

FN211: Lecture Note 1

Review of Basic Statistics and Calculus

Dr. Winai Homsombat

Bachelor of Economics, International Program

Thammasat University



Basic Statistics

Definition:

- Statistics is the science of presenting and interpreting data.

Descriptive Stats

- collecting,
- describing,
- etc.

Inferential Stats

- interpreting,
- modeling,
- Making decisions.

FN211

- FN211 lies entirely in the subset of inferential statistics (sometimes called Mathematical Statistics)
- Two stages in the mind of a statistician.
 - Design of an experiment and subsequent investigation.
 - Forming a statistical model and inference.

FN211

- Definition: Statistics provides the model that are needed to study situations involving uncertainties.
- A paramount issue is formulating appropriate and valid models depending, of course, on the proposed experiment.

Some questions and ideas we will pose and answer:

Q1: What is the probability of an event?

Q2: What is a probability model?

Q3: Discrete random variables VS Continuous random variables?

Q4: Counting and the principle of choices?

etc.

Casually speaking, the term probability is a measure of one's belief
in the occurrence of a future event.

Statistics

- In your first course in Statistics, you probably began the course by discussing data:
 1. Histograms, pie charts,
 2. Measurements of center (mean, median),
 3. Measurements of dispersion (variance, quartiles).

Then stepped back and discussed Probability, etc.

- For example, you most likely encountered the famous discrete distribution : Binomial distribution.

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \quad ; x = 0, 1, 2, \dots, n$$

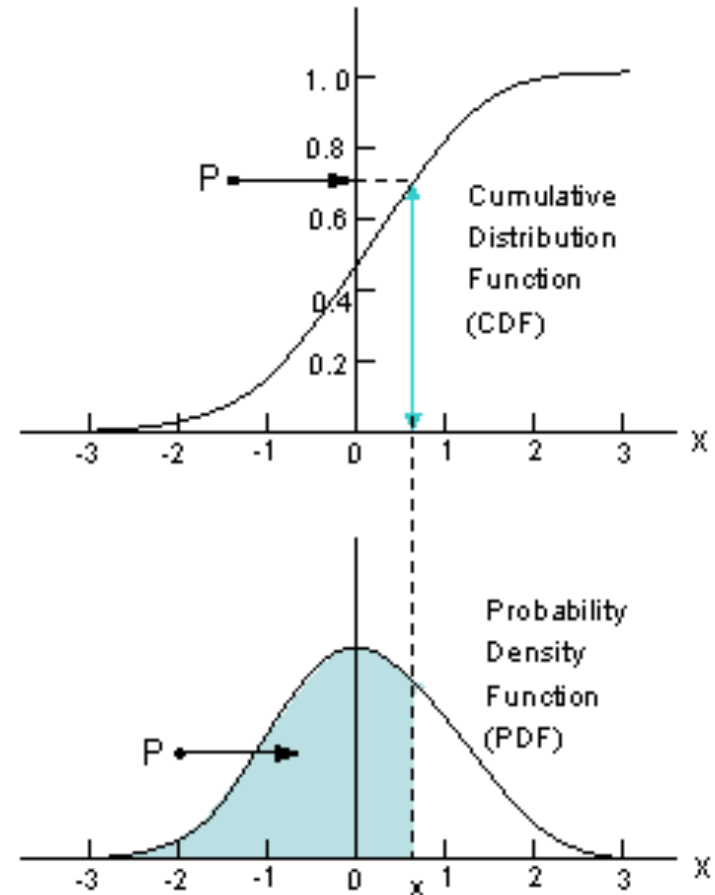
- In addition, you also discussed the normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

Normal Distribution

$$F(x) = \Phi\left(\frac{x - \mu}{\sigma}\right) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x - \mu}{\sigma\sqrt{2}}\right) \right]$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Relations Between Two Different Typical Representations of a Population

Basic Statistics: Recall

- **What is Central Tendency?**

- Statistical analyzes and interprets large sets of numbers. To make the lists of data more comprehensible, central tendencies are calculated.
- A measure of central tendency points the statistician toward a centralized, repeated, or average number.

- **What is Central Tendency?**

- A measure of difference between the observed value of a variable and some other value, often that variable's mean.

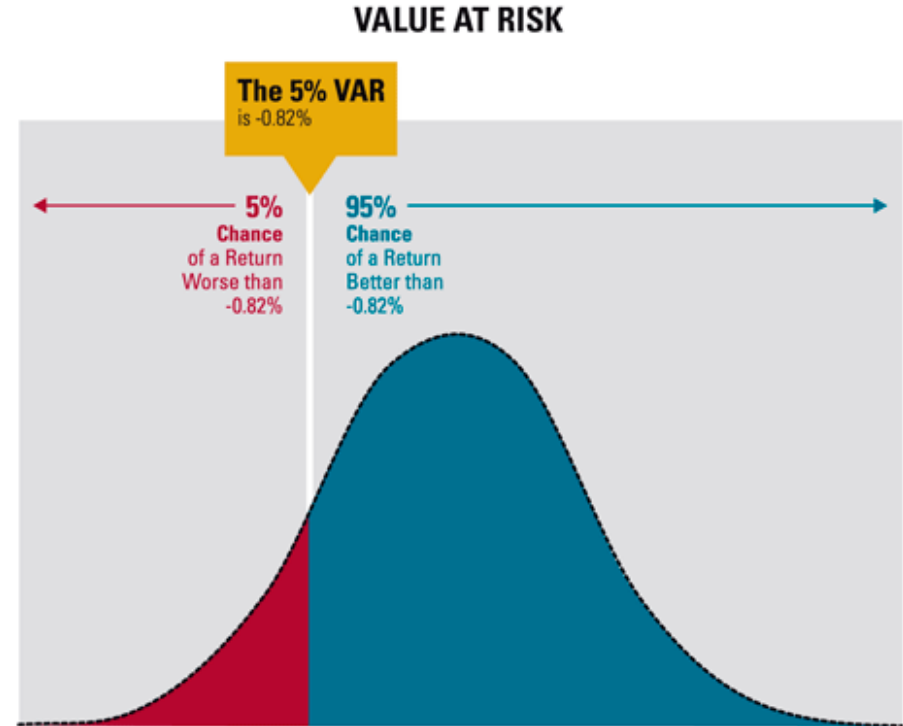
Basic Statistics and Application in Finance

Value at Risk (VaR)

“What loss level is such that we are $X\%$ confident it will not be exceeded in N business days?”

Application: VaR and Regulatory Capital

- Regulators base the capital they require banks to keep on VaR

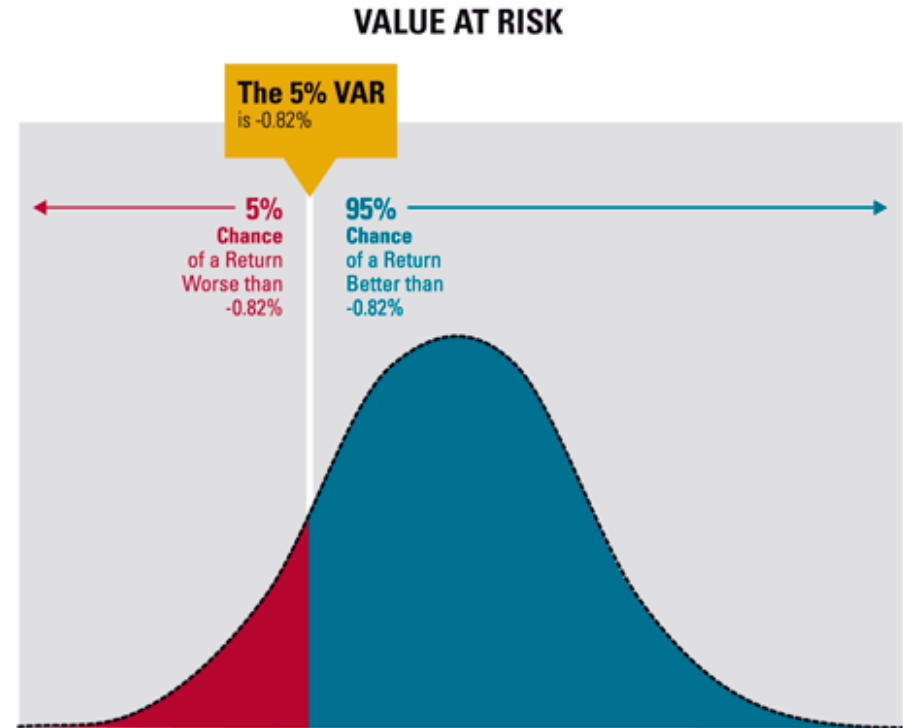


$$\text{VaR for the portfolio} = \mu + Z_{\alpha} \cdot \sigma$$

Basic Statistics and Application in Finance (2)

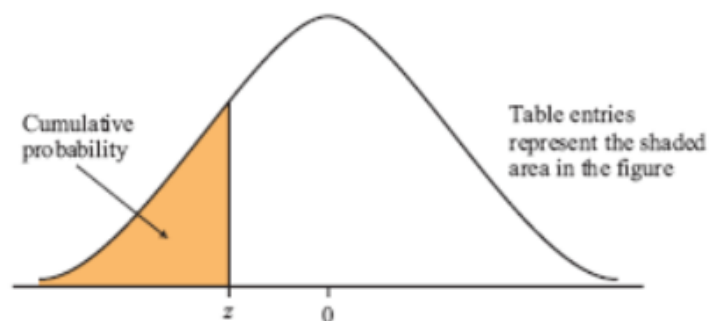
Example:

- The gain from a portfolio during six months is normally distributed with mean \$2 million and standard deviation \$10 million.
- What is the VaR for the portfolio with a six-months time horizon and a 99% confidence level?



$$\text{VaR for the portfolio} = \mu + Z_{\alpha} \cdot \sigma$$

Cumulative Probabilities for the Standard Normal Distribution



FIRST DIGIT OF z

SECOND DIGIT OF z

FIRST DIGIT OF z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	-3.0
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	-2.9
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019	-2.8
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	-2.7
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	-2.6
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048	-2.5
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064	-2.4
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	-2.3
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	-2.2
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	-2.1
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	-2.0
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	-1.9
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	-1.8
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	-1.7



Calculus

Limit

Definition of Limit

If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, then the limit of $f(x)$ as x approaches c is L . This is written as

$$\lim_{x \rightarrow c} f(x) = L.$$

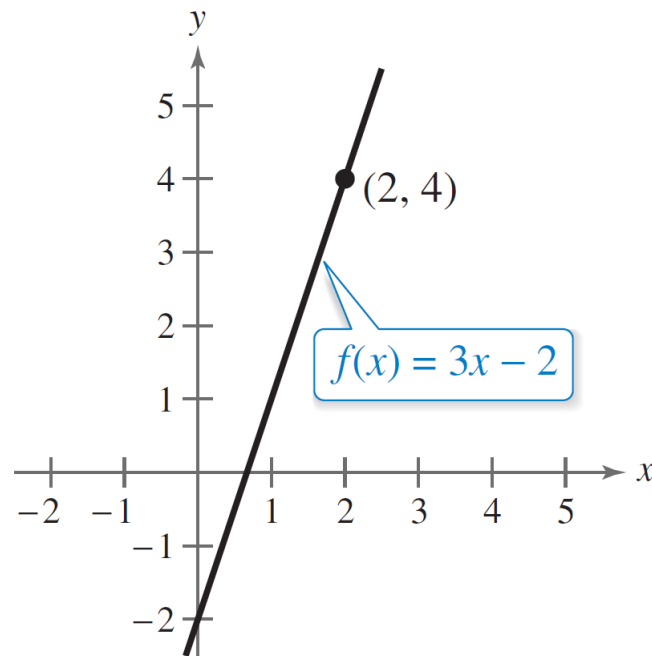
Example: Use a table to estimate numerically the limit:

$$\lim_{x \rightarrow 2} (3x - 2)$$

Limit: Example

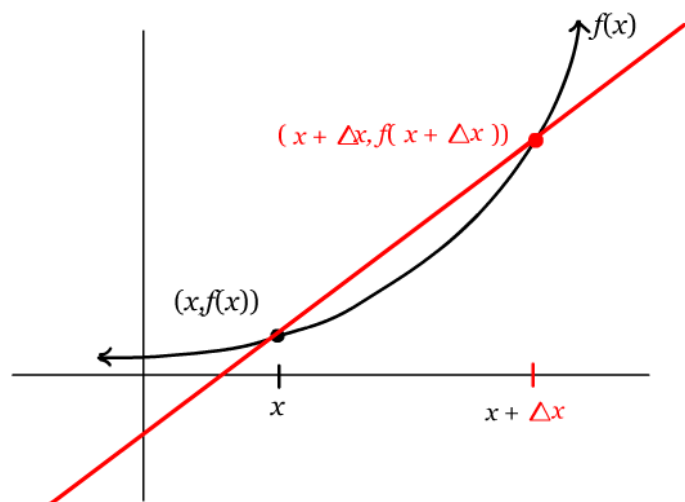
$$\text{Let } f(x) = 3x - 2$$

Then construct a table that shows values of $f(x)$ for two sets of x -values—one set that approaches 2 from the left and one that approaches 2 from the right.



x	1.9	1.99	1.999	2.0	2.001	2.01	2.1
$f(x)$	3.700	3.970	3.997	?	4.003	4.030	4.300

Limit Definition of the Derivative



The derivative is the formula which gives the slope of the tangent line at any point x for $f(x)$, and is denoted

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided this limit exists.

Example: $f(x) = x^2 - 3x + 5$

The Chain Rule

- The Chain Rule is a technique for differentiating composite functions.
- Composite functions are made up of layers of functions inside of functions.

If $y = f(u)$ and $u = g(x)$ then $y = f(g(x))$ and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{or} \quad \frac{dy}{dx} = \frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

The Chain Rule: Example

$$1) f(x) = (3x - 5x^2)^7$$

$$2) f(x) = \sqrt[3]{(x^2 - 1)^2}$$

Basic Calculus and Application in Finance

Options (derivatives product)

- Options are financial instruments that are derivatives based on the value of underlying securities such as stocks.
- Options contract offers the buyer the right to buy or sell - depending on the type of contract they hold - the underlying asset.

Call Options: Position of Buyer vs. Writer

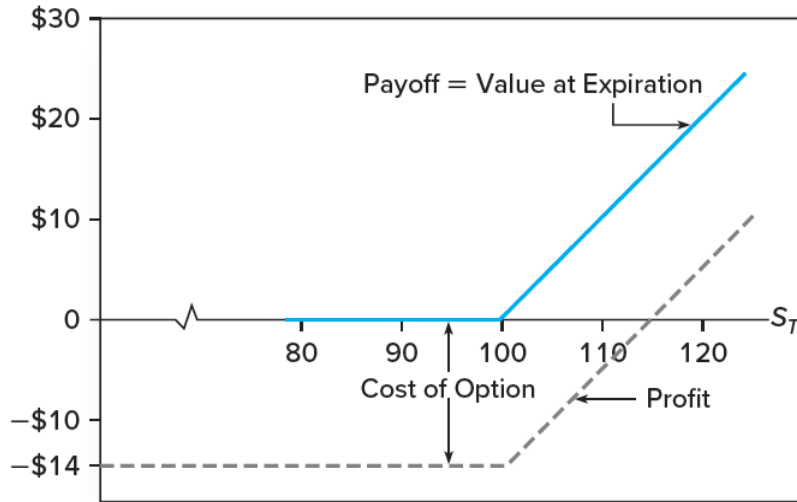


Figure 20.2 Payoff and profit to call option at expiration

Put Options: Position of Buyer vs. Writer

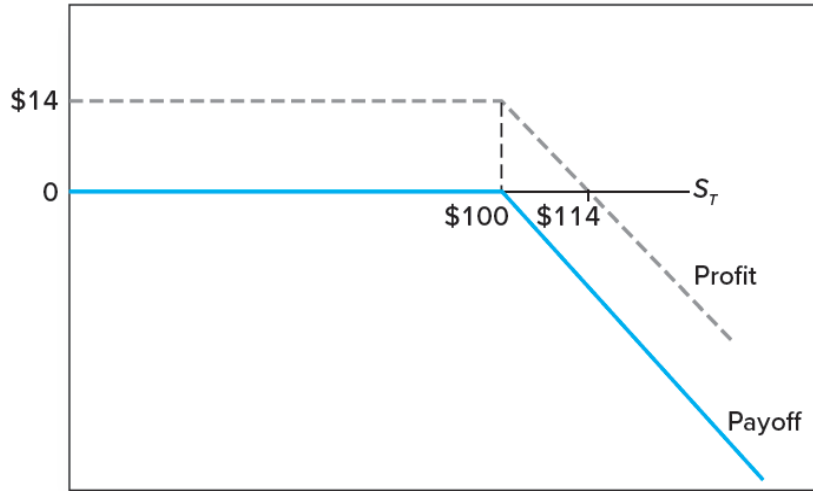


Figure 20.3 Payoff and profit to call writer at expiration

Basic Calculus and Application in Finance

- The Black-Scholes-Merton Formulas

- An investment asset provides no income

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad \text{and} \quad p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

where:

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

and

$N(x)$ is the cumulative probability distribution function for a standardized normal distribution

S_0 is today's asset price T is time to maturity of the option

K is strike price σ is stock price volatility

r is the continuously compounded risk-free rate

Basic Calculus and Application in Finance: Greek Letters

Delta (Δ)

= the rate of change of the value of the portfolio with respect to
a market variable:

$$Delta = \frac{\partial P}{\partial S}$$

where P is the value of the portfolio

S is the small increase in the value of the variable

Basic Calculus and Application in Finance: Greek Letters

Delta (Δ) Hedging

Note: Hedging is a risk management strategy employed to offset losses in investments by taking an opposite position in a related asset.

Example: The financial institution writes call options with the current delta is 0.5216 per one stock. If it writes 10,000 stocks per contract, how does it formulate hedging strategy?

Question?