**EE431: Economics of financial market and institutions**

**Semester 1/2017**

**Assignment 2 (due Thursday 26th, 2017 at the BE office; before 3 pm)**

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**<u>Instruction:</u> Use PVF and PVIFA table in your calculation**

**1.** Consider two assets with the structure of cash-flow payment given by

| Case | Asset A | Asset B |
|---|---|---|
| Raining (P=1/3) | 1 | 0 |
| Not raining (P=2/3) | 0 | 1 |

Indeed, asset A promises to deliver $1 to the holder only if there will be rain tomorrow. But, when there is no rain tomorrow, the holder of asset A receives nothing in return. On the other hand, asset B will give out $1 of cash flow if it won't rain tomorrow, and nothing if it rains. (Economically, both types of asset are considered as contingent asset whose cash-flow payments depend on some certain conditions or circumstances.)

Given that the price per unit of each asset is equal to 0.30 and 0.60 respectively. Consider the following questions. (Cash-flow given in the table is the amount of dollar received for each unit of asset.)

1.1) Calculate the market-implied discount rate for the two assets.

1.2) Suppose that a company is newly launching an IPO with the pattern of cash-flow structure given in the below table. (See the table in the next page.) That, for every single unit of stock purchased, the holder will be receiving $4 if raining occurs tomorrow, and $2 if there will be no rain.

| Case | Stock cash-flow |
|---|---|
| Raining (P=1/3) | $4 |
| No raining (P=2/3) | $2 |

Based on the information above (including what's given in 1.1), what should be the market price of this stock? Explain the reason for your answer. Justify your approach to the valuation.

1.3) What is the market-implied risk-free rate? What should be the price of risk-free asset? (Hint: An asset is considered to be a risk-free asset if the asset delivers the same fixed amount of cash-flow under any circumstances.)

**2.** Consider the following five zero-coupon bonds, each of which has the face value of $100.

| **Time to maturity** | **Spot rate (%)*** | **Price** |
|---|---|---|
| 1 year | 1 | A |
| 2 year | 2 | B |
| 3 year | 3 | C |
| 4 year | 4 | D |
| 5 year | 5 | E |

*Financially, yield to maturity of a *zero-coupon bond* is called *spot rate*

Consider the following questions

2.1) Calculate the price of each of the five zero-coupon bonds

2.2) Suppose that an investment banker is now offering you a new investment asset with the asked price of $100. The structure of cash-flow of the new asset offered is given in the below table. Should your purchase this new asset? Why or why not?

| Year | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cash-flow | 4 | 4 | 4 | 4 | 104 |

2.3) Suppose that you hold that new asset to the maturity date, what would be the yield of maturity of your investment? (Hint: State the condition that is needed for the calculation, but you don't need to calculate the numerical value of YTM.)

3. Consider the following table

| Bond | Time to maturity (years) | Payments per year | coupon (% pa) | yield (% pa) | price ($) |
|---|---|---|---|---|---|
| A | 3 | 1 | 3% | 2% | **F** |
| B | 5 | 2 | 4% | **G** | 918.24332 |
| C | 7 | 3 | **H** | 5% | 1000 |
| D | 10 | 4 | 4% | 8% | **J** |
| E | 10 | 0 | **K** | 5% | **L** |

Suppose that each of the five bonds has its face value of $1,000. Consider the following questions.

3.1) Complete the table by filling in the value of F, G, H J, K and L. Show your work and explain everything step-by-step.
3.2) Calculate the Macaulay Duration, Modified Duration, and % change the price of each of the five bonds. Which one of them has the greatest interest rate risk? Is it the one with longest remaining time to maturity? (Hint: Use the excel to calculate all the durations.)

4. A 2-year government bond with its face value of $1,000 offers the holder $100 as the coupon payment which is given out to the investor at the end of each year. Suppose that the current market price is $1,000, consider the following problems.
4.1) What is the coupon rate of this 2-year bond?
4.2) What is the yield to maturity of this bond?

4.3) The below table represents the investors' survey over that outlook of market interest next year. Calculate the 1-year expected rate of return on the 2-year bond purchased today. Suppose that you can sell off this bond after you've collected bond's coupon next year.

| Rate | 8% | 10% | 12% |
|---|---|---|---|
| Probability | 1/2 | 1/4 | ¼ |

5. Consider 3-year and 5-year bond with the face value of $1,000. Each type of the two bonds pays the holder of 2% and 5% coupon rate. The payment is given once at the end of each year. Following the information in the bond market, current price of each of the two bonds is $971.71388 and $1,044.51822. Consider the following problems.

5.1) which one of the two bonds offers the investor higher yield? (Hint: your answer may be an *approximate* to the exact solution!)

5.2) Calculate the Macaulay Duration and the Modified Duration of each bond.

5.3) which one of the two bonds has a greater degree of risk? Based on your calculation, is it true that long-term bond is riskier than short-term bond?

6. Consider two zero-coupon bonds with remaining time to maturity of 10 years. The first one is issued by a private company named "The face". The second one is issued by the Thai government. Each bond has its yield to maturity of 6% and 3%, respectively. Suppose that the face value of each of the two bonds is $1,000. (Don't get surprised why the two domestic bonds have their face value in USD. This is an example of what is called foreign-currency denominated bond. That is, a domestic entity promises investors to repay coupon and principal amount in terms of foreign currency. In this example, the payment is made in US dollar.) Form the information, consider the following problems.

6.1) Calculate the current price of each of the two bonds.

6.2) Using the exact bond pricing formula, calculate the percentage change in price of the two bonds if market interest unexpectedly increases by 1% right after the investor had purchased the two bonds.

6.3) Compare the percentage change in prices implied by the duration concept. How big is the approximation error? Would the approximate error be bigger under the high-yield bond than the low-yield bond, or vice versa?

7. As a CFO of an invest bank, you are presented with daily fact sheet that summarizes the activity in bond market. Below is an example of your fact sheet in table.

| Bond | Price | Coupon rate (%) | YTM (%) |
|---|---|---|---|
| U | 90 | 6 | 9 |
| V | 96 | 9 | 8 |
| W | 110 | 8 | 6 |
| X | 105 | 0 | 5 |
| Y | 107 | 7 | 9 |
| Z | 100 | 6 | 6 |

Do you find anything wrong with the fact sheet? If yes, please state out what they are?

8. (CFA/Level 1) A 6% coupon bond paying interest annually has a modified duration of 10 years, sells for $800, and is priced at a yield to maturity of 8%. If the YTM increases to 9%, what is the predicted change in price using the duration concept?

9. (CFA/Level 1) A 6% coupon bond with semiannual coupons has a convexity (in years) of 120, sells for 80% of par, and is priced at a yield to maturity of 8%. If the YTM increases to 9.5%, what is the predicted contribution to the percentage change in price due to convexity?

10. (CFA/Level 1) A bond with *annual* coupon payments has a coupon rate of 8%, yield to maturity of 10%, and Macaulay duration of 9 years. What is the bond's modified duration?

11. (CFA/Level 1) When interest rates decline, the duration of a 30-year bond selling at (Choose the best answer, and explain why?)
    a. premium:
    b. Increases.
    c. Decreases.
    d. Remains the same.
    e. Increases at first, then declines.

12. (CFA/level 1) Which bond has the longest duration? (Choose the best answer, and explain why?)
    a. 8-year maturity, 6% coupon.
    b. 8-year maturity, 11% coupon.
    c. 15-year maturity, 6% coupon.
    d. 15-year maturity, 11% coupon.

13. (CFA/Level 1) A newly issued bond has the following characteristics:

| Coupon | Yield | Maturity | Duration |
|---|---|---|---|
| 8% | 8% | 15 years | 10 years |

a. Calculate modified duration using the information above.

*b.* Explain why modified duration is a better measure than maturity when calculating the bond's sensitivity to changes in interest rates.

*c.* Identify the direction of change in modified duration if:

c.1) The coupon of the bond were 4%, not 8%.

c.2) The maturity of the bond were 7 years, not 15 years.

*d.* Define convexity and explain how modified duration and convexity are used to approximate the bond's percentage change in price, given a change in interest rates.

# PRESENT VALUE OF $1

| Periods | RATE PER PERIOD | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.25% | 0.50% | 0.75% | 1.00% | 1.50% | 2.00% | 2.50% | 3.00% | 4.00% | 5.00% | 6.00% | 7.00% | 8.00% | 9.00% | 10.00% | 11.00% | 12.00% |
| 1 | 0.99751 | 0.99502 | 0.99256 | 0.99010 | 0.98522 | 0.98039 | 0.97561 | 0.97087 | 0.96154 | 0.95238 | 0.94340 | 0.93458 | 0.92593 | 0.91743 | 0.90909 | 0.90090 | 0.89286 |
| 2 | 0.99502 | 0.99007 | 0.98517 | 0.98030 | 0.97066 | 0.96117 | 0.95181 | 0.94260 | 0.92456 | 0.90703 | 0.89000 | 0.87344 | 0.85734 | 0.84168 | 0.82645 | 0.81162 | 0.79719 |
| 3 | 0.99254 | 0.98515 | 0.97783 | 0.97059 | 0.95632 | 0.94232 | 0.92860 | 0.91514 | 0.88900 | 0.86384 | 0.83962 | 0.81630 | 0.79383 | 0.77218 | 0.75131 | 0.73119 | 0.71178 |
| 4 | 0.99006 | 0.98025 | 0.97055 | 0.96098 | 0.94218 | 0.92385 | 0.90595 | 0.88849 | 0.85480 | 0.82270 | 0.79209 | 0.76290 | 0.73503 | 0.70843 | 0.68301 | 0.65873 | 0.63552 |
| 5 | 0.98759 | 0.97537 | 0.96333 | 0.95147 | 0.92826 | 0.90573 | 0.88385 | 0.86261 | 0.82193 | 0.78353 | 0.74726 | 0.71299 | 0.68058 | 0.64993 | 0.62092 | 0.59345 | 0.56743 |
| 6 | 0.98513 | 0.97052 | 0.95616 | 0.94205 | 0.91454 | 0.88797 | 0.86230 | 0.83748 | 0.79031 | 0.74622 | 0.70496 | 0.66634 | 0.63017 | 0.59627 | 0.56447 | 0.53464 | 0.50663 |
| 7 | 0.98267 | 0.96569 | 0.94904 | 0.93272 | 0.90103 | 0.87056 | 0.84127 | 0.81309 | 0.75992 | 0.71068 | 0.66506 | 0.62275 | 0.58349 | 0.54703 | 0.51316 | 0.48166 | 0.45235 |
| 8 | 0.98022 | 0.96089 | 0.94198 | 0.92348 | 0.88771 | 0.85349 | 0.82075 | 0.78941 | 0.73069 | 0.67684 | 0.62741 | 0.58201 | 0.54027 | 0.50187 | 0.46651 | 0.43393 | 0.40388 |
| 9 | 0.97778 | 0.95610 | 0.93496 | 0.91434 | 0.87459 | 0.83676 | 0.80073 | 0.76642 | 0.70259 | 0.64461 | 0.59190 | 0.54393 | 0.50025 | 0.46043 | 0.42410 | 0.39092 | 0.36061 |
| 10 | 0.97534 | 0.95135 | 0.92800 | 0.90529 | 0.86167 | 0.82035 | 0.78120 | 0.74409 | 0.67556 | 0.61391 | 0.55839 | 0.50835 | 0.46319 | 0.42241 | 0.38554 | 0.35218 | 0.32197 |
| 11 | 0.97291 | 0.94661 | 0.92109 | 0.89632 | 0.84893 | 0.80426 | 0.76214 | 0.72242 | 0.64958 | 0.58468 | 0.52679 | 0.47509 | 0.42888 | 0.38753 | 0.35049 | 0.31728 | 0.28748 |
| 12 | 0.97048 | 0.94191 | 0.91424 | 0.88745 | 0.83639 | 0.78849 | 0.74356 | 0.70138 | 0.62460 | 0.55684 | 0.49697 | 0.44401 | 0.39711 | 0.35553 | 0.31863 | 0.28584 | 0.25668 |
| 13 | 0.96806 | 0.93722 | 0.90743 | 0.87866 | 0.82403 | 0.77303 | 0.72542 | 0.68095 | 0.60057 | 0.53032 | 0.46884 | 0.41496 | 0.36770 | 0.32618 | 0.28966 | 0.25751 | 0.22917 |
| 14 | 0.96565 | 0.93256 | 0.90068 | 0.86996 | 0.81185 | 0.75788 | 0.70773 | 0.66112 | 0.57748 | 0.50507 | 0.44230 | 0.38782 | 0.34046 | 0.29925 | 0.26333 | 0.23199 | 0.20462 |
| 15 | 0.96324 | 0.92792 | 0.89397 | 0.86135 | 0.79985 | 0.74301 | 0.69047 | 0.64186 | 0.55526 | 0.48102 | 0.41727 | 0.36245 | 0.31524 | 0.27454 | 0.23939 | 0.20900 | 0.18270 |
| 16 | 0.96084 | 0.92330 | 0.88732 | 0.85282 | 0.78803 | 0.72845 | 0.67362 | 0.62317 | 0.53391 | 0.45811 | 0.39365 | 0.33873 | 0.29189 | 0.25187 | 0.21763 | 0.18829 | 0.16312 |
| 17 | 0.95844 | 0.91871 | 0.88071 | 0.84438 | 0.77639 | 0.71416 | 0.65720 | 0.60502 | 0.51337 | 0.43630 | 0.37136 | 0.31657 | 0.27027 | 0.23107 | 0.19784 | 0.16963 | 0.14564 |
| 18 | 0.95605 | 0.91414 | 0.87416 | 0.83602 | 0.76491 | 0.70016 | 0.64117 | 0.58739 | 0.49363 | 0.41552 | 0.35034 | 0.29586 | 0.25025 | 0.21199 | 0.17986 | 0.15282 | 0.13004 |
| 19 | 0.95367 | 0.90959 | 0.86765 | 0.82774 | 0.75361 | 0.68643 | 0.62553 | 0.57029 | 0.47464 | 0.39573 | 0.33051 | 0.27651 | 0.23171 | 0.19449 | 0.16351 | 0.13768 | 0.11611 |
| 20 | 0.95129 | 0.90506 | 0.86119 | 0.81954 | 0.74247 | 0.67297 | 0.61027 | 0.55368 | 0.45639 | 0.37689 | 0.31180 | 0.25842 | 0.21455 | 0.17843 | 0.14864 | 0.12403 | 0.10367 |
| 21 | 0.94892 | 0.90056 | 0.85478 | 0.81143 | 0.73150 | 0.65978 | 0.59539 | 0.53755 | 0.43883 | 0.35894 | 0.29416 | 0.24151 | 0.19866 | 0.16370 | 0.13513 | 0.11174 | 0.09256 |
| 22 | 0.94655 | 0.89608 | 0.84842 | 0.80340 | 0.72069 | 0.64684 | 0.58086 | 0.52189 | 0.42196 | 0.34185 | 0.27751 | 0.22571 | 0.18394 | 0.15018 | 0.12285 | 0.10067 | 0.08264 |
| 23 | 0.94419 | 0.89162 | 0.84210 | 0.79544 | 0.71004 | 0.63416 | 0.56670 | 0.50669 | 0.40573 | 0.32557 | 0.26180 | 0.21095 | 0.17032 | 0.13778 | 0.11168 | 0.09069 | 0.07379 |
| 24 | 0.94184 | 0.88719 | 0.83583 | 0.78757 | 0.69954 | 0.62172 | 0.55288 | 0.49193 | 0.39012 | 0.31007 | 0.24698 | 0.19715 | 0.15770 | 0.12640 | 0.10153 | 0.08170 | 0.06588 |
| 25 | 0.93949 | 0.88277 | 0.82961 | 0.77977 | 0.68921 | 0.60953 | 0.53939 | 0.47761 | 0.37512 | 0.29530 | 0.23300 | 0.18425 | 0.14602 | 0.11597 | 0.09230 | 0.07361 | 0.05882 |
| 30 | 0.92783 | 0.86103 | 0.79919 | 0.74192 | 0.63976 | 0.55207 | 0.47674 | 0.41199 | 0.30832 | 0.23138 | 0.17411 | 0.13137 | 0.09938 | 0.07537 | 0.05731 | 0.04368 | 0.03338 |
| 35 | 0.91632 | 0.83982 | 0.76988 | 0.70591 | 0.59387 | 0.50003 | 0.42137 | 0.35538 | 0.25342 | 0.18129 | 0.13011 | 0.09366 | 0.06763 | 0.04899 | 0.03558 | 0.02592 | 0.01894 |
| 40 | 0.90495 | 0.81914 | 0.74165 | 0.67165 | 0.55126 | 0.45289 | 0.37243 | 0.30656 | 0.20829 | 0.14205 | 0.09722 | 0.06678 | 0.04603 | 0.03184 | 0.02209 | 0.01538 | 0.01075 |
| 50 | 0.88263 | 0.77929 | 0.68825 | 0.60804 | 0.47500 | 0.37153 | 0.29094 | 0.22811 | 0.14071 | 0.08720 | 0.05429 | 0.03395 | 0.02132 | 0.01345 | 0.00852 | 0.00542 | 0.00346 |

# PRESENT VALUE OF ORDINARY ANNUITY
## (annuity in arrears -- end of period payments)

| Periods | RATE PER PERIOD | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.25% | 0.50% | 0.75% | 1.00% | 1.50% | 2.00% | 2.50% | 3.00% | 4.00% | 5.00% | 6.00% | 7.00% | 8.00% | 9.00% | 10.00% | 11.00% | 12.00% |
| 1 | 0.99751 | 0.99502 | 0.99256 | 0.99010 | 0.98522 | 0.98039 | 0.97561 | 0.97087 | 0.96154 | 0.95238 | 0.94340 | 0.93458 | 0.92593 | 0.91743 | 0.90909 | 0.90090 | 0.89286 |
| 2 | 1.99252 | 1.98510 | 1.97772 | 1.97040 | 1.95588 | 1.94156 | 1.92742 | 1.91347 | 1.88609 | 1.85941 | 1.83339 | 1.80802 | 1.78326 | 1.75911 | 1.73554 | 1.71252 | 1.69005 |
| 3 | 2.98506 | 2.97025 | 2.95556 | 2.94099 | 2.91220 | 2.88388 | 2.85602 | 2.82861 | 2.77509 | 2.72325 | 2.67301 | 2.62432 | 2.57710 | 2.53129 | 2.48685 | 2.44371 | 2.40183 |
| 4 | 3.97512 | 3.95050 | 3.92611 | 3.90197 | 3.85438 | 3.80773 | 3.76197 | 3.71710 | 3.62990 | 3.54595 | 3.46511 | 3.38721 | 3.31213 | 3.23972 | 3.16987 | 3.10245 | 3.03735 |
| 5 | 4.96272 | 4.92587 | 4.88944 | 4.85343 | 4.78264 | 4.71346 | 4.64583 | 4.57971 | 4.45182 | 4.32948 | 4.21236 | 4.10020 | 3.99271 | 3.88965 | 3.79079 | 3.69590 | 3.60478 |
| 6 | 5.94785 | 5.89638 | 5.84560 | 5.79548 | 5.69719 | 5.60143 | 5.50813 | 5.41719 | 5.24214 | 5.07569 | 4.91732 | 4.76654 | 4.62288 | 4.48592 | 4.35526 | 4.23054 | 4.11141 |
| 7 | 6.93052 | 6.86207 | 6.79464 | 6.72819 | 6.59821 | 6.47199 | 6.34939 | 6.23028 | 6.00205 | 5.78637 | 5.58238 | 5.38929 | 5.20637 | 5.03295 | 4.86842 | 4.71220 | 4.56376 |
| 8 | 7.91074 | 7.82296 | 7.73661 | 7.65168 | 7.48593 | 7.32548 | 7.17014 | 7.01969 | 6.73274 | 6.46321 | 6.20979 | 5.97130 | 5.74664 | 5.53482 | 5.33493 | 5.14612 | 4.96764 |
| 9 | 8.88852 | 8.77906 | 8.67158 | 8.56602 | 8.36052 | 8.16224 | 7.97087 | 7.78611 | 7.43533 | 7.10782 | 6.80169 | 6.51523 | 6.24689 | 5.99525 | 5.75902 | 5.53705 | 5.32825 |
| 10 | 9.86386 | 9.73041 | 9.59958 | 9.47130 | 9.22218 | 8.98259 | 8.75206 | 8.53020 | 8.11090 | 7.72173 | 7.36009 | 7.02358 | 6.71008 | 6.41766 | 6.14457 | 5.88923 | 5.65022 |
| 11 | 10.83677 | 10.67703 | 10.52067 | 10.36763 | 10.07112 | 9.78685 | 9.51421 | 9.25262 | 8.76048 | 8.30641 | 7.88687 | 7.49867 | 7.13896 | 6.80519 | 6.49506 | 6.20652 | 5.93770 |
| 12 | 11.80725 | 11.61893 | 11.43491 | 11.25508 | 10.90751 | 10.57534 | 10.25776 | 9.95400 | 9.38507 | 8.86325 | 8.38384 | 7.94269 | 7.53608 | 7.16073 | 6.81369 | 6.49236 | 6.19437 |
| 13 | 12.77532 | 12.55615 | 12.34235 | 12.13374 | 11.73153 | 11.34837 | 10.98318 | 10.63496 | 9.98565 | 9.39357 | 8.85268 | 8.35765 | 7.90378 | 7.48690 | 7.10336 | 6.74987 | 6.42355 |
| 14 | 13.74096 | 13.48871 | 13.24302 | 13.00370 | 12.54338 | 12.10625 | 11.69091 | 11.29607 | 10.56312 | 9.89864 | 9.29498 | 8.74547 | 8.24424 | 7.78615 | 7.36669 | 6.98187 | 6.62817 |
| 15 | 14.70420 | 14.41662 | 14.13699 | 13.86505 | 13.34323 | 12.84926 | 12.38138 | 11.93794 | 11.11839 | 10.37966 | 9.71225 | 9.10791 | 8.55948 | 8.06069 | 7.60608 | 7.19087 | 6.81086 |
| 16 | 15.66504 | 15.33993 | 15.02431 | 14.71787 | 14.13126 | 13.57771 | 13.05500 | 12.56110 | 11.65230 | 10.83777 | 10.10590 | 9.44665 | 8.85137 | 8.31256 | 7.82371 | 7.37916 | 6.97399 |
| 17 | 16.62348 | 16.25863 | 15.90502 | 15.56225 | 14.90765 | 14.29187 | 13.71220 | 13.16612 | 12.16567 | 11.27407 | 10.47726 | 9.76322 | 9.12164 | 8.54363 | 8.02155 | 7.54879 | 7.11963 |
| 18 | 17.57953 | 17.17277 | 16.77918 | 16.39827 | 15.67256 | 14.99203 | 14.35336 | 13.75351 | 12.65930 | 11.68959 | 10.82760 | 10.05909 | 9.37189 | 8.75563 | 8.20141 | 7.70162 | 7.24967 |
| 19 | 18.53320 | 18.08236 | 17.64683 | 17.22601 | 16.42617 | 15.67846 | 14.97889 | 14.32380 | 13.13394 | 12.08532 | 11.15812 | 10.33560 | 9.60360 | 8.95011 | 8.36492 | 7.83929 | 7.36578 |
| 20 | 19.48449 | 18.98742 | 18.50802 | 18.04555 | 17.16864 | 16.35143 | 15.58916 | 14.87747 | 13.59033 | 12.46221 | 11.46992 | 10.59401 | 9.81815 | 9.12855 | 8.51356 | 7.96333 | 7.46944 |
| 21 | 20.43340 | 19.88798 | 19.36280 | 18.85698 | 17.90014 | 17.01121 | 16.18455 | 15.41502 | 14.02916 | 12.82115 | 11.76408 | 10.83553 | 10.01680 | 9.29224 | 8.64869 | 8.07507 | 7.56200 |
| 22 | 21.37995 | 20.78406 | 20.21121 | 19.66038 | 18.62082 | 17.65805 | 16.76541 | 15.93692 | 14.45112 | 13.16300 | 12.04158 | 11.06124 | 10.20074 | 9.44243 | 8.77154 | 8.17574 | 7.64465 |
| 23 | 22.32414 | 21.67568 | 21.05331 | 20.45582 | 19.33086 | 18.29220 | 17.33211 | 16.44361 | 14.85684 | 13.48857 | 12.30338 | 11.27219 | 10.37106 | 9.58021 | 8.88322 | 8.26643 | 7.71843 |
| 24 | 23.26598 | 22.56287 | 21.88915 | 21.24339 | 20.03041 | 18.91393 | 17.88499 | 16.93554 | 15.24696 | 13.79864 | 12.55036 | 11.46933 | 10.52876 | 9.70661 | 8.98474 | 8.34814 | 7.78432 |
| 25 | 24.20547 | 23.44564 | 22.71876 | 22.02316 | 20.71961 | 19.52346 | 18.42438 | 17.41315 | 15.62208 | 14.09394 | 12.78336 | 11.65358 | 10.67478 | 9.82258 | 9.07704 | 8.42174 | 7.84314 |
| 30 | 28.86787 | 27.79405 | 26.77508 | 25.80771 | 24.01584 | 22.39646 | 20.93029 | 19.60044 | 17.29203 | 15.37245 | 13.76483 | 12.40904 | 11.25778 | 10.27365 | 9.42691 | 8.69379 | 8.05518 |
| 35 | 33.47243 | 32.03537 | 30.68266 | 29.40858 | 27.07559 | 24.99862 | 23.14516 | 21.48722 | 18.66461 | 16.37419 | 14.49825 | 12.94767 | 11.65457 | 10.56682 | 9.64416 | 8.85524 | 8.17550 |
| 40 | 38.01986 | 36.17223 | 34.44694 | 32.83469 | 29.91585 | 27.35548 | 25.10278 | 23.11477 | 19.79277 | 17.15909 | 15.04630 | 13.33171 | 11.92461 | 10.75736 | 9.77905 | 8.95105 | 8.24378 |
| 50 | 46.94617 | 44.14279 | 41.56645 | 39.19612 | 34.99969 | 31.42361 | 28.36231 | 25.72976 | 21.48218 | 18.25593 | 15.76186 | 13.80075 | 12.23348 | 10.96168 | 9.91481 | 9.04165 | 8.30450 |