FINAL 2011 MA217

1. Suppose that a house is situated in one corner of a rectangular plot of land 300 feet by 200 feet. The contour plot of land is given by the equation $E(x, y) = 10^{-4}\left(x^2 - \frac{xy}{2}\right)$. The house is situated at the origin, the *x*-axis is the 300 feet length and the *y*-axis is the 200 feet width. It is desired to level this property at the level of the house $\left(E(x, y) = 0\right)$.

(a) How much fill has to be removed or brought in to accomplish this? **State clearly if the soil is to be removed or brought and state the unit clearly.**

(b) In addition, what is the original average elevation of the properties from the level of the house $\left(E(x, y) = 0\right)$. **State clearly if the original average elevation above or below the level of the house state the unit clearly.**

(4 marks)

5. To fine a critical point of $f(x, y, z) = x^2 + xy + 2y^2 + z^2$ subjected to $x - 3y - 4z = 16$, a Lagrange multiplier equation can be setup as follow

$$L = x^2 + xy + 2y^2 + z^2 - \lambda\{x - 3y - 4z - 16\}$$

and hence consider the first derivatives

$L_x' = 2x + y - \lambda = 0$ (1)
$L_y' = x + 4y + 3\lambda = 0$ (2)
$L_z' = 2z + 4\lambda = 0$ (3)
$x - 3y - 4z = 16$ (4)

Equations (1) – (4) are a system of linear equation. **Use matrix** to solve this system of linear equations and show that your answer is correct.

(5 marks)

6. Find $\underline{\mathbf{A}}$ if $(\underline{\mathbf{A}}^{-1} - 3\underline{I})^T = \begin{bmatrix} 5 & 10 & \frac{7}{2} \\ 3 & 1 & \frac{3}{2} \\ 1 & 1 & -\frac{5}{2} \end{bmatrix}$.

(5 marks)

7. Given $\underline{\mathbf{A}} = \begin{bmatrix} 1 & 4 & -3 & 2 & -6 & 8 \\ 3 & -5 & 7 & -9 & 1 & 2 \\ -1 & 3 & -4 & 3 & 2 & 1 \\ -3 & 6 & 5 & 4 & 9 & 8 \\ 1 & 0 & 2 & -1 & 5 & 9 \end{bmatrix}$, $\underline{\mathbf{B}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & t^2 \end{bmatrix}$, $\underline{\mathbf{C}} = \begin{bmatrix} 0 & 0 & 8 & 2 \\ 0 & 0 & 1 & 0 \\ 7 & 1 & 2 & 5 \\ 2 & 0 & 0 & 1 \end{bmatrix}$,

$\underline{\mathbf{D}} = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -3 & 3 & 0 & 1 \\ 4 & 0 & 0 & 5 \\ 2 & 0 & 0 & 0 \end{bmatrix}$

(a) Determine det $\underline{\mathbf{A}}$

(b) What values of t which make the matrix $\underline{\mathbf{B}}$ singular?

(c) If $t = 2$, find det $\underline{\mathbf{B}}$, det ($\underline{\mathbf{B}}\,\underline{\mathbf{B}}^{\mathrm{T}}$) and det($\underline{\mathbf{B}}^{-1}\underline{\mathbf{B}}^{\mathrm{T}}$)

(d) Determine det $\underline{\mathbf{E}}$ if $\det(\underline{\mathbf{B}}^2\ \underline{\mathbf{C}}\,\underline{\mathbf{D}}^{-1}\underline{\mathbf{E}}^3) = \frac{4}{9}$ where $t = 2$ for $\underline{\mathbf{B}}$.

(5 marks)

8. Given $\underline{\mathbf{A}} = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 2 & 0 \\ 0 & 2 & -9 \end{bmatrix}$ and $(\underline{\mathbf{A}}\underline{\mathbf{B}})^T = \underline{\mathbf{I}}$,

(a) Find matrix $\underline{\mathbf{B}}$.

(b) If $\underline{\mathbf{D}} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$ and $\underline{\mathbf{D}}$ is invertible, solve $\left(\underline{\mathbf{A}}\underline{\mathbf{D}}^{-1}\right)^T \underline{\mathbf{x}} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ for $\underline{\mathbf{x}}$.

(5 marks)