Solution: Quiz 7

1. Determine every solution of the following system of linear equations by using Gaussian elimination method.

$$\begin{aligned} x - 2y &= 3z \\ -x + y + 2z &= 3 \\ y + z + 3 &= 0 \end{aligned}$$

Solution: By rearranging the terms in each equation, the corresponding augmented matrix of the given linear system can be written as

$$\begin{array}{l} R_1 : \\ R_2 : \\ R_3 : \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -3 \end{array}\right] \quad \Leftrightarrow \quad \begin{array}{rcrcrcr} x & - & 2y & - & 3z & = & 0 \\ -x & + & y & + & 2z & = & 3 \\ & & y & + & z & = & -3 \end{array}$$

and we will apply the following 2 procedures in Gaussian elimination to the augmented matrix.

I. Forward elimination to obtain the upper triangular form

II. Back substitution

I. Forward elimination

Step 1: R_1 is the "*pivot row*" and $a_{11} = 1$ is the "*pivot element*".
Let $m_{21} = \frac{a_{21}}{a_{11}} = \frac{-1}{1} = -1$. Note that there is no need to change R_3.

$$\begin{array}{l} R_1 : \\ R_2 \mapsto R_2 - m_{21}R_1 : \\ R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 1 & 1 & -3 \end{array}\right]$$

Step 2: R_2 is the "*pivot row*" and $a_{22} = -1$ is the "*pivot element*".
Let $m_{32} = \frac{a_{32}}{a_{22}} = \frac{2}{1} = 2$.

$$\begin{array}{l} R_1 : \\ R_2 : \\ R_3 \mapsto R_3 - m_{32}R_2 : \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \Leftrightarrow \begin{array}{rcrcrcr} x & - & 2y & - & 3z & = & 0 \\ & & -y & - & z & = & 3 \\ & & & & 0 \cdot z & = & 0 \end{array}$$

II. Back substitution Notice that any value of $z \in \mathbb{R}$ will satisfy the last equation.

- Row $R_3 : z = t$, where $t \in \mathbb{R}$
- Row $R_2 : -y - t = 3 \quad \Rightarrow \quad y = -t - 3.$
- Row $R_1 : x - 2(-t - 3) - 3t = 0 \quad \Rightarrow \quad x = t - 6$

That is, there are infinitely many solutions for this linear system which can be written in terms of the set of solutions as $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t - 6 \\ -t - 3 \\ t \end{bmatrix} \;\middle|\; t \in \mathbb{R} \right\}$. ∎