

Outline

1. Properties of Preferences
2. From Preferences to Utility (and viceversa)
3. Common Utility Functions
4. Utility maximization

1 Properties of Preferences II

- Nicholson, Ch. 3, pp. 89-90
- Commodity set X (apples vs. strawberries, work vs. leisure, consume today vs. tomorrow)
- Preference relation \succeq over X
- A preference relation \succeq is *rational* if
 1. It is *complete*: For all x and y in X , either $x \succeq y$, or $y \succeq x$ or both
 2. It is *transitive*: For all x , y , and z , $x \succeq y$ and $y \succeq z$ implies $x \succeq z$
- Preference relation \succeq is *continuous* if for all y in X , the sets $\{x : x \succeq y\}$ and $\{x : y \succeq x\}$ are closed sets.

- Example 2: choice of combinations of apples and oranges: $X = \{(1, 0), (0, 1), (1, 1), (0, 0)\}$

- Example 2: $X = \mathbb{R}^2$ with map of indifference curves

- Indifference relation \sim : $x \sim y$ if $x \succeq y$ and $y \succeq x$
- Strict preference: $x \succ y$ if $x \succeq y$ and not $y \succeq x$
- Exercise. If \succeq is rational,
 - \succ is transitive
 - \sim is transitive
 - Reflexive property of \succeq . For all x , $x \succeq x$.

- Other features of preferences
- Preference relation \succeq is:
 - *monotonic* if $x \geq y$ implies $x \succeq y$.
 - *strictly monotonic* if $x \geq y$ and $x_j > y_j$ for some j implies $x \succ y$.
 - *convex* if for all $x, y,$ and z in X such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all t in $[0, 1]$

2 From preferences to utility

- Nicholson, Ch. 3
- Economists like to use utility functions $u : X \rightarrow R$
- $u(x)$ is 'liking' of good x
- $u(a) > u(b)$ means: I prefer a to b .
- **Def.** Utility function u represents preferences \succeq if, for all x and y in X , $x \succeq y$ if and only if $u(x) \geq u(y)$.
- **Theorem.** If preference relation \succeq is rational and continuous, there exists a continuous utility function $u : X \rightarrow R$ that represents it.

- [Skip proof]

- Example:

$$(x_1, x_2) \succeq (y_1, y_2) \text{ iff } x_1 + x_2 \geq y_1 + y_2$$

- Draw:

- Utility function that represents it: $u(x) = x_1 + x_2$

- But... Utility function representing \succeq is not unique

- Take $3u(x)$ or $\exp(u(x))$

- $u(a) > u(b) \iff \exp(u(a)) > \exp(u(b))$

- If $u(x)$ represents preferences \succeq and f is a strictly increasing function, then $f(u(x))$ represents \succeq as well.

- If preferences are represented from a utility function, are they rational?
 - completeness
 - transitivity

- Indifference curves: $u(x_1, x_2) = \bar{u}$
- They are just implicit functions! $u(x_1, x_2) - \bar{u} = 0$

$$\frac{dx_2}{dx_1} = -\frac{U'_{x_1}}{U'_{x_2}} = MRS$$

- Indifference curves for:
 - monotonic preferences;
 - strictly monotonic preferences;
 - convex preferences

3 Common utility functions

- Nicholson, Ch. 3, pp. 102-105

1. Cobb-Douglas preferences: $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$

- $MRS = -\alpha x_1^{\alpha-1} x_2^{1-\alpha} / (1-\alpha) x_1^\alpha x_2^{-\alpha} = -\frac{\alpha}{1-\alpha} \frac{x_2}{x_1}$

2. Perfect substitutes: $u(x_1, x_2) = \alpha x_1 + \beta x_2$

- $MRS = -\alpha/\beta$

3. Perfect complements: $u(x_1, x_2) = \min(\alpha x_1, \beta x_2)$

- MRS discontinuous at $x_2 = \frac{\alpha}{\beta}x_1$

4. Constant Elasticity of Substitution: $u(x_1, x_2) = (\alpha x_1^\rho + \beta x_2^\rho)^{1/\rho}$

- $MRS = -\frac{\alpha}{\beta} \left(\frac{x_1}{x_2}\right)^{\rho-1}$
- if $\rho = 1$, then...
- if $\rho = 0$, then...
- if $\rho \rightarrow -\infty$, then...

4 Utility Maximization

- Nicholson, Ch. 4, pp. 119–128
- $X = R_+^2$ (2 goods)
- Consumers: choose bundle $x = (x_1, x_2)$ in X which yields highest utility.
- Constraint: income = M
- Price of good 1 = p_1 , price of good 2 = p_2
- Bundle x is feasible if $p_1x_1 + p_2x_2 \leq M$
- Consumer maximizes

$$\begin{aligned} & \max_{x_1, x_2} u(x_1, x_2) \\ & s.t. \quad p_1x_1 + p_2x_2 \leq M \\ & \quad x_1 \geq 0, \quad x_2 \geq 0 \end{aligned}$$

- Maximization subject to inequality. How do we solve that?
- Trick: u strictly increasing in at least one dimension. (\succeq strictly monotonic)
- Budget constraint always satisfied with equality
- Ignore temporarily $x_1 \geq 0, x_2 \geq 0$ and check afterwards that they are satisfied for x_1^* and x_2^* .

- Problem becomes

$$\begin{aligned} & \max_{x_1, x_2} u(x_1, x_2) \\ & s.t. \quad p_1 x_1 + p_2 x_2 - M = 0 \end{aligned}$$