



B.E. International Program

Faculty of Economics, Thammasat University



EE 320 Introductory Mathematical Economics

Semester 1/2016

In-Class Exercise 1 - Answers

Given the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$$

1. Find the determinant of A.

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix} = (1)(-1)^{1+1} \begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} + (1)(-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = -3$$

2. Find the inverse matrix of A.

$$A^{-1} = \frac{1}{|A|} \text{Adjoint}(A) = \frac{1}{-3} \begin{bmatrix} -2 & -2 & -1 \\ -1 & -1 & 1 \\ -1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 2/3 & 1/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & -1/3 & -1/3 \end{bmatrix}$$

3. Given $\mathbf{b} = \begin{bmatrix} 15 \\ 5 \\ 0 \end{bmatrix}$, find the matrix multiplication $A^{-1}\mathbf{b}$.

$$A^{-1}\mathbf{b} = \begin{bmatrix} 2/3 & 1/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & -1/3 & -1/3 \end{bmatrix} \begin{bmatrix} 15 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 35/3 \\ 35/3 \\ 10/3 \end{bmatrix}$$

4. Use Cramer's rule to solve the following system of equation, $AX = \mathbf{b}$.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \\ 0 \end{bmatrix}$$

$$x = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 15 & 0 & 1 \\ 5 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}} = \frac{-35}{-3} = \frac{35}{3}$$

$$y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 1 & 15 & 1 \\ 0 & 5 & -2 \\ 1 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}} = \frac{-35}{-3} = \frac{35}{3}$$

$$z = \frac{|A_3|}{|A|} = \frac{\begin{vmatrix} 1 & 0 & 15 \\ 0 & 1 & 5 \\ 1 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}} = \frac{-10}{-3} = \frac{10}{3}$$