

# **FN 312 Investment**

## **Exercise for Mid-term Examination 2019**

### *Guided Solution (Selected)*

## Lecture 1 Introduction and Asset Classes and Financial Instruments

7. Lanni Products is a start-up computer software development firm. It currently owns computer equipment worth \$30,000 and has cash on hand of \$20,000 contributed by Lanni's owners. For each of the following transactions, identify the real and/or financial assets that trade hands. Are any financial assets created or destroyed in the transaction?
- Lanni takes out a bank loan. It receives \$50,000 in cash and signs a note promising to pay back the loan over 3 years.
  - Lanni uses the cash from the bank plus \$20,000 of its own funds to finance the development of new financial planning software.
  - Lanni sells the software product to Microsoft, which will market it to the public under the Microsoft name. Lanni accepts payment in the form of 2,500 shares of Microsoft stock.
  - Lanni sells the shares of stock for \$50 per share and uses part of the proceeds to pay off the bank loan.

a. The bank loan is a financial liability for Lanni, and a financial asset for the bank. The cash Lanni receives is a financial asset. The new financial asset created is Lanni's promissory note to repay the loan.

b. Lanni transfers financial assets (cash) to the software developers. In return, Lanni receives the completed software package, which is a real asset. No financial assets are created or destroyed; cash is simply transferred from one party to another.

c. Lanni exchanges the real asset (the software) for a financial asset, which is 2,500 shares of Microsoft stock. If Microsoft issues new shares in order to pay Lanni, then this would represent the creation of new financial assets.

d. By selling its shares in Microsoft, Lanni exchanges one financial asset (2,500 shares of stock) for another (\$125,000 in cash). Lanni uses the financial asset of \$50,000 in cash to repay the bank and retire its promissory note. The bank must return its financial asset to Lanni. The loan is "destroyed" in the transaction, since it is retired when paid off and no longer exists.

3. Which of the following *correctly* describes a repurchase agreement?

- The sale of a security with a commitment to repurchase the same security at a specified future date and a designated price.
- The sale of a security with a commitment to repurchase the same security at a future date left unspecified, at a designated price.
- The purchase of a security with a commitment to purchase more of the same security at a specified future date.

(a) A repurchase agreement is an agreement whereby the seller of a security agrees to "repurchase" it from the buyer on an agreed upon date at an agreed upon price. Repos are typically used by securities dealers as a means for obtaining funds to purchase securities.

8. Suppose investors can earn a return of 2% per 6 months on a Treasury note with 6 months remaining until maturity. What price would you expect a 6-month maturity Treasury bill to sell for?

9. Find the after-tax return to a corporation that buys a share of preferred stock at \$40, sells it at year-end at \$40, and receives a \$4 year-end dividend. The firm is in the 30% tax bracket.

8)  $P = \$10,000/1.02 = \$9,803.92$

9) Taxes are:  $0.30 \times \$1.20 = \$0.36$

After-tax income is:  $\$4.00 - \$0.36 = \$3.64$

Rate of return is:  $\$3.64/\$40.00 = 9.10\%$

13. An investor is in a 30% tax bracket. If corporate bonds offer 9% yields, what must municipals offer for the investor to prefer them to corporate bonds?

14. Find the equivalent taxable yield of a short-term municipal bond currently offering yields of 4% for tax brackets of (a) zero, (b) 10%, (c) 20%, and (d) 30%.

13) The after-tax yield on the corporate bonds is:  $0.09 \times (1 - 0.30) = 0.063 = 6.30\%$

Therefore, municipals must offer a yield to maturity of at least 6.30%.

14) a. 4.00%

b. 4.44%

c. 5.00%

d. 5.71%

16. Which security should sell at a greater price?

a. A 10-year Treasury bond with a 4% coupon rate versus a 10-year T-bond with a 5% coupon.

b. A 3-month expiration call option with an exercise price of \$40 versus a 3-month call on the same stock with an exercise price of \$35.

c. A put option on a stock selling at \$50, or a put option on another stock selling at \$60 (all other relevant features of the stocks and options may be assumed to be identical).

16) a. The ten-year Treasury bond with the higher coupon rate will sell for a higher price because its bondholder receives higher interest payments.

b. The call option with the lower exercise price has more value than one with a higher exercise price.

c. The put option written on the lower priced stock has more value than one written on a higher priced stock.

19. Why do call options with exercise prices greater than the price of the underlying stock sell for positive prices?
20. Both a call and a put currently are traded on stock XYZ; both have strike prices of \$50 and expirations of 6 months. What will be the profit to an investor who buys the call for \$4 in the following scenarios for stock prices in 6 months? What will be the profit in each scenario to an investor who buys the put for \$6?
- \$40
  - \$45
  - \$50
  - \$55
  - \$60

20)

	Value of Call at Expiration	Initial Cost	Profit
a.	0	4	-4
b.	0	4	-4
c.	0	4	-4
d.	5	4	1
e.	10	4	6
	Value of Put at Expiration	Initial Cost	Profit
a.	10	6	4
b.	5	6	-1
c.	0	6	-6
d.	0	6	-6
e.	0	6	-6

2. A municipal bond carries a coupon of 6.75% and is trading at par. What is the equivalent taxable yield to a taxpayer in a combined federal plus state 34% tax bracket?
3. Which is the *most risky* transaction to undertake in the stock index option markets if the stock market is expected to increase substantially after the transaction is completed?
  - a. Write a call option.
  - b. Write a put option.
  - c. Buy a call option.
  - d. Buy a put option.
4. Short-term municipal bonds currently offer yields of 4%, while comparable taxable bonds pay 5%. Which gives you the higher after-tax yield if your tax bracket is:
  - a. Zero
  - b. 10%
  - c. 20%
  - d. 30%
5. The coupon rate on a tax-exempt bond is 5.6%, and the rate on a taxable bond is 8%. Both bonds sell at par. At what tax bracket (marginal tax rate) would an investor be indifferent between the two bonds?

2. The equivalent taxable yield is:  $6.75\% / (1 - 0.34) = 10.23\%$

3. (a) Writing a call entails unlimited potential losses as the stock price rises.

4. a. The taxable bond. With a zero tax bracket, the after-tax yield for the taxable bond is the same as the before-tax yield (5%), which is greater than the yield on the municipal bond.

b. The taxable bond. The after-tax yield for the taxable bond is:  
 $0.05 \times (1 - 0.10) = 4.5\%$

c. You are indifferent. The after-tax yield for the taxable bond is:  
 $0.05 \times (1 - 0.20) = 4.0\%$

The after-tax yield is the same as that of the municipal bond.

d. The municipal bond offers the higher after-tax yield for investors in tax brackets above 20%.

5. If the after-tax yields are equal, then:  $0.056 = 0.08 \times (1 - t)$   
 This implies that  $t = 0.30 = 30\%$ .

## Lecture 2 How Securities are Traded

4. A market order has:

- a. Price uncertainty but not execution uncertainty.
- b. Both price uncertainty and execution uncertainty.
- c. Execution uncertainty but not price uncertainty.

(a) A market order is an order to execute the trade immediately at the best possible price. The emphasis in a market order is the speed of execution (the reduction of execution uncertainty). The disadvantage of a market order is that the price at which it will be executed is not known ahead of time; it thus has price uncertainty.

6. Dée Trader opens a brokerage account and purchases 300 shares of Internet Dreams at \$40 per share. She borrows \$4,000 from her broker to help pay for the purchase. The interest rate on the loan is 8%.

- a. What is the margin in Dée's account when she first purchases the stock?
- b. If the share price falls to \$30 per share by the end of the year, what is the remaining margin in her account? If the maintenance margin requirement is 30%, will she receive a margin call?
- c. What is the rate of return on her investment?

7. Old Economy Traders opened an account to short sell 1,000 shares of Internet Dreams from the previous problem. The initial margin requirement was 50%. (The margin account pays no interest.) A year later, the price of Internet Dreams has risen from \$40 to \$50, and the stock has paid a dividend of \$2 per share.

- a. What is the remaining margin in the account?
- b. If the maintenance margin requirement is 30%, will Old Economy receive a margin call?
- c. What is the rate of return on the investment?

8. Consider the following limit-order book for a share of stock. The last trade in the stock occurred at a price of \$50.

Limit Buy Orders		Limit Sell Orders	
Price	Shares	Price	Shares
\$49.75	500	\$50.25	100
49.50	800	51.50	100
49.25	500	54.75	300
49.00	200	58.25	100
48.50	600		

- a. If a market buy order for 100 shares comes in, at what price will it be filled?
- b. At what price would the next market buy order be filled?
- c. If you were a security dealer, would you want to increase or decrease your inventory of this stock?

9. You are bullish on Telecom stock. The current market price is \$50 per share, and you have \$5,000 of your own to invest. You borrow an additional \$5,000 from your broker at an interest rate of 8% per year and invest \$10,000 in the stock.

- a. What will be your rate of return if the price of Telecom stock goes up by 10% during the next year? The stock currently pays no dividends.
- b. How far does the price of Telecom stock have to fall for you to get a margin call if the maintenance margin is 30%? Assume the price fall happens immediately.

- 6)
  - a. \$8,000
  - b. 52% Therefore, the investor will not receive a margin call.
  - c. -41.5%
- 7)
  - a. The remaining margin is:  $\$20,000 - \$10,000 - \$2,000 = \$8,000$
  - b.  $\$8,000/\$50,000 = 0.16$ , or 16%. So there will be a margin call.
  - c.  $(-\$12,000/\$20,000) = -0.60$ , or -60%

10. You are bearish on Telecom and decide to sell short 100 shares at the current market price of \$50 per share.
- a. How much in cash or securities must you put into your brokerage account if the broker's initial margin requirement is 50% of the value of the short position?
  - b. How high can the price of the stock go before you get a margin call if the maintenance margin is 30% of the value of the short position?
11. Suppose that Xtel currently is selling at \$20 per share. You buy 1,000 shares using \$15,000 of your own money, borrowing the remainder of the purchase price from your broker. The rate on the margin loan is 8%.
- a. What is the percentage increase in the net worth of your brokerage account if the price of Xtel *immediately* changes to: (i) \$22; (ii) \$20; (iii) \$18? What is the relationship between your percentage return and the percentage change in the price of Xtel?
  - b. If the maintenance margin is 25%, how low can Xtel's price fall before you get a margin call?
  - c. How would your answer to (b) change if you had financed the initial purchase with only \$10,000 of your own money?
  - d. What is the rate of return on your margined position (assuming again that you invest \$15,000 of your own money) if Xtel is selling *after 1 year* at: (i) \$22; (ii) \$20; (iii) \$18? What is the relationship between your percentage return and the percentage change in the price of Xtel? Assume that Xtel pays no dividends.
  - e. Continue to assume that a year has passed. How low can Xtel's price fall before you get a margin call?
12. Suppose that you sell short 1,000 shares of Xtel, currently selling for \$20 per share, and give your broker \$15,000 to establish your margin account.
- a. If you earn no interest on the funds in your margin account, what will be your rate of return after one year if Xtel stock is selling at: (i) \$22; (ii) \$20; (iii) \$18? Assume that Xtel pays no dividends.
  - b. If the maintenance margin is 25%, how high can Xtel's price rise before you get a margin call?
  - c. Redo parts (a) and (b), but now assume that Xtel also has paid a year-end dividend of \$1 per share. The prices in part (a) should be interpreted as ex-dividend, that is, prices after the dividend has been paid.
13. Here is some price information on Marriott:

	Bid	Ask
Marriott	69.95	70.05

You have placed a stop-loss order to sell at \$70. What are you telling your broker? Given market prices, will your order be executed?

- 10) a. 50% of \$5,000, or \$2,500.  
 b. A margin call will be issued when:

$$\frac{\$7,500 - 100P}{100P} = 0.30 \Rightarrow \text{when } P = \$57.69 \text{ or higher}$$

11) The total cost of the purchase is:  $\$20 \times 1,000 = \$20,000$   
 You borrow \$5,000 from your broker and invest \$15,000 of your own funds. Your margin account starts out with equity of \$15,000.

- a. (i) Equity increases to:  $(\$22 \times 1,000) - \$5,000 = \$17,000$   
 Percentage gain =  $\$2,000/\$15,000 = 0.1333$ , or 13.33%  
 (ii) With price unchanged, equity is unchanged.  
 Percentage gain = zero  
 (iii) Equity falls to  $(\$18 \times 1,000) - \$5,000 = \$13,000$   
 Percentage gain =  $(-\$2,000/\$15,000) = -0.1333$ , or -13.33%

The relationship between the percentage return and the percentage change in the price of the stock is given by:

$$\% \text{ return} = \% \text{ change in price} \times \frac{\text{Total investment}}{\text{Investor's initial equity}} = \% \text{ change in price} \times 1.333$$

For example, when the stock price rises from \$20 to \$22, the percentage change in price is 10%, while the percentage gain for the investor is:

$$\% \text{ return} = 10\% \times \frac{\$20,000}{\$15,000} = 13.33\%$$

- b. The value of the 1,000 shares is  $1,000P$ . Equity is  $(1,000P - \$5,000)$ . You will receive a margin call when:

$$\frac{1,000P - \$5,000}{1,000P} = 0.25 \Rightarrow \text{when } P = \$6.67 \text{ or lower}$$

- c. The value of the 1,000 shares is  $1,000P$ . But now you have borrowed \$10,000 instead of \$5,000. Therefore, equity is  $(1,000P - \$10,000)$ . You will receive a margin call when:

$$\frac{1,000P - \$10,000}{1,000P} = 0.25 \Rightarrow \text{when } P = \$13.33 \text{ or lower}$$

With less equity in the account, you are far more vulnerable to a margin call.

- b. By the end of the year, the amount of the loan owed to the broker grows to:

$$\$5,000 \times 1.08 = \$5,400$$

The equity in your account is  $(1,000P - \$5,400)$ . Initial equity was \$15,000. Therefore, your rate of return after one year is as follows:

- (i)  $\frac{(1,000 \times \$22) - \$5,400 - \$15,000}{\$15,000} = 0.1067$ , or 10.67%
- (ii)  $\frac{(1,000 \times \$20) - \$5,400 - \$15,000}{\$15,000} = -0.0267$ , or -2.67%
- (iii)  $\frac{(1,000 \times \$18) - \$5,400 - \$15,000}{\$15,000} = -0.1600$ , or -16.00%

The relationship between the percentage return and the percentage change in the price of Xtel is given by:

$$\% \text{ return} = \left( \% \text{ change in price} \times \frac{\text{Total investment}}{\text{Investor's initial equity}} \right) - \left( 8\% \times \frac{\text{Funds borrowed}}{\text{Investor's initial equity}} \right)$$

For example, when the stock price rises from \$40 to \$44, the percentage change in price is 10%, while the percentage gain for the investor is:

$$\left( 10\% \times \frac{\$20,000}{\$15,000} \right) - \left( 8\% \times \frac{\$5,000}{\$15,000} \right) = 10.67\%$$

- e. The value of the 1000 shares is  $1,000P$ . Equity is  $(1,000P - \$5,400)$ . You will receive a margin call when:

$$\frac{1,000P - \$5,400}{1,000P} = 0.25 \Rightarrow \text{when } P = \$7.20 \text{ or lower}$$

12)

- a. The gain or loss on the short position is:  $(-1,000 \times \Delta P)$   
Invested funds = \$15,000

Therefore: rate of return =  $(-1,000 \times \Delta P)/15,000$

The rate of return in each of the three scenarios is:

- (i) Rate of return =  $(-1,000 \times \$2)/\$15,000 = -0.1333$ , or -13.33%
- (ii) Rate of return =  $(-1,000 \times \$0)/\$15,000 = 0\%$
- (iii) Rate of return =  $[-1,000 \times (-\$2)]/\$15,000 = +0.1333$ , or +13.33%

- b. Total assets in the margin account equal:

\$20,000 (from the sale of the stock) + \$15,000 (the initial margin) = \$35,000

Liabilities are  $500P$ . You will receive a margin call when:

$$\frac{\$35,000 - 1,000P}{1,000P} = 0.25 \Rightarrow \text{when } P = \$28 \text{ or higher}$$

- c. With a \$1 dividend, the short position must now pay on the borrowed shares:  $(\$1/\text{share} \times 1000 \text{ shares}) = \$1000$ . Rate of return is now:

$$[(-1,000 \times \Delta P) - 1,000]/15,000$$

- (i) Rate of return =  $[(-1,000 \times \$2) - \$1,000]/\$15,000 = -0.2000$ , or  $-20.00\%$
- (ii) Rate of return =  $[(-1,000 \times \$0) - \$1,000]/\$15,000 = -0.0667$ , or  $-6.67\%$
- (iii) Rate of return =  $[(-1,000) \times (-\$2) - \$1,000]/\$15,000 = +0.067$ , or  $+6.67\%$

Total assets are \$35,000, and liabilities are  $(1,000P + 1,000)$ . A margin call will be issued when:

$$\frac{35,000 - 1,000P - 1,000}{1,000P} = 0.25 \Rightarrow \text{when } P = \$27.2 \text{ or higher}$$

13) The broker is instructed to attempt to sell your Marriott stock as soon as the Marriott stock trades at a bid price of \$70 or less. Here, the broker will attempt to execute but may not be able to sell at \$70, since the bid price is now \$69.95. The price at which you sell may be more or less than \$70 because the stop-loss becomes a market order to sell at current market prices.

14. Here is some price information on FinCorp stock. Suppose that FinCorp trades in a dealer market.

Bid	Ask
55.25	55.50

- a. Suppose you have submitted an order to your broker to buy at market. At what price will your trade be executed?
  - b. Suppose you have submitted an order to sell at market. At what price will your trade be executed?
  - c. Suppose you have submitted a limit order to sell at \$55.62. What will happen?
  - d. Suppose you have submitted a limit order to buy at \$55.37. What will happen?
15. You've borrowed \$20,000 on margin to buy shares in Ixnay, which is now selling at \$40 per share. Your account starts at the initial margin requirement of 50%. The maintenance margin is 35%. Two days later, the stock price falls to \$35 per share.
- a. Will you receive a margin call?
  - b. How low can the price of Ixnay shares fall before you receive a margin call?
16. On January 1, you sold short one round lot (i.e., 100 shares) of Four Sisters stock at \$21 per share. On March 1, a dividend of \$2 per share was paid. On April 1, you covered the short sale by buying the stock at a price of \$15 per share. You paid 50 cents per share in commissions for each transaction. What is the value of your account on April 1?

14) a. \$55.50

b. \$55.25

c. The trade will not be executed because the bid price is lower than the price specified in the limit-sell order.

d. The trade will not be executed because the asked price is greater than the price specified in the limit-buy order.

15) a. You will not receive a margin call. You borrowed \$20,000 and with another \$20,000 of your own equity you bought 1,000 shares of Ixnay at \$40 per share. At

\$35 per share, the market value of the stock is \$35,000, your equity is \$15,000, and the percentage margin is:  $\$15,000/\$35,000 = 42.9\%$   
 Your percentage margin exceeds the required maintenance margin.

b. You will receive a margin call when:

$$\frac{1,000P - \$20,000}{1,000P} = 0.35 \Rightarrow \text{when } P = \$30.77 \text{ or lower}$$

16) The value of your account is equal to the net profit on the transaction:

$$\$2,050 - \$200 - \$1,550 = \$300$$

1. FBN Inc. has just sold 100,000 shares in an initial public offering. The underwriter's explicit fees were \$70,000. The offering price for the shares was \$50, but immediately upon issue, the share price jumped to \$53.
  - a. What is your best guess as to the total cost to FBN of the equity issue?
  - b. Is the entire cost of the underwriting a source of profit to the underwriters?
2. If you place a stop-loss order to sell 100 shares of stock at \$55 when the current price is \$62, how much will you receive for each share if the price drops to \$50?
  - a. \$50.
  - b. \$55.
  - c. \$54.87.
  - d. Cannot tell from the information given.
3. Specialists on the New York Stock Exchange do all of the following *except*:
  - a. Act as dealers for their own accounts.
  - b. Execute limit orders.
  - c. Help provide liquidity to the marketplace.
  - d. Act as odd-lot dealers.

1) a. Total costs of \$370,000.  
 b. No. The underwriters do not capture the part of the costs corresponding to the underpricing. The underpricing may be a rational marketing strategy. Without it, the underwriters would need to spend more resources in order to place the issue with the public. The underwriters would then need to charge higher explicit fees to the issuing firm. The issuing firm may be just as well off paying the implicit issuance cost represented by the underpricing.

2) (d) The broker will sell, at current market price, after the first transaction at \$55 or less.

3) (d)

### Lecture 3 Mutual funds and other investment companies

7. An open-end fund has a net asset value of \$10.70 per share. It is sold with a front-end load of 6%. What is the offering price?
8. If the offering price of an open-end fund is \$12.30 per share and the fund is sold with a front-end load of 5%, what is its net asset value?
9. The composition of the Fingroup Fund portfolio is as follows:

Stock	Shares	Price
A	200,000	\$35
B	300,000	40
C	400,000	20
D	600,000	25

The fund has not borrowed any funds, but its accrued management fee with the portfolio manager currently totals \$30,000. There are 4 million shares outstanding. What is the net asset value of the fund?

$$7) \text{ Offering price} = \frac{\text{NAV}}{1 - \text{Load}} = \frac{\$10.70}{1 - 0.06} = \$11.38$$

$$8) \$12.30 \times .95 = \$11.69$$

$$9) \text{ Net asset value} = \frac{\$42,000,000 - \$30,000}{4,000,000} = \$10.49$$

11. The Closed Fund is a closed-end investment company with a portfolio currently worth \$200 million. It has liabilities of \$3 million and 5 million shares outstanding.
  - a. What is the NAV of the fund?
  - b. If the fund sells for \$36 per share, what is its premium or discount as a percent of net asset value?
12. Corporate Fund started the year with a net asset value of \$12.50. By year-end, its NAV equaled \$12.10. The fund paid year-end distributions of income and capital gains of \$1.50. What was the (pretax) rate of return to an investor in the fund?
13. A closed-end fund starts the year with a net asset value of \$12.00. By year-end, NAV equals \$12.10. At the beginning of the year, the fund was selling at a 2% premium to NAV. By the end of the year, the fund is selling at a 7% discount to NAV. The fund paid year-end distributions of income and capital gains of \$1.50.
  - a. What is the rate of return to an investor in the fund during the year?
  - b. What would have been the rate of return to an investor who held the same securities as the fund manager during the year?
14.
  - a. Impressive Fund had excellent investment performance last year, with portfolio returns that placed it in the top 10% of all funds with the same investment policy. Do you expect it to be a top performer next year? Why or why not?
  - b. Suppose instead that the fund was among the poorest performers in its comparison group. Would you be more or less likely to believe its relative performance will persist into the following year? Why?

15. Consider a mutual fund with \$200 million in assets at the start of the year and 10 million shares outstanding. The fund invests in a portfolio of stocks that provides dividend income at the end of the year of \$2 million. The stocks included in the fund's portfolio increase in price by 8%, but no securities are sold and there are no capital gains distributions. The fund charges 12b-1 fees of 1%, which are deducted from portfolio assets at year-end. What is the fund's net asset value at the start and end of the year? What is the rate of return for an investor in the fund?

$$11) \text{ Premium (or discount)} = \frac{\text{Price} - \text{NAV}}{\text{NAV}} = \frac{\$36 - \$39.40}{\$39.40} = -0.086, \text{ or } -8.6\%$$

12)

$$\frac{\text{NAV}_1 - \text{NAV}_0 + \text{Distributions}}{\text{NAV}_0} = \frac{\$12.10 - \$12.50 + \$1.50}{\$12.50} = 0.088, \text{ or } 8.8\%$$

13)

a. Rate of return =

$$\frac{P_1 - P_0 + \text{Distributions}}{P_0} = \frac{\$11.25 - \$12.24 + \$1.50}{\$12.24} = 0.042, \text{ or } 4.2\%$$

b. 
$$\frac{\text{NAV}_1 - \text{NAV}_0 + \text{Distributions}}{\text{NAV}_0} = \frac{\$12.10 - \$12.00 + \$1.50}{\$12.00} = 0.133, \text{ or } 13.3\%$$

$$15) \text{ Rate of return} = \frac{\$21.384 - \$20 + \$0.20}{\$20} = 0.0792, \text{ or } 7.92\%$$

18. You purchased 1,000 shares of the New Fund at a price of \$20 per share at the beginning of the year. You paid a front-end load of 4%. The securities in which the fund invests increase in value by 12% during the year. The fund's expense ratio is 1.2%. What is your rate of return on the fund if you sell your shares at the end of the year?

19. Loaded-Up Fund charges a 12b-1 fee of 1.0% and maintains an expense ratio of .75%. Economy Fund charges a front-end load of 2% but has no 12b-1 fee and an expense ratio of .25%. Assume the rate of return on both funds' portfolios (before any fees) is 6% per year. How much will an investment in each fund grow to after:

- a. 1 year?  
b. 3 years?  
c. 10 years?

18) The rate of return is:  $(\$22,160 - \$20,833)/\$20,833 = 6.37\%$ .

19)

Assume \$1,000 investment	Loaded-Up Fund	Economy Fund
Yearly growth ( <i>r</i> is 6%)	$(1 + r - .01 - .0075)$	$(.98) \times (1 + r - .0025)$
<i>t</i> = 1 year	\$1,042.50	\$1,036.35
<i>t</i> = 3 years	\$1,133.00	\$1,158.96
<i>t</i> = 10 years	\$1,516.21	\$1,714.08

21. The Investments Fund sells Class A shares with a front-end load of 6% and Class B shares with 12b-1 fees of .5% annually as well as back-end load fees that start at 5% and fall by 1% for each full year the investor holds the portfolio (until the fifth year). Assume the portfolio rate of return net of operating expenses is 10% annually. If you plan to sell the fund after 4 years, are Class A or Class B shares the better choice for you? What if you plan to sell after 15 years?
22. You are considering an investment in a mutual fund with a 4% load and an expense ratio of .5%. You can invest instead in a bank CD paying 6% interest.
- If you plan to invest for 2 years, what annual rate of return must the fund portfolio earn for you to be better off in the fund than in the CD? Assume annual compounding of returns.
  - How does your answer change if you plan to invest for 6 years? Why does your answer change?
  - Now suppose that instead of a front-end load the fund assesses a 12b-1 fee of .75% per year. What annual rate of return must the fund portfolio earn for you to be better off in the fund than in the CD? Does your answer in this case depend on your time horizon?

- 21) Suppose you have \$1,000 to invest. The initial investment in Class A shares is \$940 net of the front-end load. After four years, your portfolio will be worth:

$$\$940 \times (1.10)^4 = \$1,376.25$$

Class B shares allow you to invest the full \$1,000, but your investment performance net of 12b-1 fees will be only 9.5%, and you will pay a 1% back-end load fee if you sell after four years. Your portfolio value after four years will be:

$$\$1,000 \times (1.095)^4 = \$1,437.66$$

After paying the back-end load fee, your portfolio value will be:

$$\$1,437.66 \times .99 = \$1,423.28$$

Class B shares are the better choice if your horizon is four years.

With a 15-year horizon, the Class A shares will be worth:

$$\$940 \times (1.10)^{15} = \$3,926.61$$

For the Class B shares, there is no back-end load in this case since the horizon is greater than five years. Therefore, the value of the Class B shares will be:

$$\$1,000 \times (1.095)^{15} = \$3,901.32$$

At this longer horizon, Class B shares are no longer the better choice. The effect of Class B's 0.5% 12b-1 fees accumulates over time and finally overwhelms the 6% load charged to Class A investors.

- 22) a. After two years, each dollar invested in a fund with a 4% load and a portfolio return equal to  $r$  will grow to:  $\$0.96 \times (1 + r - 0.005)^2$ .

Each dollar invested in the bank CD will grow to:  $\$1 \times 1.06^2$ .

If the mutual fund is to be the better investment, then the portfolio return ( $r$ ) must satisfy:

$$0.96 \times (1 + r - 0.005)^2 > 1.06^2$$

$$0.96 \times (1 + r - 0.005)^2 > 1.1236$$

$$(1 + r - 0.005)^2 > 1.1704$$

$$1 + r - 0.005 > 1.0819$$

$$1 + r > 1.0869$$

Therefore:  $r > 0.0869 = 8.69\%$

- b. If you invest for six years, then the portfolio return must satisfy:

$$0.96 \times (1 + r - 0.005)^6 > 1.06^6 = 1.4185$$

$$(1 + r - 0.005)^6 > 1.4776$$

$$1 + r - 0.005 > 1.0672$$

$$r > 7.22\%$$

The cutoff rate of return is lower for the six-year investment because the “fixed cost” (the one-time front-end load) is spread over a greater number of years.

- c. With a 12b-1 fee instead of a front-end load, the portfolio must earn a rate of return ( $r$ ) that satisfies:

$$1 + r - 0.005 - 0.0075 > 1.06$$

In this case,  $r$  must exceed 7.25% regardless of the investment horizon.

## Lecture 4 Learning about Return and Risk from the Historical Record

3. You are considering two alternative two-year investments: You can invest in a risky asset with a positive risk premium and returns in each of the two years that will be identically distributed and uncorrelated, or you can invest in the risky asset for only one year and then invest the proceeds in a risk-free asset. Which of the following statements about the first investment alternative (compared with the second) are true?

- Its two-year risk premium is the same as the second alternative.
  - The standard deviation of its two-year return is the same.
  - Its annualized standard deviation is lower.
  - Its Sharpe ratio is higher.
  - It is relatively more attractive to investors who have lower degrees of risk aversion.
6. You are considering the choice between investing \$50,000 in a conventional 1-year bank CD offering an interest rate of 5% and a 1-year "Inflation-Plus" CD offering 1.5% per year plus the rate of inflation.
- Which is the safer investment?
  - Can you tell which offers the higher expected return?
  - If you expect the rate of inflation to be 3% over the next year, which is the better investment? Why?
  - If we observe a risk-free nominal interest rate of 5% per year and a risk-free real rate of 1.5% on inflation-indexed bonds, can we infer that the market's expected rate of inflation is 3.5% per year?

7. Suppose your expectations regarding the stock price are as follows:

State of the Market	Probability	Ending Price	HPR (including dividends)
Boom	0.35	\$140	44.5%
Normal growth	0.30	110	14.0
Recession	0.35	80	-16.5

Use Equations 5.11 and 5.12 to compute the mean and standard deviation of the HPR on stocks.

8. Derive the probability distribution of the 1-year HPR on a 30-year U.S. Treasury bond with an 8% coupon if it is currently selling at par and the probability distribution of its yield to maturity a year from now is as follows:

State of the Economy	Probability	YTM
Boom	0.20	11.0%
Normal growth	0.50	8.0
Recession	0.30	7.0

For simplicity, assume the entire 8% coupon is paid at the end of the year rather than every 6 months.

9. Determine the standard deviation of a random variable  $q$  with the following probability distribution:

Value of $q$	Probability
0	0.25
1	0.25
2	0.50

$$7) \quad E(r) = [0.35 \times 44.5\%] + [0.30 \times 14.0\%] + [0.35 \times (-16.5\%)] = 14\%$$

$$\sigma^2 = [0.35 \times (44.5 - 14)^2] + [0.30 \times (14 - 14)^2] + [0.35 \times (-16.5 - 14)^2] = 651.175$$

$$\sigma = 25.52\%$$

- 8) Probability distribution of price and one-year holding period return for a 30-year U.S. Treasury bond (which will have 29 years to maturity at year-end):

Economy	Probability	YTM	Price	Capital Gain	Coupon Interest	HPR
Boom	0.20	11.0%	\$ 74.05	-\$25.95	\$8.00	-17.95%
Normal growth	0.50	8.0	100.00	0.00	8.00	8.00
Recession	0.30	7.0	112.28	12.28	8.00	20.28

9)  $E(q) = (0 \times 0.25) + (1 \times 0.25) + (2 \times 0.50) = 1.25$

$$\sigma_q = [0.25 \times (0 - 1.25)^2 + 0.25 \times (1 - 1.25)^2 + 0.50 \times (2 - 1.25)^2]^{1/2} = 0.8292$$

13. During a period of severe inflation, a bond offered a nominal HPR of 80% per year. The inflation rate was 70% per year.
- What was the real HPR on the bond over the year?
  - Compare this real HPR to the approximation  $r_{\text{real}} \approx r_{\text{nom}} - i$ .
14. Suppose that the inflation rate is expected to be 3% in the near future. Using the historical data provided in this chapter, what would be your predictions for:
- The T-bill rate?
  - The expected rate of return on the Big/Value portfolio?
  - The risk premium on the stock market?
15. An economy is making a rapid recovery from steep recession, and businesses foresee a need for large amounts of capital investment. Why would this development affect real interest rates?
16. You are faced with the probability distribution of the HPR on the stock market index fund given in Spreadsheet 5.1 of the text. Suppose the price of a put option on a share of the index fund with exercise price of \$110 and time to expiration of 1 year is \$12.
- What is the probability distribution of the HPR on the put option?
  - What is the probability distribution of the HPR on a portfolio consisting of one share of the index fund and a put option?
  - In what sense does buying the put option constitute a purchase of insurance in this case?
17. Take as given the conditions described in the previous problem, and suppose the risk-free interest rate is 6% per year. You are contemplating investing \$107.55 in a 1-year CD and simultaneously buying a call option on the stock market index fund with an exercise price of \$110 and expiration of 1 year. What is the probability distribution of your dollar return at the end of the year?

13) a  $r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + i} - 1 = \frac{r_{\text{nominal}} - i}{1 + i} = \frac{0.80 - 0.70}{1.70} = 0.0588, \text{ or } 5.88\%$

b.  $r_{\text{nominal}} - i = .80 - .70 = .10 \approx r_{\text{real}}$

Clearly, the approximation gives a real HPR that is too high.

- 14) From Table 5.2, the average real rate on T-bills has been 0.56%.

a. T-bills:  $0.56\% \text{ real rate} + 3\% \text{ inflation} = 3.56\%$

b. Expected return on Big/Value:

$3.56\% \text{ T-bill rate} + 11.67\% \text{ historical risk premium} = 15.23\%$

c. The risk premium on stocks remains unchanged. A premium, the difference between two rates, is a real value, unaffected by inflation.

15) Real interest rates are expected to rise. The investment activity will shift the demand for funds curve (in Figure 5.1) to the right. Therefore the equilibrium real interest rate will increase.

17) The probability distribution of the dollar return on CD plus call option is:

State of the Economy	Probability	Ending Value of CD	Ending Value of Call	Combined Value
Excellent	0.25	\$ 114.00	\$16.50	\$130.50
Good	0.45	114.00	0.00	114.00
Poor	0.25	114.00	0.00	114.00
Crash	0.05	114.00	0.00	114.00

1. Given \$100,000 to invest, what is the expected risk premium in dollars of investing in equities versus risk-free T-bills (U.S. Treasury bills) based on the following table?

Action	Probability	Expected Return
Invest in equities	0.6	\$50,000
	0.4	-\$30,000
Invest in risk-free T-bill	1.0	\$ 5,000

2. Based on the scenarios below, what is the expected return for a portfolio with the following return profile?

	Bear Market	Normal Market	Bull Market
Probability	0.2	0.3	0.5
Rate of return	-25%	10%	24%

Use the following scenario analysis for Stocks X and Y to answer CFA Problems 3 through 6 (round to the nearest percent).

	Bear Market	Normal Market	Bull Market
Probability	0.2	0.5	0.3
Stock X	-20%	18%	50%
Stock Y	-15%	20%	10%

- What are the expected rates of return for Stocks X and Y?
- What are the standard deviations of returns on Stocks X and Y?
- Assume that of your \$10,000 portfolio, you invest \$9,000 in Stock X and \$1,000 in Stock Y. What is the expected return on your portfolio?
- Probabilities for three states of the economy and probabilities for the returns on a particular stock in each state are shown in the table below.

State of Economy	Probability of Economic State	Stock Performance	Probability of Stock Performance in Given Economic State
Good	0.3	Good	0.6
		Neutral	0.3
		Poor	0.1
Neutral	0.5	Good	0.4
		Neutral	0.3
		Poor	0.3
Poor	0.2	Good	0.2
		Neutral	0.3
		Poor	0.5

What is the probability that the economy will be neutral *and* the stock will experience poor performance?

- An analyst estimates that a stock has the following probabilities of return depending on the state of the economy:

State of Economy	Probability	Return
Good	0.1	15%
Normal	0.6	13
Poor	0.3	7

What is the expected return of the stock?

- The expected risk premium is \$13,000.
- $E(r) = [0.2 \times (-25\%)] + [0.3 \times 10\%] + [0.5 \times 24\%] = 10\%$
- $E(r_X) = [0.2 \times (-20\%)] + [0.5 \times 18\%] + [0.3 \times 50\%] = 20\%$   
 $E(r_Y) = [0.2 \times (-15\%)] + [0.5 \times 20\%] + [0.3 \times 10\%] = 10\%$

4)  $\sigma_x^2 = [0.2 \times (-20 - 20)^2] + [0.5 \times (18 - 20)^2] + [0.3 \times (50 - 20)^2] = 592$

$\sigma_x = 24.33\%$

$\sigma_y^2 = [0.2 \times (-15 - 10)^2] + [0.5 \times (20 - 10)^2] + [0.3 \times (10 - 10)^2] = 175$

$\sigma_y = 13.23\%$

5.  $E(r) = (0.9 \times 20\%) + (0.1 \times 10\%) = 19\% \rightarrow \$1,900$  in returns

6. The probability of both poor stock performance and a neutral economy is therefore:

$0.30 \times 0.50 = 0.15 = 15\%$

7.  $E(r) = (0.1 \times 15\%) + (0.6 \times 13\%) + (0.3 \times 7\%) = 11.4\%$

## Lecture 5 Risk Aversion and Capital Allocation to Risky Assets

1. Which of the following choices best completes the following statement? Explain. An investor with a higher degree of risk aversion, compared to one with a lower degree, will most prefer investment portfolios
  - a. with higher risk premiums.
  - b. that are riskier (with higher standard deviations).
  - c. with lower Sharpe ratios.
  - d. with higher Sharpe ratios.

1) (d)

2. Which of the following statements are true? Explain.
  - a. A lower allocation to the risky portfolio reduces the Sharpe (reward-to-volatility) ratio.
  - b. The higher the borrowing rate, the lower the Sharpe ratios of levered portfolios.
  - c. With a fixed risk-free rate, doubling the expected return and standard deviation of the risky portfolio will double the Sharpe ratio.
  - d. Holding constant the risk premium of the risky portfolio, a higher risk-free rate will increase the Sharpe ratio of investments with a positive allocation to the risky asset.

2) (b)

4. Consider a risky portfolio. The end-of-year cash flow derived from the portfolio will be either \$70,000 or \$200,000 with equal probabilities of .5. The alternative risk-free investment in T-bills pays 6% per year.
  - a. If you require a risk premium of 8%, how much will you be willing to pay for the portfolio?
  - b. Suppose that the portfolio can be purchased for the amount you found in (a). What will be the expected rate of return on the portfolio?
  - c. Now suppose that you require a risk premium of 12%. What is the price that you will be willing to pay?
  - d. Comparing your answers to (a) and (c), what do you conclude about the relationship between the required risk premium on a portfolio and the price at which the portfolio will sell?
5. Consider a portfolio that offers an expected rate of return of 12% and a standard deviation of 18%. T-bills offer a risk-free 7% rate of return. What is the maximum level of risk aversion for which the risky portfolio is still preferred to T-bills?
6. Draw the indifference curve in the expected return–standard deviation plane corresponding to a utility level of .05 for an investor with a risk aversion coefficient of 3. (*Hint*: Choose several possible standard deviations, ranging from 0 to .25, and find the expected rates of return providing a utility level of .05. Then plot the expected return–standard deviation points so derived.)
7. Now draw the indifference curve corresponding to a utility level of .05 for an investor with risk aversion coefficient  $A = 4$ . Comparing your answer to Problem 6, what do you conclude?
8. Draw an indifference curve for a risk-neutral investor providing utility level .05.
9. What must be true about the sign of the risk aversion coefficient,  $A$ , for a risk lover? Draw the indifference curve for a utility level of .05 for a risk lover.

- 4) a. The present value of the portfolio is:  $\$135,000/1.14 = \$118,421$   
 b.  $E(r) = 14\%$ .  
 c. The required return is:  $6\% + 12\% = 18\%$   
 d. For a given expected cash flow, portfolios that command greater risk premiums must sell at lower prices. The extra discount from expected value is a penalty for risk.

5) In order for the risky portfolio to be preferred to bills, the following must hold:

$$0.12 - 0.0162A > 0.07 \Rightarrow A < 0.05/0.0162 = 3.09$$

**For Problems 10 through 12:** Consider historical data showing that the average annual rate of return on the S&P 500 portfolio over the past 90 years has averaged roughly 8% more than the Treasury bill return and that the S&P 500 standard deviation has been about 20% per year. Assume these values are representative of investors' expectations for future performance and that the current T-bill rate is 5%.

10. Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 index with weights as follows:

$W_{\text{Bills}}$	$W_{\text{Index}}$
0	1.0
0.2	0.8
0.4	0.6
0.6	0.4
0.8	0.2
1.0	0

11. Calculate the utility levels of each portfolio of Problem 10 for an investor with  $A = 2$ . What do you conclude?  
 12. Repeat Problem 11 for an investor with  $A = 3$ . What do you conclude?

10) The portfolio expected return and variance are computed as follows:

(1) $W_{\text{Bills}}$	(2) $r_{\text{Bills}}$	(3) $W_{\text{Index}}$	(4) $r_{\text{Index}}$	$r_{\text{Portfolio}}$ (1)×(2)+(3)×(4)	$\sigma_{\text{Portfolio}}$ (3) × 20%	$\sigma^2_{\text{Portfolio}}$
0.0	5%	1.0	13.0%	13.0% = 0.130	20% = 0.20	0.0400
0.2	5	0.8	13.0	11.4% = 0.114	16% = 0.16	0.0256
0.4	5	0.6	13.0	9.8% = 0.098	12% = 0.12	0.0144
0.6	5	0.4	13.0	8.2% = 0.082	8% = 0.08	0.0064
0.8	5	0.2	13.0	6.6% = 0.066	4% = 0.04	0.0016
1.0	5	0.0	13.0	5.0% = 0.050	0% = 0.00	0.0000

- 11) Computing utility from  $U = E(r) - 0.5 \times A\sigma^2 = E(r) - \sigma^2$ , we arrive at the values in the column labeled  $U(A = 2)$  in the following table:

$W_{\text{Bills}}$	$W_{\text{Index}}$	$r_{\text{Portfolio}}$	$\sigma_{\text{Portfolio}}$	$\sigma^2_{\text{Portfolio}}$	$U(A = 2)$	$U(A = 3)$
0.0	1.0	0.130	0.20	0.0400	0.0900	.0700
0.2	0.8	0.114	0.16	0.0256	0.0884	.0756
0.4	0.6	0.098	0.12	0.0144	0.0836	.0764
0.6	0.4	0.082	0.08	0.0064	0.0756	.0724
0.8	0.2	0.066	0.04	0.0016	0.0644	.0636
1.0	0.0	0.050	0.00	0.0000	0.0500	.0500

The column labeled  $U(A = 2)$  implies that investors with  $A = 2$  prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

- 12) The column labeled  $U(A = 3)$  in the table above is computed from:

$$U = E(r) - 0.5A\sigma^2 = E(r) - 1.5\sigma^2$$

The more risk averse investors prefer the portfolio that is invested 40% in the market, rather than the 100% market weight preferred by investors with  $A = 2$ .

**For Problems 23 through 26:** Suppose that the borrowing rate that your client faces is 9%. Assume that the equity market index has an expected return of 13% and standard deviation of 25%, that  $r_f = 5\%$ , and that your fund has the parameters given in Problem 21.

23. Draw a diagram of your client's CML, accounting for the higher borrowing rate. Superimpose on it two sets of indifference curves, one for a client who will choose to borrow, and one for a client who will invest in both the index fund and a money market fund.
24. What is the range of risk aversion for which a client will neither borrow nor lend, that is, for which  $y = 1$ ?
25. Solve Problems 23 and 24 for a client who uses your fund rather than an index fund.
26. What is the largest percentage fee that a client who currently is lending ( $y < 1$ ) will be willing to pay to invest in your fund? What about a client who is borrowing ( $y > 1$ )?

**For Problems 27 through 29:** You estimate that a passive portfolio, for example, one invested in a risky portfolio that mimics the S&P 500 stock index, yields an expected rate of return of 13% with a standard deviation of 25%. You manage an active portfolio with expected return 18% and standard deviation 28%. The risk-free rate is 8%.

27. Draw the CML and your funds' CAL on an expected return–standard deviation diagram.
  - a. What is the slope of the CML?
  - b. Characterize in one short paragraph the advantage of your fund over the passive fund.
28. Your client ponders whether to switch the 70% that is invested in your fund to the passive portfolio.
  - a. Explain to your client the disadvantage of the switch.
  - b. Show him the maximum fee you could charge (as a percentage of the investment in your fund, deducted at the end of the year) that would leave him at least as well off investing in your fund as in the passive one. (*Hint:* The fee will lower the slope of his CAL by reducing the expected return net of the fee.)
29. Consider again the client in Problem 19 with  $A = 3.5$ .
  - a. If he chose to invest in the passive portfolio, what proportion,  $y$ , would he select?
  - b. Is the fee (percentage of the investment in your fund, deducted at the end of the year) that you can charge to make the client indifferent between your fund and the passive strategy affected by his capital allocation decision (i.e., his choice of  $y$ )?

27) a. Slope of the CML =  $\frac{.13 - .08}{.25} = 0.20$

- 28) a. With 70% of his money invested in my fund's portfolio, the client's expected return is 15% per year with a standard deviation of 19.6% per year. If he shifts that money to the passive portfolio (which has an expected return of 13% and standard deviation of 25%), his overall expected return becomes:

$$E(r_C) = r_f + 0.7 \times [E(r_M) - r_f] = .08 + [0.7 \times (.13 - .08)] = .115, \text{ or } 11.5\%$$

The standard deviation of the complete portfolio using the passive portfolio would be:

$$\sigma_C = 0.7 \times \sigma_M = 0.7 \times 25\% = 17.5\%$$

Therefore, the shift entails a decrease in mean from 15% to 11.5% and a decrease in standard deviation from 19.6% to 17.5%. Since both mean return *and* standard deviation decrease, it is not yet clear whether the move is beneficial. The disadvantage of the shift is that, if the client is willing to accept a mean return on his total portfolio of 11.5%, he can achieve it with a lower standard deviation using my fund rather than the passive portfolio.

To achieve a target mean of 11.5%, we first write the mean of the complete portfolio as a function of the proportion invested in my fund ( $y$ ):

$$E(r_C) = .08 + y \times (.18 - .08) = .08 + .10 \times y$$

Our target is:  $E(r_C) = 11.5\%$ . Therefore, the proportion that must be invested in my fund is determined as follows:

$$.115 = .08 + .10 \times y \Rightarrow y = \frac{.115 - .08}{.10} = 0.35$$

The standard deviation of this portfolio would be:

$$\sigma_C = y \times 28\% = 0.35 \times 28\% = 9.8\%$$

Thus, by using my portfolio, the same 11.5% expected return can be achieved with a standard deviation of only 9.8% as opposed to the standard deviation of 17.5% using the passive portfolio.

- b. The fee would reduce the reward-to-volatility ratio, i.e., the slope of the CAL. The client will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let  $f$  denote the fee:

$$\text{Slope of CAL with fee} = \frac{.18 - .08 - f}{.28} = \frac{.10 - f}{.28}$$

$$\text{Slope of CML (which requires no fee)} = \frac{.13 - .08}{.25} = 0.20$$

Setting these slopes equal we have:

$$\frac{.10 - f}{.28} = 0.20 \Rightarrow f = 0.044 = 4.4\% \text{ per year}$$

29. a. The formula for the optimal proportion to invest in the passive portfolio is:

$$y^* = \frac{E(r_M) - r_f}{A\sigma_M^2}$$

Substitute the following:  $E(r_M) = 13\%$ ;  $r_f = 8\%$ ;  $\sigma_M = 25\%$ ;  $A = 3.5$ :

$$y^* = \frac{0.13 - 0.08}{3.5 \times 0.25^2} = 0.2286, \text{ or } 22.86\% \text{ in the passive portfolio}$$

- b. The answer here is the same as the answer to Problem 28(b). The fee that you can charge a client is the same regardless of the asset allocation mix of the client's portfolio. You can charge a fee that will equate the reward-to-volatility *ratio* of your portfolio to that of your competition.

## Lecture The Capital Asset Pricing Model

9. Consider the following table, which gives a security analyst's expected return on two stocks in two particular scenarios for the rate of return on the market:

Market Return	Aggressive Stock	Defensive Stock
5%	-2%	6%
25	38	12

- What are the betas of the two stocks?
  - What is the expected rate of return on each stock if the two scenarios for the market return are equally likely?
  - If the T-bill rate is 6% and the market return is equally likely to be 5% or 25%, draw the SML for this economy.
  - Plot the two securities on the SML graph. What are the alphas of each?
  - What hurdle rate should be used by the management of the aggressive firm for a project with the risk characteristics of the defensive firm's stock?
- 9) a. Call the aggressive stock A and the defensive stock D. Beta is the sensitivity of the stock's return to the market return, i.e., the change in the stock return per unit change in the market return. Therefore, we compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in the market return:

$$\beta_A = \frac{-0.02 - .38}{.05 - .25} = 2.00 \quad \beta_D = \frac{.06 - .12}{.05 - .25} = 0.30$$

- b. With the two scenarios equally likely, the expected return is an average of the two possible outcomes:

$$E(r_A) = 0.5 \times (-.02 + .38) = .18 = 18\%$$

$$E(r_D) = 0.5 \times (.06 + .12) = .09 = 9\%$$

- c. The SML is determined by the market expected return of  $[0.5 \times (.25 + .05)] = 15\%$ , with  $\beta_M = 1$ , and  $r_f = 6\%$  (which has  $\beta_f = 0$ ). The equation for the security market line is:

$$E(r) = .06 + \beta \times (.15 - .06)$$

**For Problems 10 through 16:** If the simple CAPM is valid, which of the following situations are possible? Explain. Consider each situation independently.

10. Portfolio	Expected Return	Beta
A	20%	1.4
B	25%	1.2

11. Portfolio	Expected Return	Standard Deviation
A	30%	35%
B	40%	25%

12. Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18%	24%
A	16%	12%

13. Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18%	24%
A	20%	22%

10) Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower than the expected return for Portfolio B. Thus, these two portfolios cannot exist in equilibrium.

11) Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk, represented by beta, rather than for the standard deviation, which includes nonsystematic risk. Thus, Portfolio A's lower rate of return can be paired with a higher standard deviation, as long as A's beta is less than B's.

12) Not possible. The reward-to-variability ratio for Portfolio A is better than that of the market. This scenario is impossible according to the CAPM because the CAPM predicts that the market is the most efficient portfolio. Using the numbers supplied:

$$S_A = \frac{.16 - .10}{.12} = 0.5 \quad S_M = \frac{.18 - .10}{.24} = 0.33$$

Portfolio A provides a better risk-reward trade-off than the market portfolio.

13) Not possible. Portfolio A clearly dominates the market portfolio. Portfolio A has both a lower standard deviation and a higher expected return.

**For Problems 17 through 19:** Assume that the risk-free rate of interest is 6% and the expected rate of return on the market is 16%.

17. A share of stock sells for \$50 today. It will pay a dividend of \$6 per share at the end of the year. Its beta is 1.2. What do investors expect the stock to sell for at the end of the year?
18. I am buying a firm with an expected perpetual cash flow of \$1,000 but am unsure of its risk. If I think the beta of the firm is .5, when in fact the beta is really 1, how much *more* will I offer for the firm than it is truly worth?
19. A stock has an expected rate of return of 4%. What is its beta?
20. Two investment advisers are comparing performance. One averaged a 19% rate of return and the other a 16% rate of return. However, the beta of the first investor was 1.5, whereas that of the second investor was 1.
  - a. Can you tell which investor was a better selector of individual stocks (aside from the issue of general movements in the market)?
  - b. If the T-bill rate was 6% and the market return during the period was 14%, which investor would be considered the superior stock selector?
  - c. What if the T-bill rate was 3% and the market return was 15%?

17) Since the stock's beta is equal to 1.2, its expected rate of return is:

$$.06 + [1.2 \times (.16 - .06)] = 18\%$$

$$E(r) = \frac{D_1 + P_1 - P_0}{P_0} \rightarrow 0.18 = \frac{P_1 - \$50 + \$6}{\$50} \rightarrow P_1 = \$53$$

18) The series of \$1,000 payments is a perpetuity. If beta is 0.5, the cash flow should be discounted at the rate:

$$.06 + [0.5 \times (.16 - .06)] = .11 = 11\%$$

$$PV = \$1,000/0.11 = \$9,090.91$$

If, however, beta is equal to 1, then the investment should yield 16%, and the price paid for the firm should be:

$$PV = \$1,000/0.16 = \$6,250$$

The difference, \$2,840.91, is the amount you will overpay if you erroneously assume that beta is 0.5 rather than 1.

19) Using the SML:  $.04 = .06 + \beta \times (.16 - .06) \Rightarrow \beta = -.02/.10 = -0.2$

20)  $r_1 = 19\%$ ;  $r_2 = 16\%$ ;  $\beta_1 = 1.5$ ;  $\beta_2 = 1$

a. To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b. If  $r_f = 6\%$  and  $r_M = 14\%$ , then (using the notation alpha for the abnormal return):

$$\alpha_1 = .19 - [.06 + 1.5 \times (.14 - .06)] = .19 - .18 = 1\%$$

$$\alpha_2 = .16 - [.06 + 1 \times (.14 - .06)] = .16 - .14 = 2\%$$

Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

- c. If  $r_f = 3\%$  and  $r_M = 15\%$ , then:

$$\alpha_1 = .19 - [.03 + 1.5 \times (.15 - .03)] = .19 - .21 = -2\%$$

$$\alpha_2 = .16 - [.03 + 1 \times (.15 - .03)] = .16 - .15 = 1\%$$

Here, not only does the second investor appear to be the superior stock selector, but the first investor's predictions appear valueless (or worse).