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EE431 Economics of Financial Markets and Institutions

Exercise 3: Debt Market and Structure of Interest Rate

1. Based on economists' s forecasts and analysis, one-year Treasury bill rates for the next three years are expected to be as follows:

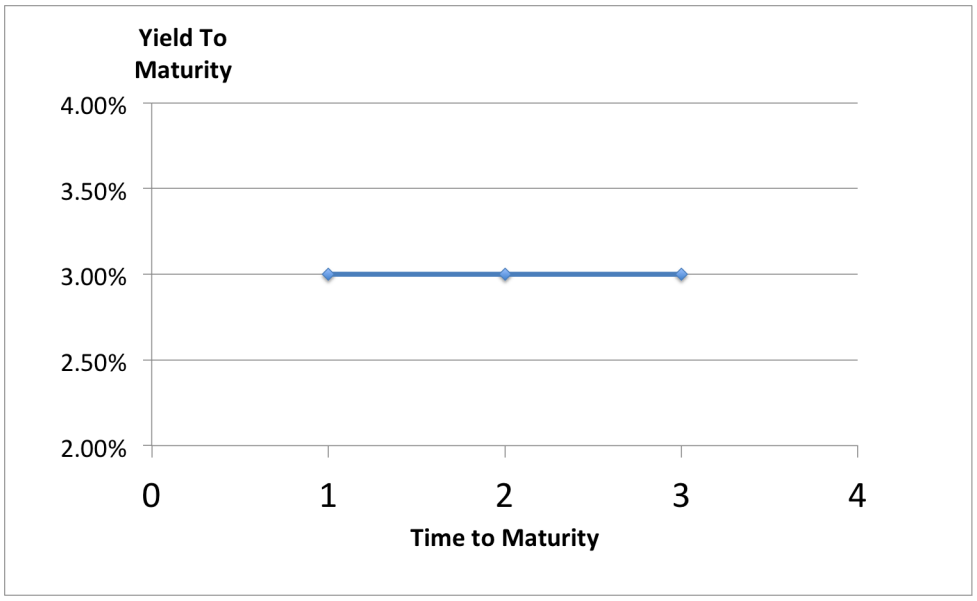
interest rate on one-year Treasury bill this year (2015)	3.00%
interest rate on one-year Treasury bill this year (2016)	3.00%
interest rate on one-year Treasury bill this year (2017)	3.00%

   (a) Suppose the **pure expectation theory** holds, what are the interest rates on one-year, two-year and three-year government bond rate in 2015? Plot the current yield curve.

   Answer. According to expectation theory, $i_{n,t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + ... + i^e_{1,t+n-1}}{n}$. In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} \\
i_{2,2015} &= \frac{i_{1,2015} + i^e_{1,2016}}{2} \\
&= \frac{3.00\% + 3.00\%}{2} \\
&= 3.00\%
\end{aligned}
$$

$$
\begin{aligned}
i_{3,2015} &= \frac{i_{1,2015} + i^e_{1,2016} + i^e_{1,2017}}{3} \\
&= \frac{3\% + 3.00\% + 3.00\%}{3} \\
&= 3.00\%
\end{aligned}
$$



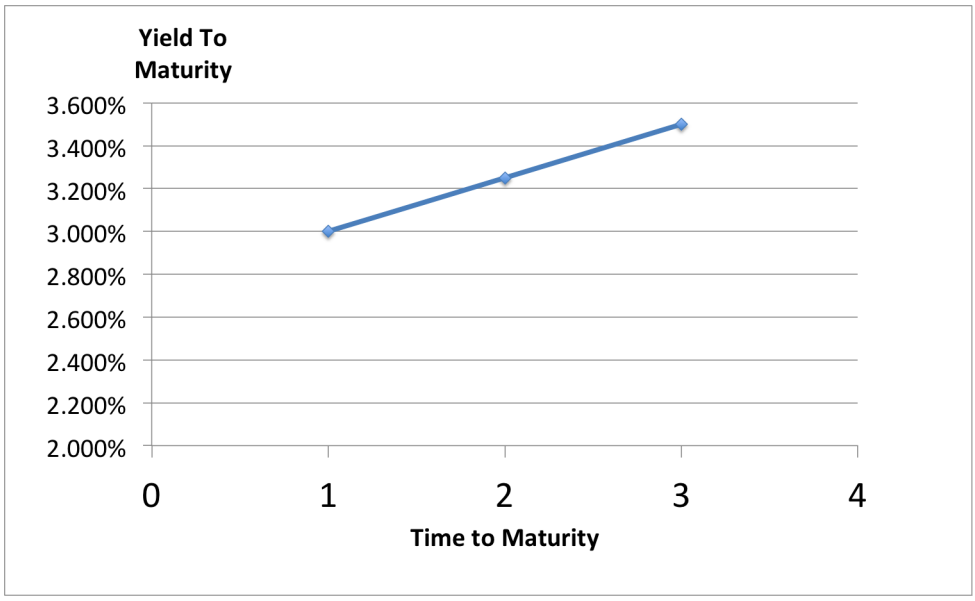
(b) Let liquidity premium are as follows:

liquidity premium for one year bond	0.00%
liquidity premium for two year bond	0.25%
liquidity premium for three year bond	0.50%

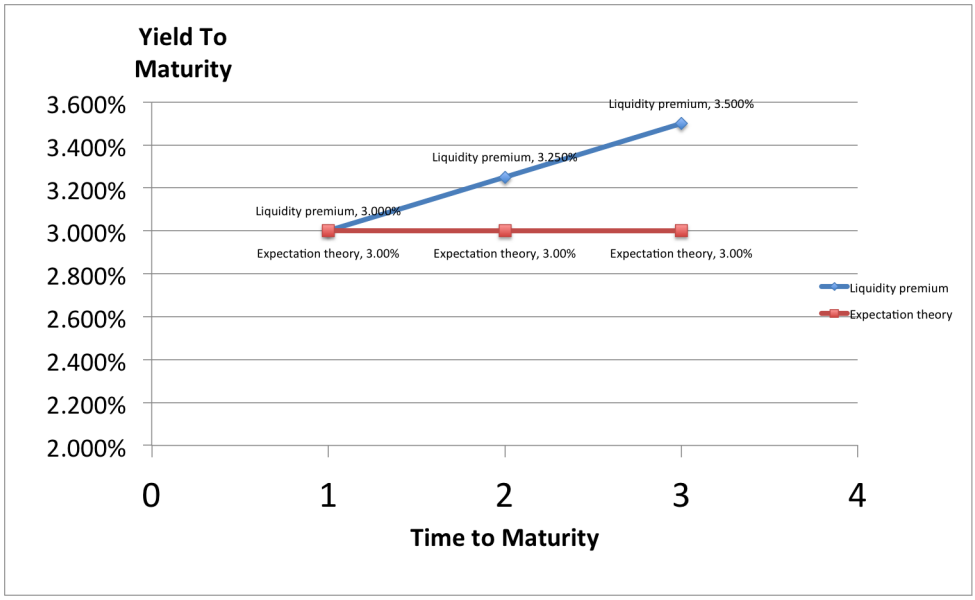
Suppose the **liquidity premium hypothesis** holds, what are the interest rates on one-year, two-year and three-year government bond rate in 2015? Plot the current yield curve.

According to liquidity premium theory, $i_{n.t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n} + \eta_{n,t}$. In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond plus term premium.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} \\
i_{2,2015} &= \frac{i_{1,2015} + i^e_{1,2016}}{2} + \eta_{2,2015} \\
&= \frac{3.00\% + 3.00\%}{2} + 0.25\% \\
&= 3.00\% + 0.25\% \\
&= 3.25\% \\
i_{3,2015} &= \frac{i_{1,2015} + i^e_{1,2016} + i^e_{1,2017}}{3} + \eta_{3,2015} \\
&= \frac{3\% + 3.00\% + 3.00\%}{3} + 0.50\% \\
&= 3.00\% + 0.50\% \\
&= 3.50\%
\end{aligned}
$$



(c) Plot yield curve in question (a) and (b) in the same graph.



2. Based on economists' s forecasts and analysis, one-year Treasury bill rates and liquidity premiums for the next four years are expected to be as follows:

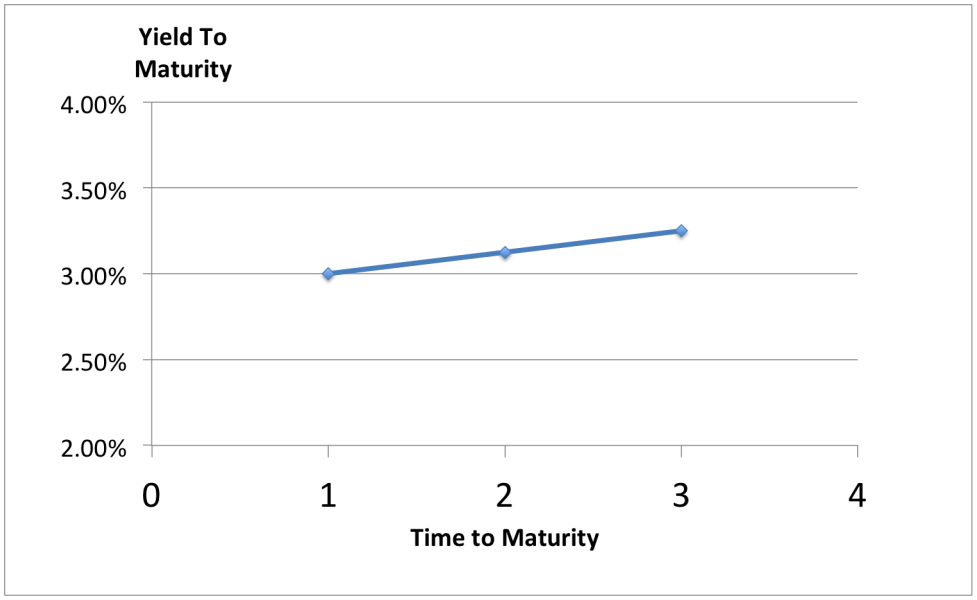
interest rate on one-year Treasury bill this year (2015)	3.00%
interest rate on one-year Treasury bill this year (2016)	3.25%
interest rate on one-year Treasury bill this year (2017)	3.50%

(a) Suppose the **pure expectation theory** holds, what are the interest rates on one-year, two-year and three-year government bond rate in 2015? Plot the current yield curve.

Answer. According to expectation theory, $i_{n,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + ... + i^e_{1,t+n-1}}{n}$.
In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} \\
i_{2,2015} &= \frac{i_{1,2015} + i^e_{1,2016}}{2} \\
&= \frac{3.00\% + 3.25\%}{2} \\
&= 3.125\%
\end{aligned}
$$

$$
\begin{aligned}
i_{3,2015} &= \frac{i_{1,2015} + i^e_{1,2016} + i^e_{1,2017}}{3} \\
&= \frac{3\% + 3.25\% + 3.50\%}{3} \\
&= 3.25\%
\end{aligned}
$$



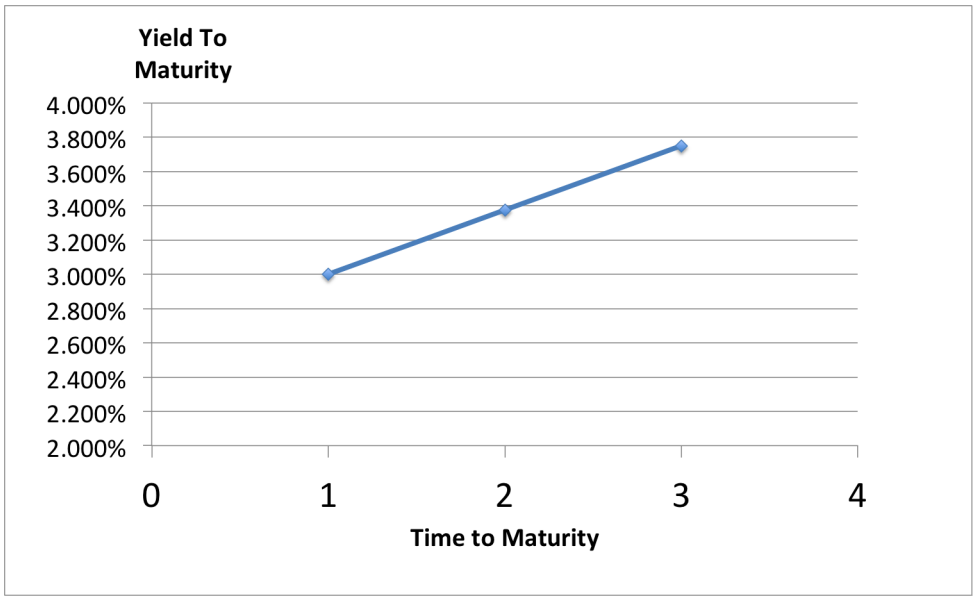
(b) Let liquidity premium are as follows:

liquidity premium for one year bond	0.00%
liquidity premium for two year bond	0.25%
liquidity premium for three year bond	0.50%

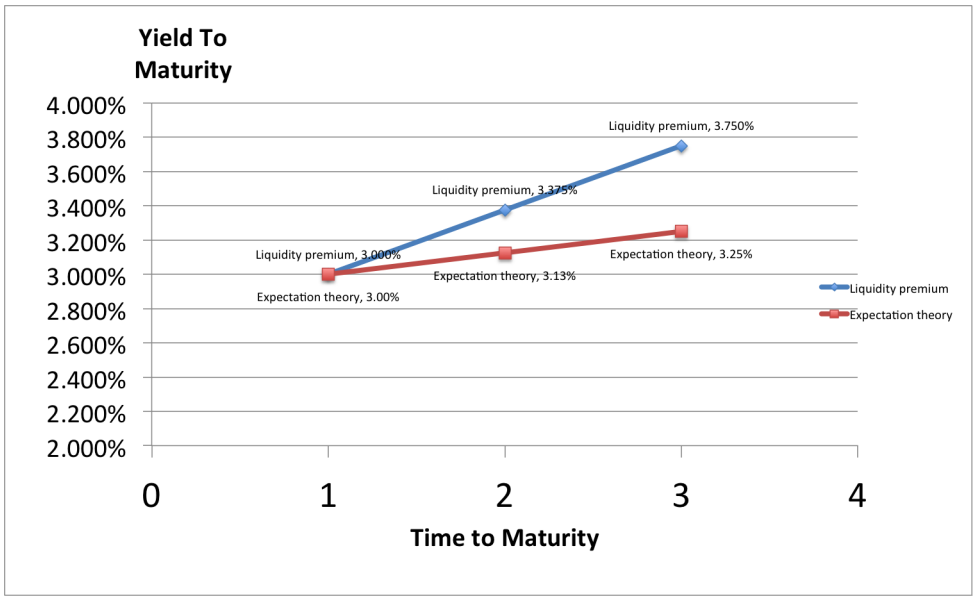
Suppose the **liquidity premium hypothesis** holds, what are the interest rates on one-year, two-year and three-year government bond rate in 2015? Plot the current yield curve.

According to liquidity premium theory, $i_{n.t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n} + \eta_{n,t}$. In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond plus term premium.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} \\
i_{2,2015} &= \frac{i_{1,2015} + i^e_{1,2016}}{2} + \eta_{2,2015} \\
&= \frac{3.00\% + 3.25\%}{2} + 0.25\% \\
&= 3.125\% + 0.25\% \\
&= 3.375\% \\
i_{3,2015} &= \frac{i_{1,2015} + i^e_{1,2016} + i^e_{1,2017}}{3} + \eta_{3,2015} \\
&= \frac{3\% + 3.25\% + 3.50\%}{3} + 0.50\% \\
&= 3.25\% + 0.50\% \\
&= 3.75\%
\end{aligned}
$$



(c) Plot yield curve in question (a) and (b) in the same graph.

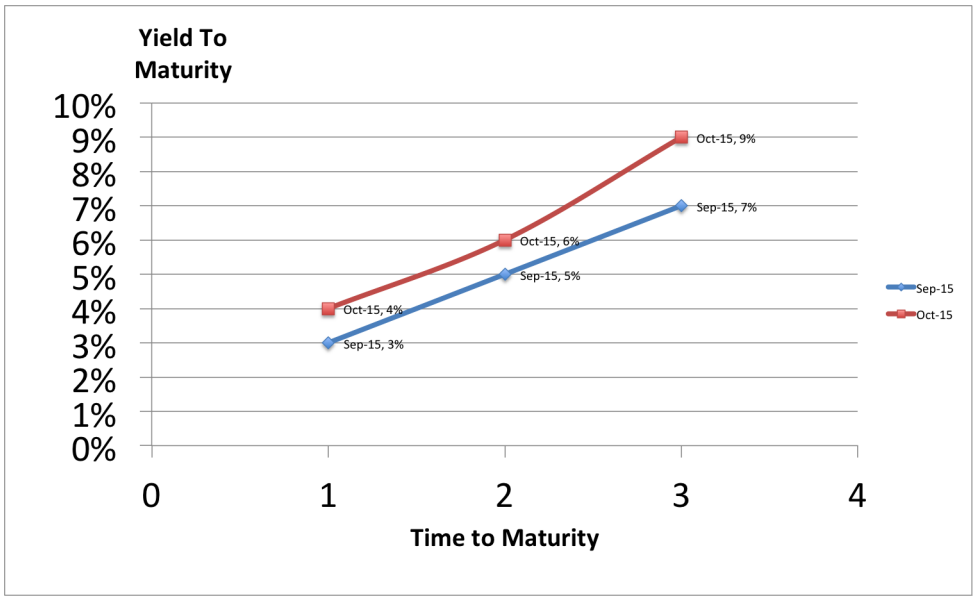


3. Use the following information. Answer all parts of this question.

Government Bond Yield

TTM (Time To Maturity)	September 2015	October2015
1	3%	4%
2	5%	6%
3	7%	9%

(a) Sketch the yield curves in September 2015 and October 2015 in the same graph.



(b) Use the government bond yield data in **September 2015**. Suppose the expectation theory of the yield curve holds, what are the expected one-year government bond rate for 2016 and 2017?

ANSWER. According to expectation theory, $i_{n,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + ... + i^e_{1,t+n-1}}{n}$.

In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} \\
i_{2,2015} &= \frac{i_{1,2015} + i^e_{1,2016}}{2} \\
5\% &= \frac{3\% + i^e_{1,2016}}{2} \\
i^e_{1,2016} &= 7\%
\end{aligned}
$$

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} \\
i_{3,2015} &= \frac{i_{1,2015} + i^e_{1,2016} + i^e_{1,2017}}{3} \\
7\% &= \frac{3\% + 7\% + i^e_{1,2017}}{3} \\
i^e_{1,2017} &= 11\%
\end{aligned}
$$

The expected one-year government bond rate for 2016 and 2017 are 7% and 11%, respectively. Market expects interest rate to increase.

(c) Use the government bond yield in **September 2015**. Suppose liquidity premium for 1 year and 2 years are 0% and 0.5% respectively. According to the liquidity premium theory of the yield curve, what is your prediction for the interest rate on one-year bonds in 2016?

ANSWER. According to liquidity premium theory, $i_{n.t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + + i^e_{1,t+n-1}}{n} + \eta_{n,t}$. In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond plus term premium.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} \\
i_{2,2015} &= \frac{i_{1,2015} + i^e_{1,2016}}{2} + \eta_{2,2015} \\
5\% &= \frac{3\% + i^e_{1,2016}}{2} + 0.5\% \\
i^e_{1,2016} &= 6\%
\end{aligned}
$$

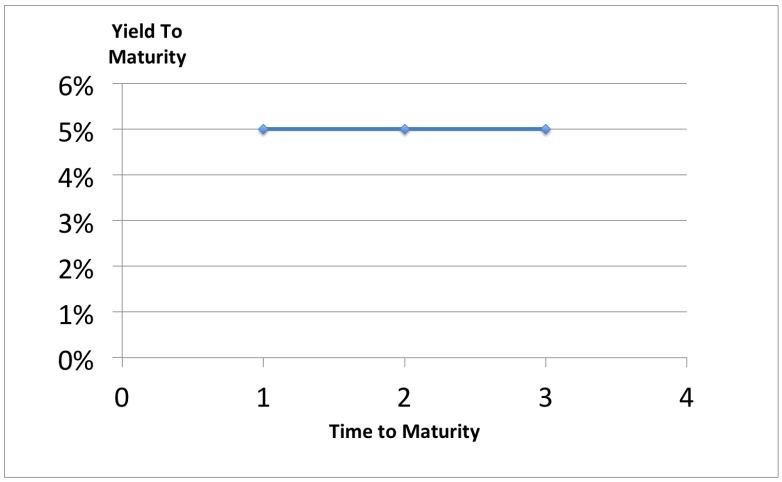
The interest rate on one-year bonds in 2016 is expected to be equal to 6%.

4. Use the following information. Answer all parts of this question.

Government Bond Yield

TTM (Time To Maturity)	**September** 2015
1	5%
2	5%
3	5%

(a) Sketch the yield curves in September 2015



(b) Use the government bond yield data above. Suppose the expectation theory of the yield curve holds, what is the expected one-year government bond rate for 2016 and 2017?

ANSWER. According to expectation theory, $i_{n,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + ... + i^e_{1,t+n-1}}{n}$. In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond.

$$i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2}$$

$$i_{2,2015} = \frac{i_{1,2015} + i^e_{1,2016}}{2}$$

$$5\% = \frac{5\% + i^e_{1,2016}}{2}$$

$$i^e_{1,2016} = 5\%$$

$$i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2}$$

$$i_{3,2015} = \frac{i_{1,2015} + i^e_{1,2016} + i^e_{1,2017}}{3}$$

$$5\% = \frac{5\% + 5\% + i^e_{1,2017}}{3}$$

$$i^e_{1,2017} = 5\%$$

The expected one-year government bond rate for 2016 and 2017 are 5% and 5%, respectively. Market expects interest rate to stay the same.

(c) Suppose liquidity premium for 1 year and 2 years are 0% and 0.5% respectively. According to the liquidity premium theory of the yield curve, what is your prediction for the interest rate on one-year bonds in 2016?

According to liquidity premium theory, $i_{n.t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n} + \eta_{n,t}$. In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond plus term premium.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} \\
i_{2,2015} &= \frac{i_{1,2015} + i^e_{1,2016}}{2} + \eta_{2,2015} \\
5\% &= \frac{5\% + i^e_{1,2016}}{2} + 0.5\% \\
i^e_{1,2016} &= 4\%
\end{aligned}
$$

The interest rate on one-year bonds in 2016 is expected to be equal to 4%. Market expects interest rate to decrease.