

Limited Dependent Variable Models

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Part 3

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Marginal effects

- ▶ For nonlinear models, the marginal effect (ME) varies with the point of evaluation
- ▶ 3 common choices of evaluation:
 1. AME: average marginal effect (average of ME at each $x = x_i$)
 2. MEM: marginal effect at mean (ME at $x = \bar{x}$)
 3. MER: marginal effect at a representative value (ME at $x = x^*$)
- ▶ AME: margins, dydx^*
- ▶ MEM: margins, dydx^* atmean
- ▶ MER: margins, dydx^* at(female =1 age=40)

Marginal effects

- ▶ If the conditional mean is of the single-index form:
 $E(y|x) = m(x\beta)$, then $ME_j = m'(x\beta) \cdot \beta_j$
 - ▶ If $m(x\beta)$ is monotonically increasing, $m'(x\beta) > 0$. The sign of $\hat{\beta}_j$ gives the sign of the ME.
 - ▶ For any function $m(\cdot)$ and at any value of x , we have $\frac{ME_j}{ME_k} = \frac{\beta_j}{\beta_k}$

Multinomial models

Probabilities and MEs

- ▶ The outcome, y_i , for individual i is one of m alternatives.
- ▶ Set $y_j = j$ if the outcome is the j th alternative, $j = 1, 2, \dots, m$
- ▶ The probability that the outcome for individual i is alternative j , conditional on the regressors $\mathbf{x}_j$, is
$$P_{ij} = P(y_i = j) = F_j(\mathbf{x}_i, \beta), j=1, \dots, m, i = 1, \dots, n$$
 - ▶ where functional form, $F_j(\cdot)$, corresponds to multinomial models, e.g. multinomial logit model
- ▶ Only $m-1$ of the probabilities can be freely specified because probabilities sum to 1. Multinomial models therefore require a normalization

Multinomial models

Probabilities and MEs

- ▶ The parameters of multinomial models are generally not directly interpretable.
- ▶ We need marginal effects (MEs). For individual i , the ME of a change in the k th regressor on the probability that alternative j is the outcome is
$$ME_{ijk} = \frac{\partial P(y_i=j)}{\partial x_{ik}} = \frac{\partial F_j(x_i, \beta)}{\partial x_{ik}}$$
- ▶ For each regressor, there will be m MEs corresponding to the m probabilities, and these m MEs sum to zero because probabilities sum to one.

Additive random-utility model

Unordered multinomial outcomes

- ▶ For individual i and alternative j , we suppose that utility U_{ij} is the sum of deterministic component, V_{ij} , that depends on regressors and unknown parameters, and an unobserved random component ε_{ij} :

$$U_{ij} = V_{ij} + \varepsilon_{ij}$$

- ▶ This is called an additive random-utility model (ARUM).
- ▶ We observed the outcome $y_i = j$ if alternative j has the highest utility of the alternatives.

$$\begin{aligned} P(y_i = j) &= P(U_{ij} > U_{ik}), \text{ for all } k \\ &= P(\varepsilon_{ik} - \varepsilon_{ij} \leq V_{ij} - V_{ik}), \text{ for all } k \end{aligned}$$

- ▶ Different assumptions about the joint distribution of $\varepsilon_{i1}, \dots, \varepsilon_{im}$ lead to different multinomial models with different specifications for $F_j(\mathbf{x}_i, \beta)$

Multinomial logit model

- ▶ Datasets have only case-specific variables.
- ▶ Multinomial logit model (MNL): $P_{ij} = \frac{\exp(x_i \beta_j)}{\sum_{l=1}^m \exp(x_i \beta_l)}$, $j = 1, \dots, m$
 - ▶ $\sum_{j=1}^m P_{ij} = 1$
 - ▶ To ensure model identification, β_j is set to zero for one of the categories, and coefficients are then interpreted with respect to the based category
- ▶ For simplicity, we set the base category to be the first category. Then, the MNL model:
$$P(y_i = j | y_i = 1) = \frac{\exp(x_i \beta_j)}{1 + \sum_{l=2}^m \exp(x_i \beta_l)}$$

Multinomial logit model

- ▶ Thus, $\hat{\beta}_j$ can be viewed as parameters of a binary logit model between alternative j and alternative 1.
- ▶ So, a positive coefficient means that as the regressor increases, we are more likely to choose alternative j than alternative 1
- ▶ We can transform to odds ratios or relative-risk ratio of choosing alternative j rather than alternative 1 is
$$\frac{P(y_i=j)}{P(y_i=1)} = \exp(\mathbf{x}_i\beta_j)$$
- ▶ So, $e^{\beta_{ir}}$ gives the proportionate change in the relative risk of choosing alternative j rather than alternative 1 when x_{ir} changes by one unit.

Multinomial logit model

- ▶ For the MNL model, the MEs can be shown to be $\frac{\partial P_{ij}}{\partial x_i} = P_{ij} \cdot (\beta_j - \bar{\beta}_i)$
 - ▶ where $\bar{\beta}_i = \sum_l P_{il} \beta_l$ is a probability weighted average of the β_l
- ▶ The MEs vary with the point of evaluation, x_i , because P_{ij} varies with x_i
- ▶ The signs of the β_j s do not give the signs of the MEs.
- ▶ For a variable x , the ME is positive if $\beta_j > \bar{\beta}_i$

Ordered outcomes

- ▶ A latent variable crosses progressively higher thresholds.
- ▶ For an m -alternative ordered model, we define
$$y_i = j \text{ if } \alpha_{j-1} < y_i^* \leq \alpha_j, j = 1, \dots, m$$
where $\alpha_0 = -\infty$ and $\alpha_m = \infty$
- ▶ Then,
$$\begin{aligned} P(y_i = j) &= P(\alpha_{j-1} < x_i\beta + u_i \leq \alpha_j) \\ &= P(\alpha_{j-1} - x_i\beta < u_i \leq \alpha_j - x_i\beta) \\ &= F(\alpha_j - x_i\beta) - F(\alpha_{j-1} - x_i\beta) \end{aligned}$$
- ▶ The $m-1$ threshold parameters, $\alpha_1, \dots, \alpha_{m-1}$, are obtained by maximizing the log likelihood with $P(y_i = j)$

Ordered outcomes

- ▶ Ordered logit model: u is logistically distributed with $F(z) = e^z / (1 + e^z)$
- ▶ Ordered probit model: u is standard normally distributed with $F(\cdot) = \Phi(\cdot)$, the standard normal cdf
- ▶ The sign of β can be interpreted as determining whether the latent variable increases with the regressor.
- ▶ If $\beta_j > 0$, then an increase in x_{ij} will decrease the probability of being in the lowest category and increase the probability of being in the highest category.
- ▶ ME: $\frac{\partial P(y_i=j)}{\partial x_{ri}} = [F'(\alpha_j - x_i\beta) - F'(\alpha_{j-1} - x_i\beta)]\beta_r$