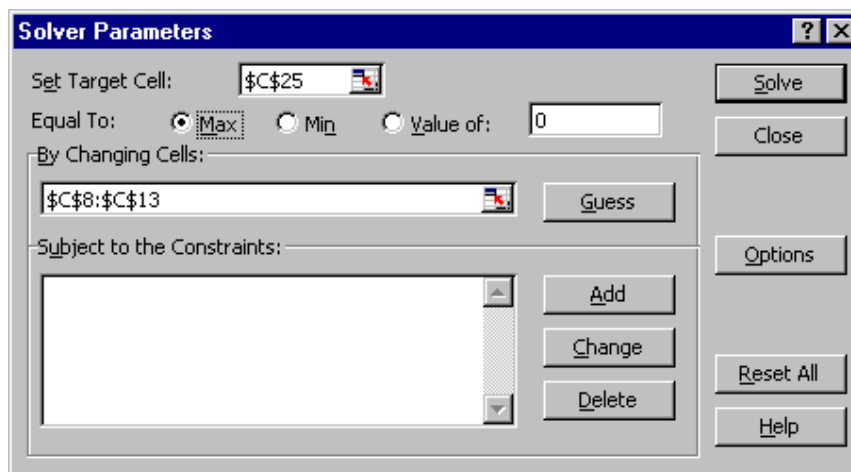


3C.2. In the Solver Parameters window, you choose "maximize" to find the portfolio that has the steepest capital allocation line CAL -- i.e., maximize the reward-to-variability ratio.

Set Target Cell: **\$C\$25** the cell recording the slope of CAL

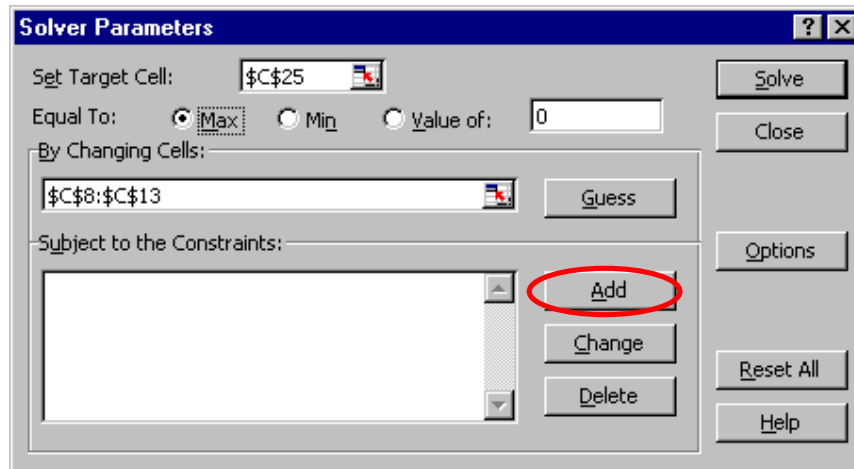
Equal To: **Max**

By Changing Cells: **\$C\$8:\$C\$13** the cells containing the weights for the 6 assets.



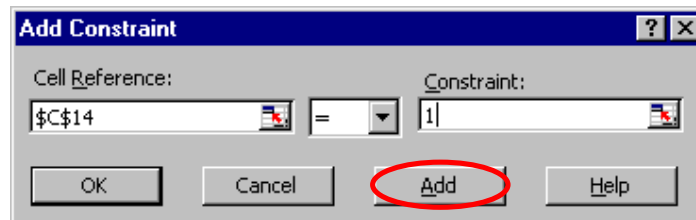
Note on Using Excel to Compute Optimal Risky Portfolios

3C.3. You have to provide the Solver with the constraints by **adding constraints (do not forget to delete constraints that may be there already if you are not going to need them)**.



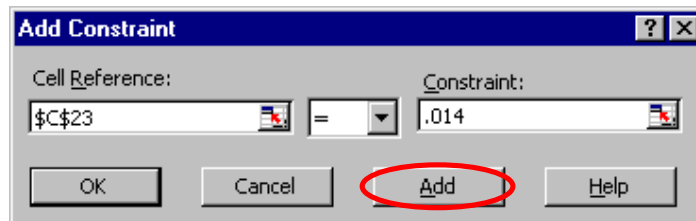
Constraint 1: **\$C\$14** the cell recording the sum of the weights ($=\text{sum}(\$C\$8:\$C\$13)$) equals ($=$) **1**.

Press the button **Add** to add another parameter.



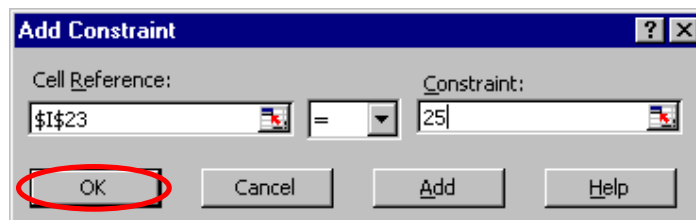
Constraint 2: **\$C\$23** the cell recording the risk-free rate **equals** ($=$) **0.014**.

Press the button **Add** to add the third constraint (another parameter).



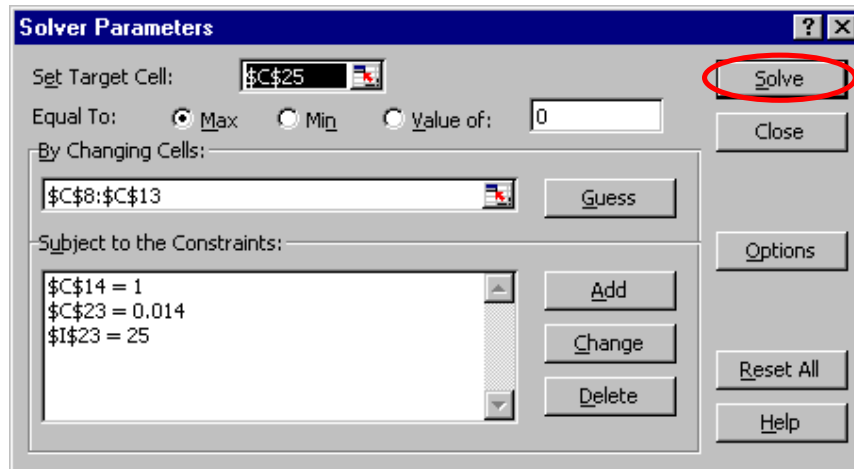
Constraint 3: **\$I\$23** the cell recording the risk aversion coefficient **equals** ($=$) **25**.

Press the button **OK** to return to the Solver Parameters window.

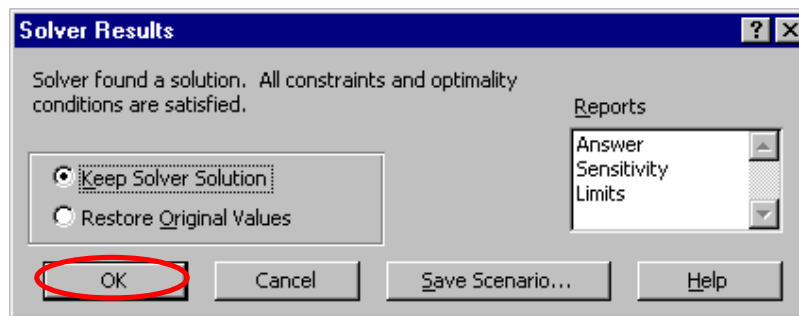


Note on Using Excel to Compute Optimal Risky Portfolios

3C.4. The following window will appear. Press the button **Solve** to obtain the desired optimal portfolio.



3C.5. To keep the results, press the button **OK**.



Note on Using Excel to Compute Optimal Risky Portfolios

The optimal risky portfolio is given in \$C\$8:\$C\$13 and the corresponding expected return and standard deviation in \$D\$19 and \$D\$20 whereas the optimal weight on the tangency portfolio is given in \$I\$25 for the risk aversion coefficient of 25.

Microsoft Excel - Markowitz

File Edit View Insert Format Tools Data Window Help

Q3 = 25

Number of securities: Construct Tables Fill in Names

No	Name	Fraction	Expected Return	Standard Deviation
1	IBX	0.05434853	0.075	0.139
2	COLEX	0.53923596	0.109	0.090
3	ATX	0.08095722	0.169	0.180
4	HPX	-0.02906374	0.204	0.452
5	IRWINX	0.20845228	0.223	0.148
6	BORYLAND	0.14666973	0.161	0.258

Correlations

	2 COLEX	3 ATX	4 HPX	5 IRWINX
1 IBX	0.00	-0.21	0.13	0.39
2 COLEX	1.00	-0.05	0.54	-0.09
3 ATX		1.00	-0.01	0.45
4 HPX			1.00	0.03
5 IRWINX				1.00

Results:

Portfolio's Expected Return	0.1422
Portfolio's Standard Deviation	0.0517

Risk Free Rate Risk Aversion Coefficient

Slope of CAL y^* =

Main

Ready

Note on Using Excel to Compute Optimal Risky Portfolios

3C.6. You can also study how the optimal weight for the risky assets and the slope of CAL change with the risk-free rate or the risk aversion coefficient. For instance, **increase** the risk-free rate to **0.05**, the optimal weight on the risky assets recording in the cell \$I\$25 immediately decreases from 1.92 to 1.38 and the slope of CAL recording in the cell \$C\$25 drops from 2.48 to 1.78.

The screenshot shows an Excel spreadsheet titled "Markowitz" with the following data:

Number of securities: 6

No	Name	Fraction	Expected Return	Standard Deviation
1	IBX	0.05434853	0.075	0.139
2	COLEX	0.53923596	0.109	0.090
3	ATX	0.08036722	0.189	0.180
4	HPX	-0.02906374	0.204	0.452
5	IRWINX	0.20845228	0.223	0.148
6	BORYLAND	0.14666973	0.161	0.258

Correlations Matrix:

	2 COLEX	3 ATX	4 HPX	5 IRWINX
1 IBX	0.00	-0.21	0.13	0.39
2 COLEX	1.00	-0.05	0.54	-0.09
3 ATX		1.00	-0.01	0.45
4 HPX			1.00	0.03
5 IRWINX				1.00

Results:

Portfolio's Expected Return	0.1422
Portfolio's Standard Deviation	0.0517

Input Parameters:

- Risk Free Rate: 0.0500
- Risk Aversion Coefficient: 25.00
- Slope of CAL: 1.78
- y^* : 1.38

