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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$\Sigma_i X_i = 621$ $\qquad \Sigma_i Y_i = 25.7$

$\bar{X} = \frac{621}{8} = 77.625$ $\qquad \bar{Y} = \frac{25.7}{8} = 3.2125$

$\Sigma_i X_i^2 = 48,717$

$\Sigma_i X_i Y_i = 2012.4$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\Sigma_i (Y_i - \bar{Y})(X_i - \bar{X})}{\Sigma_i (X_i - \bar{X})^2}$$

$$= \frac{\Sigma_i X_i (Y_i - \bar{Y})}{\Sigma_i X_i^2 - n\bar{X}^2}$$

$$= \frac{2012.4 - 3.2125(621)}{48717 - 8(77.625)^2}$$

$$= 0.0341$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= 3.2125 - 0.0341(77.625)$$

$$= 0.5655$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_i = 0.5655 + 0.0341 X_i$

Student	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$
1	2.7138	0.0862
2	3.0207	0.3793
3	3.2253	−0.2253
4	3.3276	0.1724
5	3.5322	0.0678
6	3.123	−0.123
7	3.123	−0.423
8	3.6345	0.0655

$\Sigma_i \hat{u}_i = 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0678 - 0.123 - 0.423 + 0.0655$

$= -0.0001 \approx 0$

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1.3 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \sigma^2 = \frac{\Sigma_i u_i^2}{n-2} = \frac{0.434726}{8-2} = \frac{0.434726}{6} = 0.0725$$

$$Var(\hat{\beta}_0) = \frac{\Sigma_i X_i^2}{n\,\Sigma_i (X_i-\bar{X})^2}\sigma^2 = \frac{48717}{8(511.875)}(0.0725) = 0.8625$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\Sigma_i (X_i-\bar{X})^2} = \frac{0.0725}{511.875} = 0.000142$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$\Sigma X_i = 200$

$\bar{X} = 20$

$\Sigma Y_i = 91$

$\bar{Y} = 9.1$

$\Sigma X_i^2 = 4440$

$\Sigma X_i Y_i = 2214$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\Sigma X_i (Y_i - \bar{Y})}{\Sigma X_i^2 - n\bar{X}^2}$$

$$= \frac{\Sigma_i X_i Y_i - \bar{Y}\,\Sigma_i X_i}{\Sigma X_i^2 - n\bar{X}^2}$$

$$= \frac{2214 - 9.1(200)}{4440 - 10(20)^2}$$

$$= 0.8955$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= 9.1 - (0.8955)(20)$$

$$= -8.81$$

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2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_i = -8.81 + 0.8955 X_i$

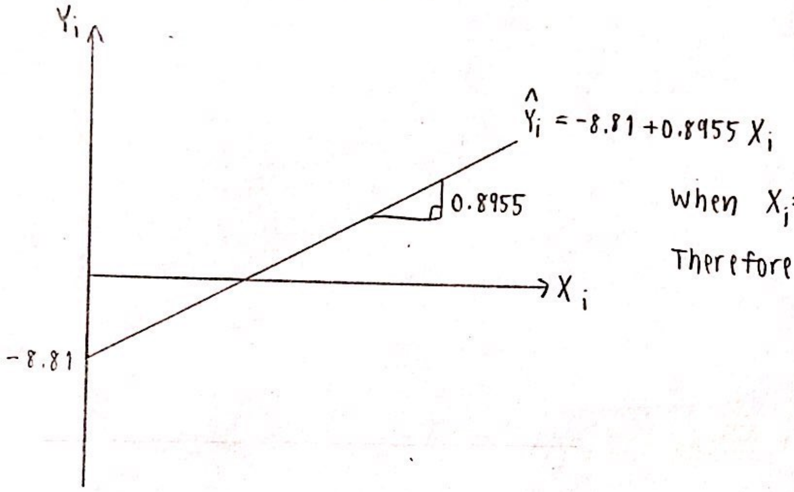
X_i	$\hat{Y}_i$	$\hat{u}_i$
10	0.195	-0.195
12	1.936	0.064
14	3.727	1.273
16	5.518	0.482
18	7.309	-0.309
22	10.891	-0.891
24	12.682	-2.682
26	14.473	0.527
28	16.264	-0.264
30	18.055	1.945

$\Sigma_i \hat{u}_i = -0.195 + 0.064 + 1.273 + 0.482 - 0.309 - 0.891 - 2.682 + 0.527 - 0.264 + 1.945$

$\Sigma_i \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

SRF : $\hat{Y}_i = -8.81 + 0.8955 X_i$



When $X_i = \bar{X} = 20$, $\hat{Y}_i = 9.1 = \bar{Y}$

Therefore, the regression line pass $(\bar{X}, \bar{Y})$.

2.4 If $X_i = 16$, what is the predicted Y?

$\hat{Y}_i = -8.81 + 0.8955(16) = 5.518$

2.5 Find $var(\hat{u}_i), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$Var(\hat{u}_i) = \sigma^2 = \frac{\Sigma_i u_i^2}{n-2} = \frac{14.09091}{10-2} = \frac{14.09091}{8} = 1.7614$

$Var(\hat{\beta}_0) = \frac{\Sigma_i X_i^2}{n \Sigma_i (X_i - \bar{X})^2} \sigma^2 = \frac{4440}{10(440)} (1.76137) = 1.7774$

$Var(\hat{\beta}_1) = \frac{\sigma^2}{\Sigma_i (X_i - \bar{X})^2} = \frac{1.7614}{440} = 0.004$

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3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\underset{\beta_1}{\text{argmin}} \sum_i \hat{u}_i^2 = \underset{\beta_1}{\text{argmin}} \sum_i (Y_i - \beta_1 X_i)^2$$

F.O.C.

$$0 = 2(-X_i)\sum_i (Y_i - \beta_1 X_i)$$

$$0 = -2 \sum_i (X_i Y_i - \beta_1 X_i^2)$$

$$0 = \sum_i X_i Y_i - \beta_1 \sum_i X_i^2$$

$$\boxed{\beta_1 = \frac{\sum_i X_i Y_i}{\sum_i X_i^2}}$$

From $\beta_1 = \frac{\sum_i X_i Y_i}{\sum_i X_i^2}$

$$E(\beta_1) = E\left(\frac{\sum_i X_i Y_i}{\sum_i X_i^2}\right)$$

$$= E\left(\frac{\sum_i X_i (\beta_1 X_i + u_i)}{\sum_i X_i^2}\right)$$

$$= E\left(\frac{\beta_1 \sum X_i^2 + u_i \sum_i X_i}{\sum_i X_i^2}\right)$$

$$= E\left(\frac{\beta_1 \sum_i X_i^2}{\sum_i X_i^2}\right) + E\left(\frac{u_i \sum_i X_i}{\sum_i X_i^2}\right) \quad ; \text{ use SLR 4}$$

$$= \beta_1 + 0$$

$$E(\beta_1) = \beta_1$$

Therefore, β_1 is an unbiased estimator.