Assignment 4

DUE DATE: Tuesday 9nd, March 2021.

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

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Question 1 (50 points) Your score......................

$\lambda^2 - \phi_1 \lambda - \phi_2 = 0$

Given the daily log returns : (R_t) can be explained by the AR(2) model as following:

$$(1 - 1.5B + 0.9B^2)R_t = 0.25 + \varepsilon_t$$

ϕ_1 ϕ_2

$R_t - 1.5\phi_1 R_{t-1} + 0.9\phi_2 R_{t-2} = 0.25 + \varepsilon_t$

where ε_t is distributed as the Gaussian White Noise with mean (μ) = 0 and variance (σ^2) =0.25

B lag-operator

$R_t = 1.5\phi_1 R_{t-1} - 0.9\phi_2 R_{t-2} + 0.25\phi_0 + \varepsilon_t \approx$

Question 1.1 (10 points) Your score......................

$$\lambda_i = \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}$$

From the above AR(2) model, Is the model weakly stationary? Write down the reverse characteristic equation and find out the conditions to support your answer.

$$\lambda_i = \frac{1.5 \pm \sqrt{(-1.5)^2 + 4(-0.9)}}{2}$$

$$= \frac{1.5 \pm \sqrt{2.25 - 3.6}}{2}$$

$$= \frac{1.5}{2} + \frac{1.161895i}{2}, \quad \frac{1.5}{2} - \frac{1.161895i}{2}$$

$R = \sqrt{a^2 + b^2}$ $\lambda_i = a + bi$

$$\sqrt{\left(\frac{1.5}{2}\right)^2 + \left(\frac{1.161895}{2}\right)^2} = -0.018447$$

since the R is less than 1 in the reverse characteristic equation. It is weak stationarity

$r_t = \phi_0 + \phi_1 r_{t-1} + \dots + \phi_p r_{t-p} + a_t$ $\hat{a}_t = r_t \hat{r}_t$

Question 1.2 (10 points) $\hat{\sigma}_a^2 = \frac{\sum_{t=p+1}^{T} \hat{a}_a^2}{T-2p-1}$ Your score......................

ϕ_1 ϕ_2

$$(1-1.5B+0.9B^2)R_t = 0.25+\varepsilon_t$$

Calculate the unconditional mean: $E(R_t)$ of R_t and the conditional mean: $E(R_t|F_{t-1})$

conditional mean

$R_t = 0.25 + 1.5 r_{t-1} - 0.9 r_{t-2} + a_t$

$E[R_t|F_{t-1}] = E[0.25 + 1.5 r_{t-1} - 0.9 r_{t-2} + a_t | F_{t-1}]$

$= 0.25 + 1.5 E[r_{t-1}|F_{t-1}] - 0.9 E[r_{t-2}|F_{t-1}] + 0$

$= 0.25 + 1.5 r_{t-1} - 0.9 r_{t-2}$

$R_t = 1.5\phi_1 R_{t-1} - 0.9\phi_2 R_{t-2}$
$+ 0.25\phi_0 + \varepsilon_t \approx$

unconditional mean

$R_t = 0.25 + 1.5 R_{t-1} - 0.9 R_{t-2} + a$

$E[R_t] = E[0.25 + 1.5 R_{t-1} - 0.9 R_{t-2} + a]$

$= 0.25 + 1.5 E[R_{t-1}] - 0.9 E[R_{t-2}] + E[a]$

since, $E[R_t] = E(R_{t-1}) = E(R_{t-2}) = \mu$; $E[a] = 0$

$\therefore$ $E(R_t)(1 - 1.5 + 0.9) = 0.25$

$0.4 E(R_t) = 0.25$

$E(R_t) = \frac{0.25}{0.4} = 5/8$

Question 1.3 (10 points) Your score......................

γ_1 ϕ

$+1.5$ -0.9

Find out the unconditional variance: $Var(R_t)$ of R_t and conditional variance $Var(R_t|F_{t-1})$ of R_t

Unconditional varianu $R_t = 0.25 + 1.5 R_{t-1} - 0.9 R_{t-2} + a$

$Var(R_t) = Var[0.25 + 1.5 R_{t-1} - 0.9 R_{t-2} + a_t]$

$E[(r_t - a)]^2 = Var(0.25) + 1.5^2 Var(R_{t-1}) - (0.9)^2 Var R_{t-2} + Var(a_t)$

$+ 2 Cov(1.5 R_{t-1} \cdot -0.9 R_{t-2}) + 2 Cov(1.5 R_{t-1} \cdot a_t)$

$+ 2 Cov(-0.9 R_{t-2} \cdot a_t)$

$(1 - 1.5 + 0.9) Var(R_t) = 0 + 2.25 E(r_{t-1} - \mu)^2 + 0.81 E(r_{t-2} - \mu)^2 + \sigma_a^2$

$- 2.7 \gamma_1 + 0 + 0$

$= \dfrac{\sigma_a^2 - 2.7\gamma_1}{-0.44}$

$Var(r_t) = \dfrac{\sigma_a^2 + 2\phi_1\phi_2\gamma_1}{1 - \phi_1^2 - \phi_2^2}$

$\gamma = (\phi_1 \times \phi_2) - (\phi_1^2 \times \phi_2^2)$

$= 1.35 - 1.8225$

$= -0.4725$

$= \dfrac{\sigma_a^2 - 2.7(-0.4725)}{-0.44}$

$\gamma_i = E[(r_t - \mu)(r_{t-j} - \mu)]$

$1 - 1.5^2 + 0.9^2 = -0.44$

$= \dfrac{\sigma_a^2 + 1.2755}{-0.44}$

$= \dfrac{0.25 + 1.2755}{-0.44}$

$\sigma^2 = 0.25$

$= -3.467$

Question 1.4 (10 points) $E[(r_t-\mu)(r_{t-j}-\mu)] = Cov(r_t, r_{t-j}) = \gamma_j$ Your score......................

Calculate the autocorrelation: ρ_l for l=1 and 2 of R_t. Also, write down the autocorrelation: ρ_l when l $\geq$ 2.

$R_t = 0.25 + 1.5 R_{t-1} - 0.9 R_{t-2} + a$

$= 1.5 Var(R_{t-1}) - 0.9\gamma_1 - \sigma_a^2$

$\gamma_1 = 1.5(0.2631 - 2.7\gamma_1) - 0.9\gamma_1 - 0.25$

$\gamma_1 = 0.39465 - 4.05\gamma_1 - 0.9\gamma_1 - 0.28$

$\gamma_1 + 4.05\gamma_1 + 0.9 = 0.39465 - 0.28$

$\gamma_1 = -0.4725$

$\rho_1 = \frac{-0.4725}{-3.467} = 0.13628$

$\rho_2 = \frac{0.13628}{-3.467} = -0.0393$

$\gamma_l; l > 2 = 1.5\gamma_{l-1} - 0.9\gamma_{l-2}$

$\rho_l; l > 2 = \frac{1.5\gamma_{l-1} - 0.9\gamma_{l-2}}{-3.467}$

Question 1.5 (10 points) Your score......................

$var(e_n(1)|.) =$

$var(a_{n+1}|.) = \sigma_a^2 = 0.25$

Given $R_{1000} = 0.01$ $R_{999} = 0.02$ $R_{998} = 0.03$ $\varepsilon_{1000} = -0.01$ $\varepsilon_{999} = -0.02$ $\varepsilon_{998} = -0.03$ Obtain 1-step, 2-setp 95 % interval forecasts for R_t at the forecast origin t = 1000. Also the ∞-step 95 % interval forecasts for R_t. Draw these intervals.

$R_t = 0.25 + 1.5R_{t-1} - 0.9R_{t-2} + a$

n n+1

1 - step

$\hat{R}_t(1) = 0.25 + 1.5(0.01) - 0.9(0.02) - 0.02$

$= 0.227$

sd is 0.25 ; $e_t(1) = R_{t+1} - \hat{R}_t(1) = a_{t+1}$

2 - step

$\hat{R}_t(2) = 0.25 + 1.5(0.227) - 0.9(0.01) - 0.01$

$= 0.5715$

$\sqrt{var(e_t(2))} = \hat{r}(2) \pm \sqrt{(1+\phi_1^2) + \sigma_a^2} \cdot 1.96 + 0.25$

$= 0.5715 - \sqrt{(1+1.5^2) + 0.25} \cdot 1.96,\ 0.5715 - \sqrt{(1+1.5^2) + 0.25}$

$= -3.229,\ 4.367$

∞ - step $r_{h+l} = \phi_0 + \sum_{i=1}^{p} \phi_i r_{n+l-i} + a_{h+l} =$

$\hat{r}_n(l) = \phi_0 + \sum_{i=1}^{n} \phi_i \hat{r}_h(l-i)$

$\hat{r}_h(l-i) = r_{n-l-i}$

when $l \to \infty$ $\hat{r}_n(l) = E(r_t)$ $var(e_n(l)) = -3.467$

$= 0.01$