4 (20 points) Consider the following cost function for electricity generation:

$$Y = AX^{\beta}P^{\alpha_1}P^{\alpha_2}P^{\alpha_3}u \tag{Eq.4}$$

where
Y=total cost of production
X=output in kilowatt hours
P_1=price of labor input
P_2=price of capital input
P_3=price of fuel input
u=disturbance term

Theoretically, the sum of the price elasticities is expected to be unity, i.e., $\alpha_1 + \alpha_2 + \alpha_3 = 1$. By imposing this restriction, the preceding cost function can be written as

$$(Y/P_3) = AX^{\beta}(P_1/P_3)^{\alpha_1}(P_2/P_3)^{\alpha_2}u \tag{Eq.5}$$

In other words, Eq.4 is an unrestricted and Eq.5 is the restricted cost function.
On the basis of a sample of 29 medium-sized firms, and after logarithmic transformation, the estimated regression results are following:

$$\begin{aligned}\widehat{ln(Y_i)} &= \underset{(1.96)}{-4.93} + \underset{(0.11)}{0.94}\ ln(X_i) + \underset{(0.23)}{0.31}\ ln(P_1) - \underset{(0.29)}{0.26}\ ln(P_2) + \underset{(0.07)}{0.44}\ ln(P_3)\end{aligned} \tag{Eq.6}$$

$$RSS = 0.336 \quad n = 29$$

$$\widehat{ln(Y/P_3)} = \underset{(0.16)}{-6.55} + \underset{(0.11)}{0.91}\ ln(X) + \underset{(0.19)}{0.51}\ ln(P_1/P_3) + \underset{(0.16)}{0.09}\ ln(P_2/P_3) \tag{Eq.7}$$

(4) $\alpha_1 + \alpha_2 + \alpha_3 = 1 \quad \} \quad 1 - \alpha_1 - \alpha_2 = \alpha_3$

$$Y = Ax^{\beta} P_1^{\alpha_1} P_2^{\alpha_2} P_3^{1-\alpha_1-\alpha_2}$$

$$\frac{Y}{P_3} = Ax^{\beta}\left(\frac{P_1}{P_3}\right)^{\alpha_1}\left(\frac{P_2}{P_3}\right)^{\alpha_2}$$

4.1 (10 points) Interprete Eq.6 and Eq.7

4.1

First Eq. (Eq.6)

$\ln(Y_i)$; When output in kilowatt hours increase by 1% total cost of production increase by 0.[illegible]%, ceteris paribus.

$\ln(P_1)$; Price of labor Input — increase 0.31%,

$\ln(P_2)$; Price of Capital Input — decrease 0.26%,

$\ln(P_3)$ Price of fuel Input — increase 0.44%,

Second Eq. (Eq.7)

$\ln(Y_i)$; when output in kilowatt hours increase by 1% total cost of production ration of to price of fuel increase by 0.91%, ceteris paribus

$\ln(P_1/P_3)$; relative price of labour to fuel — increase by 0.51

$\ln(P_2/P_3)$; relative price of Capital to fuel — increase by 0.09

4.2 (10 points) How would you find out if the restriction $(\alpha_1 + \alpha_2 + \alpha_3) = 1$ is valid? Show your calculations.

4.2

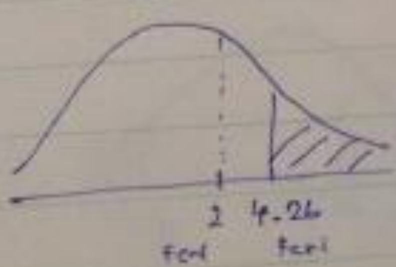
$H_0 : \alpha_1 + \alpha_2 + \alpha_3 = 1$

$: \alpha_1 + \alpha_2 + \alpha_3 \neq 1$

$$F_{cal} = \frac{(RSS_R - RSS_{UR})/m\ (\text{number of Restriction})}{RSS_{UR}/n-k}$$

$$= \frac{(0.364 - 0.336)/1}{0.336/24} = 2$$

$F_{cri} = F_{.05,1,24} = 4.26$



Since F cal does not fall into rejection region we cannot reject $H_0: \alpha_1 + \alpha_2 + \alpha_3 = 1$ at 0.05 level of significance, or In other word it is statistically significant the sum of the price elasticities is unity at 0.05 level of significant.

Model 2

$$\ln(P_i) = \alpha_1 + \alpha_2 \ln(HS_i) + \alpha_3 \ln(YS_i) + \alpha_4 DC_i + u_i$$

$$\widehat{\ln(P_i)} = 2.639 + 0.750 \ \ln(HS_i) + 0.168 \ \ln(YS_i) + 0.066 \ DC_i$$
$$se = (0.244) \quad (0.081) \qquad (0.038) \qquad (0.043) \qquad \text{(Eq.9)}$$

$$RSS = 2.843 \quad TSS = 8.018 \quad N = 88$$

5.1 (2.5 points) Interpret each of the slope coefficient estimates β_2 and β_4 in regression model 1 (Eq.1).

$\hat{\beta}_2$; when the house size increase by 100 ft^2 the selling price of house will increase by $ 13,186 , ceteris paribus

$\hat{\beta}_4$; when the house is colonial style house the selling price of house will increase by $ 19,183 , ceteris paribus

$\hat{\alpha}_2$; when the house size increase by 1% the selling price of house will increase by 0.75% , ceteris paribus

$\hat{\alpha}_4$; when the house is colonial style the selling price of house will increase by 6.6% , ceteris paribus

5.2 (2.5 points) Interpret each of the slope coefficient estimates α_2 and α_4 in regression model 2 (Eq.2).

from $\hat{\alpha}_4$; $\frac{\partial \widehat{\ln(P_i)}}{\partial DC_i} = 0.066$ $\qquad \frac{\partial \widehat{\ln(P_i)}}{\partial P_i} \cdot \frac{\partial P_i}{\partial DC_i} \approx \frac{\frac{\Delta P_i}{P_i}}{\Delta DC_i} = 0.066$

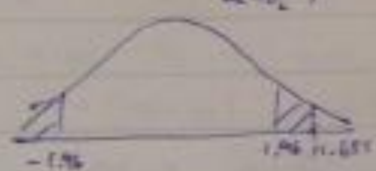
$\frac{\frac{\Delta P_i}{P_i} \times 100}{\Delta DC_i} = 0.066 \times 100 = 6.6$ #

5.3 (5 points) Use the estimation results for regression model 1 to test the individual significance of each of the slope coefficient estimates β_2 for HS_1 and β_4 for DC_1. For each test, state the null and alternative hypotheses, and show how you calculate the required test statistic. Which of these two slope coefficient estimates are individually significant at the 5 percent significance level?

$H_0: B_2 = 0 \qquad \alpha = 0.05$

$H_a: B_2 \neq 0$

$t_{cal} = \dfrac{\hat{B}_2 - B_2}{se(\hat{B}_2)} = \dfrac{13.236 - 0}{1.136} = 11.651 \qquad t_{cri} = t_{\frac{0.05}{2}, 84} = 1.96$



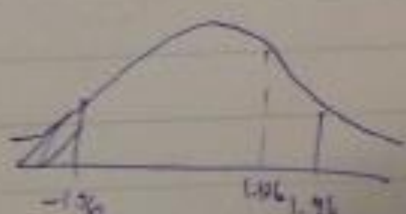
$t_{cal} > t_{cri}$ ∴ we reject H_0 $B_2 = 0$ at 0.05 level of significant. In other word It is not statistically significant that $B_2 = 0$ #

$H_0: B_4 = 0 \qquad \alpha = 0.05$

$H_a: B_4 \neq 0$

$t_{cal} = \dfrac{\hat{B}_4 - B_4}{se(\hat{B}_4)} = \dfrac{19.175 - 0}{12.44} = 1.576$

$t_{cri} = t_{\frac{0.05}{2}, 84} = 1.96$



$t_{cal} < t_{cri}$ ∴ we cannot reject $H_0: B_4 = 0$ at 0.05 level of significant. It is statistically significant that $B_4 = 0$ #.

5.4 (5 points) Use the estimation results for regression model 1 to test the joint significance of all the slope coefficient estimates at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$).

5.4

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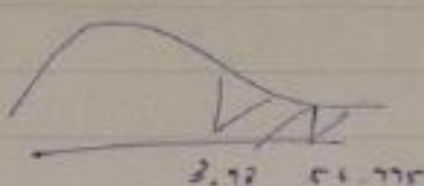
H_0 : $\beta_2 = \beta_3 = \beta_4 = 0$

H_a : At least one of the coefficient not equal to zero

$\alpha = 0.01$

$F_{cal} = \dfrac{MS \text{ of ESS}}{MS \text{ of RSS}} = \dfrac{(TSS - RSS)/3}{302,371/84} = 56.775$

$F_{cri} = F_{.01, 3, 84} \approx F_{.01, 3, 100} = 3.98$



$F_{cal} > F_{cri}$

$\therefore$ We reject H_0 : $\beta_2 = \beta_3 = \beta_4 = 0$ with 0.01 level of significant. That is at least one of the coef. not equal to zero.

5.5 (5 points) Use the estimation results for regression equation model 2 to test the proposition that $\alpha_2 > \alpha_3$, i.e., to test the proposition that the marginal effect of $\ln(HS_t)$ on $\ln P_t$ is greater than the marginal effect of $\ln(YS_t)$ on $\ln P_t$ at the 5 percent significance level. [Note: $\text{cov}(\alpha_2,\alpha_3) = -0.001$]

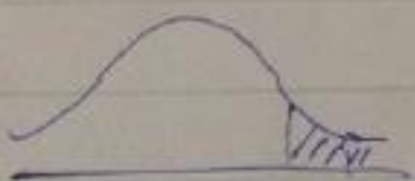
5.5 $H_0: \alpha_2 \le \alpha_3$; $\alpha_2 - \alpha_3 \le 0$; $\text{var}(\hat{\alpha}_2 - \hat{\alpha}_3) = 0.006561 + 0.001444 + 0.002$

$\alpha = 0.05$ $H_a: \alpha_2 > \alpha_3$; $\alpha_2 - \alpha_3 > 0$; $se(\hat{\alpha}_2 - \hat{\alpha}_3) = .10002$

$$\text{var}(ax - by) = a^2(\text{var}(x)) + b^2(\text{var}(y)) - 2\,\text{cov}(x,y)\,ab$$

$$t_{cal} = \frac{(\hat{\alpha}_2 - \hat{\alpha}_3) - (\alpha_2 - \alpha_3)}{se(\hat{\alpha}_2 - \hat{\alpha}_3)} = \frac{(0.75 - 0.161) - 0}{0.10002} = 5.814$$

$$t_{crit} = t_{.05,84} = 1.645$$



1.645 5.814

$\therefore$ we reject $H_0: \alpha_2 \le \alpha_3$ with 0.05 level of significant. that is the $\alpha_2 > \alpha_3$ is statistically significant at 0.05 level of significant.