Not in the final exam

9 Binary Dependent Variable (y variable): Logit and Probit Models

Reading: In the supplemental part (Blue book)

- Many research questions on consumer choice involve analyzing qualitative dependent variables such as

$$Y = \begin{cases} 1 & \text{if eaten meals at 7-Eleven} \\ 0 & \text{otherwise} \end{cases}$$

$$Y = \begin{cases} 1 & \text{if use iPad to take notes} \\ 0 & \text{otherwise} \end{cases}$$

→ The prediction of Y will be "probability" $\in [0, 1]$

- Using the linear probability model (LPM) is unrealistic because the model can result in predicted probability greater or less than one.

$$P(y=1|\mathbf{x}) = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \ldots + \hat{\beta}_k x_k$$

x can be any value BUT → $G(x) \in [0, 1]$

Can result in

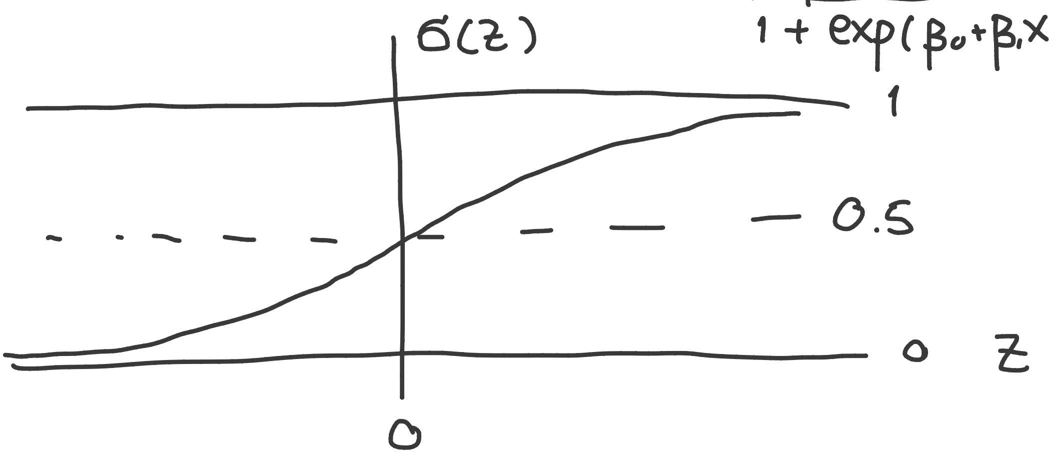
$$P(y = 1|\boldsymbol{x}) > \mathbf{1} \text{ or } P(y = 1|\boldsymbol{x}) < \mathbf{0}$$

- The Logit and Probit model introduce a function (call it "G") to restrict the probability value between zero and one.
  - Logit Model – Function G(•) is the logistic function

$$\Lambda(z) \rightarrow G(z) = \frac{\exp(z)}{1+\exp(z)} \; ; \text{ where } z = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k$$

→ Once you applies the function $G(\cdot)$ over z (regression function), the value of $G(z) \in [0, 1]$.

$$\rightarrow \text{Prob}(Y=1|X) = G(z) = \frac{\exp(\beta_0 + \beta_1 X_1 + \ldots \beta_k X_k)}{1 + \exp(\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k)}$$

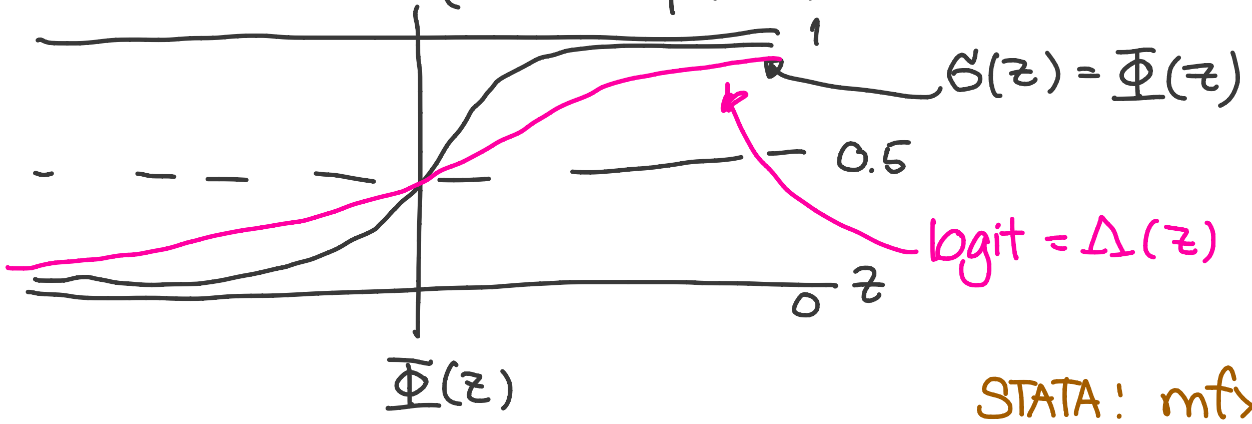


- Probit model - Function $G(\cdot)$ is the standard normal cumulative distribution function (cdf).

$$G(z) = \Phi(z) = \int_{-\infty}^{z} \phi(v)\,dv$$

where $\phi(z)$ is the standard normal density function

$$\phi(z) = (2\pi)^{-1/2} \exp(-z^2/2)$$



STATA: mfx command

NOW, the β in the logit and probit models are not marginal impact on probability!! To find the partial effect of each X variable on the response probability, we must rely on calculation. To find the partial impact of X_2, we calculate the following difference:

- Partial impact of X_2 on $Pr(y=1 \mid \mathbf{X})$ when X_2 increases by 1 unit is

Either $\Lambda(z), \Phi(z)$ →

$$G[\beta_0 + \beta_1 X_1 + \beta_2 (C_2 + 1) + \ldots + \beta_k X_k] -$$
$$G[\beta_0 + \beta_1 X_1 + \beta_2 (C_2) + \ldots + \beta_k X_k]$$

→ What should be the value of $X_1, \ldots X_k$ and C_2 that we use??

⇒ generally we use mean value of each $X_1, \ldots X_k$

- STATA provides a post-estimation command called “mfx” to calculate such marginal effects. For instance:

```
. probit eat_rte gender age income product price place promotion

Iteration 0:   log likelihood = -99.699949
Iteration 1:   log likelihood =  -63.03342
Iteration 2:   log likelihood = -59.735085
Iteration 3:   log likelihood =  -59.68356
Iteration 4:   log likelihood = -59.683526
Iteration 5:   log likelihood = -59.683526

Probit regression                                 Number of obs   =        252
                                                  LR chi2(7)      =      80.03
                                                  Prob > chi2     =     0.0000
Log likelihood = -59.683526                       Pseudo R2       =     0.4014

------------------------------------------------------------------------------
     eat_rte |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      gender |   .3639538   .3001825     1.21   0.225    -.2243931    .9523007
         age |   -.035382   .0130412    -2.71   0.007    -.0609422   -.0098218
      income |  -.0000126   8.44e-06    -1.49   0.137    -.0000291    3.99e-06
     product |   .4826703   .1645111     2.93   0.003     .1602344    .8051062
       price |   -.291123   .1639812    -1.78   0.076    -.6125202    .0302742
       place |   1.128479   .2874352     3.93   0.000     .5651166    1.691842
   promotion |   .0168971   .1699563     0.10   0.921    -.3162112    .3500054
       _cons |  -2.638742   1.207165    -2.19   0.029    -5.004742   -.2727424
------------------------------------------------------------------------------

. mfx

Marginal effects after probit
      y  = Pr(eat_rte) (predict)
         =  .95502682
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 gender* |   .0364636      .03047    1.20   0.231  -.023265  .096193   .575397
     age |  -.0033521      .00142   -2.36   0.018  -.006136 -.000569   27.7976
  income |  -1.19e-06      .00000   -1.44   0.151  -2.8e-06  4.3e-07     22501
 product |   .0457288      .01681    2.72   0.007   .012777   .07868   3.36984
   price |  -.0275814      .01534   -1.80   0.072  -.057638  .002476   3.55952
   place |   .1069136      .03163    3.38   0.001   .044928    .1689   4.20933
 promot~n |   .0016009     .01615    0.10   0.921  -.030062  .033263   2.99524
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```