1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

HW

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^*.

$$G_d = G_s$$
$$a - bP - cY = d + eP - fT$$
$$-cY = d - a + P(e-b) - fT$$
$$Y^* = -\frac{d-a+P(e-b)-fT}{c}$$
$$Y_d^* = Y^* - T_0$$
$$= -\frac{d-a+P(e-b)-fT}{c} - T_0$$
$$C^* = a + bY_d^*$$
$$C^* = a + -(b)\frac{d-a+P(e-b)-fT}{c} - T_0$$

$$Y^* = C + I + G$$
$$Y^* = a + bY_d + I_0 + G_0$$
$$Y_d^* = Y^* - T_0$$
$$C^* = a + bY_d^*$$

HW

- Numerically, if a = 1, T_0 = \$0 I_0 = \$1, G_0 = \$1 and b = 0.5, this yields us,

$$C^* = a + bY_d^*$$
$$C^* = 1 + (0.5)Y_d^* \quad \text{— (1)}$$
$$Y_d^* = Y^* - T_0$$
$$Y_d^* = Y^* - 0$$
$$Y^* = Y_d \quad \text{— (2)}$$

$$Y^* = C + I + G$$
$$Y^* = 1 + (0.5)Y^* + (1) + (1)$$
$$0.5Y^* = 3$$
$$Y^* = 6$$

* ANS

Question: What if G is now changed to \$2, how big is the change in Y^* and C^*?

HW (ส่งเมล/ทักตา .หนี้

Answer:

New Y^* = 8 ➔ $Y^* = 1 + 0.5Y^* + (1) + (2) = 8$

A \$1 increase in G causes an increase in Y^* by 2

In the later part, we will try to figure out the value of *multiplier* when we don't assign any numerical values of exogenous variables.

- The idea is simple. *We just apply the derivative method to the equilibrium solution function.*

- That is, we calculate the value of $\frac{\partial Y^*}{\partial I_0}, \frac{\partial Y^*}{\partial G_0}, \frac{\partial Y^*}{\partial a}, \frac{\partial Y^*}{\partial b}$.

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that
(i) elasticity of supply is always greater than 1 if $c > 0$,
(ii) elasticity of supply is always eqaul to 1 if $c = 0$,
(iii) elasticity of supply is always less to 1 if $c < 0$.

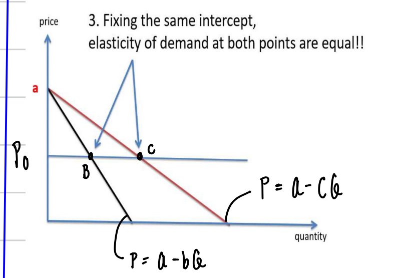
2.A.1) $P = a - bQ \rightarrow$ inverse demand.

$$Q = \left(-\frac{1}{b}\right)(P) + \frac{a}{b}$$

$$PED = \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$$

$$= \left(-\frac{1}{b}\right) \times \frac{P_0}{Q_0} \;//$$

: show the third property hold.



$$PED_B = PED_C$$

$$\left(-\frac{1}{b}\right)\left(\frac{P_0}{Q_0}\right) = \left(-\frac{1}{c}\right)\left(\frac{P_0}{Q_0}\right) \text{ ; At same } P_0$$

$$\left(-\frac{1}{\cancel{b}}\right)\left(\cancel{P_0} \times \frac{\cancel{b}}{\cancel{(a-P_0)}}\right) = \left(-\frac{1}{\cancel{c}}\right)\left(\cancel{P_0} \times \frac{\cancel{c}}{\cancel{(a-P_0)}}\right)$$

$$-1 = -1$$

// Ans

sol $P = a - bQ \rightarrow$ "black line"

$P = a - cQ \rightarrow$ "red line"

to prof the 3rd property $\rightarrow PED_B = PED_C$

PED_B: $P = a - bQ$

$$Q = \left(-\frac{1}{b}\right)(P) + \frac{a}{b}$$

$$\therefore PED_B = \frac{\Delta Q}{\Delta P} \cdot \frac{P_0}{Q_0}$$

$$= \left(-\frac{1}{b}\right)\left(\frac{P_0}{Q_0}\right)$$

PED_C: $P = a - cQ$

$$Q = \left(-\frac{1}{c}\right)(P) + \frac{a}{b}$$

$$PED_C = \left(-\frac{1}{c}\right) \cdot \frac{P_0}{Q_0}$$

2.A.2 supply $= P = C + dQ$; $d \geq 0$

P intercept

(i) elasticity of supply is always greater than 1 if $c > 0$,

$$PES \text{ ; } Q = \frac{P - C}{d} = \frac{1}{d}(P) - \frac{c}{d}$$

if $c > 0$; $PES = \frac{\Delta Q}{\Delta P} \cdot \frac{P_0}{Q_0}$

$$Q_0 = \frac{P - C}{d}$$

$$= \left(\frac{1}{d}\right) \cdot \frac{P_0}{\frac{P_0 - C}{d}}$$

$$= \left(\frac{1}{\cancel{d}}\right)(P_0)\left(\frac{\cancel{d}}{(P_0 - C)}\right) \; (+)$$

$$PES = \frac{P_0}{P_0 - C}$$

$\therefore P_0 > P_0 - C$ $\therefore$ elasticity of supply is greater than 1 $c > 0$

(ii) elasticity of supply is always eqaul to 1 if $c = 0$,

if $c > 0$; $PES = \frac{\Delta Q}{\Delta P} \cdot \frac{P_0}{Q_0}$

$$= \left(\frac{1}{d}\right) \cdot \frac{P_0}{\frac{P_0 - C}{d}}$$

$$= \left(\frac{1}{\cancel{d}}\right)(P_0)\left(\frac{\cancel{d}}{(P_0 - C)}\right) \; C = 0$$

$$PES = \frac{P_0}{P_0} = 1$$

$\therefore$ elasticity of supply is equal to 1 when $c = 0$

(iii) elasticity of supply is always less to 1 if $c < 0$.

if $c > 0$; $PES = \frac{\Delta Q}{\Delta P} \cdot \frac{P_0}{Q_0}$

$$= \left(\frac{1}{d}\right) \cdot \frac{P_0}{\frac{P_0 - C}{d}}$$

$$= \left(\frac{1}{\cancel{d}}\right)(P_0)\left(\frac{\cancel{d}}{(P_0 - C)}\right) \; (-)$$

$$PES = \frac{P_0}{P_0 + C}$$

; $P_0 + C > P_0$

$\therefore$ Elasticity of supply is lower than 1 when $c < 0$

Example 2.I: A monopolist firm faces the market demand given by $\underbrace{P = 10 - Q}_{P(Q)}$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

revenue = $TR(Q) = P(Q) \times Q$

$= (10 - Q) \times Q$

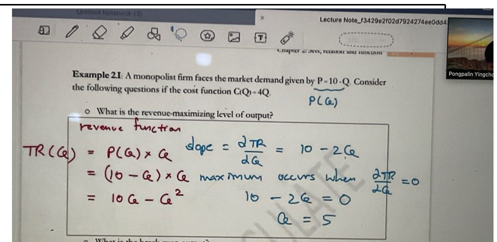
$TR(Q) = 10Q - Q^2 \leftarrow$ Diff.

$0 = 10 - 2Q \leftarrow$ slope $= 0$

$-10 = -2Q$

$Q = 5 \rightarrow \therefore$ MAX Revenue : $P = 10 - 5 = 5$

$TR = P(Q)(Q) = rev = 25$ //



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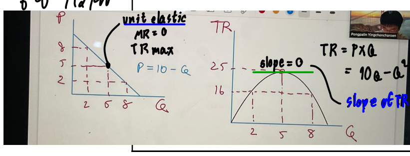


slope of TR $= \frac{\Delta TR}{\Delta Q} = MR$ | Thus, revenue max condition is $MR = 0$ //

- What is the break-even output?

คือ $\rightarrow \pi = 0 \rightarrow TR = TC$

$TR = TC$

$P(Q)(Q) = C(Q)(Q)$

$(10 - Q)\cancel{(Q)} = 4(Q)\cancel{(Q)}$

$10 - Q = 4Q$

$10 = 5Q$

$Q = 2$ //

- What is the profit-maximizing level of output?

sol^1

$MR = MC$

$MR \rightarrow \frac{\partial TR}{\partial Q} = 10Q - Q^2 = 10 - 2Q$

$MC = \frac{\partial TC}{\partial Q} = 4Q^2 = 8Q$

$\therefore MR = MC$

$10 - 2Q = 8Q$

$10 = 10Q$

$Q = 1$

sol^2

$\pi = TR - TC$

$= P(Q)(Q) - C(Q)(Q)$

$= (10 - Q)(Q) - 4Q(Q)$

$= 10Q - Q^2 - 4Q^2$

$\pi = 10Q - 5Q^2$

max $\pi \rightarrow \frac{\partial \pi}{\partial Q} = 10 - 10Q = 0$

$10 = 10Q$

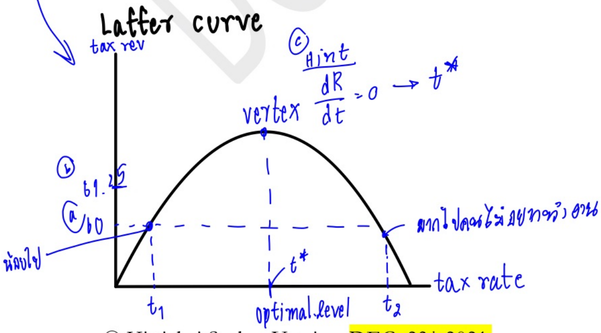
$Q = 1$ //

Exrcise 2B. Consider a function that relates tax revenues R, in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to $60 billion?

(b) What tax rate(s) is consistent with raising tax revenues equal to $61.25 billion?

(c) What tax rate is consistent with the maximum tax revenue?



a) $R = 350t - 500t^2$

$60 \text{ billion} = 350t - 500t^2$

$= 500t^2 - 350t + 60$

$= 500t^2 - 350t + 60$

$= 100t^2 - 70t + 12$

$= (10t - 4)(10t - 3)$

$\therefore t = \frac{4}{10}, \frac{3}{10}$

$t_1 = \frac{3}{10} = 0.3 \quad t_2 = \frac{4}{10} = 0.4$

b) $R = 350t - 500t^2$

$61.25 = 350t - 500t^2$

$= 500t^2 - 350t + 61.25$

$= 500t^2 - 350t + \frac{245}{4}$

$= \frac{1}{4}(2000t^2 - 1400t + 245)$

$= \frac{1}{4} \times 5(400t^2 - 280t + 49)$

$= \frac{1}{4} \times 5(20t - 7)^2$

$= 1.25(20t - 7)^2$

$t = \frac{7}{20}$

Maximize tax revenue.

c) $R = 350t - 500t^2$

$\frac{dR}{dt} = 350 - 1000t = 0$

$350 = 1000t$

$t^* = \frac{35\cancel{0}}{100\cancel{0}}$

$t^* = 0.35$