

ESS = Explained Sum of Squares =  $\sum (\hat{y}_i - \bar{y})^2$

RSS = Residual Sum of Squares =  $\sum u_i^2 = \sum (y_i - \hat{y}_i)^2$

~~= Regression Sum of Squares = RSS~~  
~~= Error Sum of Squares = ESS~~

$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_k X_{ki} + u_i$   $i=1, 2, \dots, n$

$i=1$	$Y_1$	$=$	$\beta_1 + \beta_2 X_{21} + \beta_3 X_{31} + \dots + \beta_k X_{k1} + u_1$
$i=2$	$Y_2$	$=$	$\beta_1 + \beta_2 X_{22} + \beta_3 X_{32} + \dots + \beta_k X_{k2} + u_2$
$\vdots$	$\vdots$		$\vdots$
$i=n$	$Y_n$	$=$	$\beta_1 + \beta_2 X_{2n} + \beta_3 X_{3n} + \dots + \beta_k X_{kn} + u_n$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{21} & X_{31} & \dots & X_{k1} \\ 1 & X_{22} & X_{32} & \dots & X_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{2n} & X_{3n} & \dots & X_{kn} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$\Rightarrow Y = X\beta + u$

$\hat{\beta} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = (X'X)^{-1} X'Y$

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Nested Models.

$$\begin{aligned} \checkmark \textcircled{1} & Y_i = \beta_1 + \beta_2 X_{2i} + u_i \\ \textcircled{2} & Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \\ \checkmark \textcircled{3} & Y_i = \beta_1 + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i \end{aligned} \quad \left. \vphantom{\begin{aligned} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{aligned}} \right\} R_{\textcircled{2}}^2 \geq R_{\textcircled{1}}^2$$

$$\begin{aligned} \textcircled{4} & Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \\ \textcircled{5} & Y_i = \beta_1 + \beta_2 X_{2i} + \beta_4 X_{4i} + \beta_5 X_{5i} + u_i \\ \textcircled{6} & Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i \end{aligned}$$

$$R_{\textcircled{6}}^2 \geq R_{\textcircled{4}}^2$$

$$\textcircled{1} \quad Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \quad \leftarrow \text{Variation}$$

$$\textcircled{1} \quad \underline{Y_i} = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \quad \leftarrow \text{Variation in } Y_i$$

$$\textcircled{2} \quad \underline{\ln Y_i} = \beta_1 + \beta_2 X_{2i} + \beta_4 X_{4i} + \beta_5 X_{5i} + u_i$$

$\downarrow$  Variation in  $\ln Y_i$

$$\textcircled{3} \quad Y_i = \beta_1 + \beta_2 X_{2i} + \beta_4 X_{4i} + u_i$$