EE 320 Introductory Mathematical Economics

Semester 1/2016

Homework 1

Due 13 September 2016

There are six questions in total. Each of them is worth equally.

1. Suppose that total cost function of Good Z is given by $C(Q) = 270 + 24Q$, where Q is the output level.

a. (4 points) If the price of Good Z is \$30 per unit. Derive the profit function of Good Z, and determine the break-even quantity, and illustrate by graph.

Ans. $\pi(Q) = 6Q - 270$

$Q_{BE} = 45$ units.

b. (2 points) Suppose that the producer of Good Z requires a minimum profit of \$2130. How many units of Good Z should the producer sell in the market?

Ans. Q = 400 units.

c. (4 points) Suppose now that the producer of Good Z becomes a monopolist and faces a given demand curve: $P = 61 - Q$. Derive the new total revenue and profit functions. What is the range of output should the producer produce in order to gain positive profits?

Ans. $TR(Q) = 61Q - Q^2$; $\pi(Q) = 37Q - Q^2 - 270$

For positive profit, $10 < Q < 27$.

2. General market equilibrium.

a. Consider the following system of equations:

$$Q_{d1} = a_0 + a_1 P_1 + a_2 P_2 + a_3 P_3 \; ; \qquad Q_{s1} = \alpha_0 + \alpha_1 P_1$$
$$Q_{d2} = b_0 + b_1 P_1 + b_2 P_2 \; ; \qquad Q_{s2} = \beta_0 + \beta_2 P_2$$
$$Q_{d3} = c_0 + c_1 P_1 + c_3 P_3 \; ; \qquad Q_{s3} = \gamma_0 + \gamma_3 P_3$$

where $a_i, b_i, c_i \; (for\; i = 1,2,3)$ and $\alpha_0, \alpha_1, \beta_0, \beta_2, \gamma_0, \gamma_3$ are parameters. If $a_2 > 0$ and $a_3 < 0$, what would be the signs of $b_1, b_2, c_1,$ and c_3? What are the relationships among the three goods?

Ans. Since $a_2 > 0$ and $a_3 < 0$, $b_1 > 0$ and $c_1 < 0$. Also, by the law of demand, both b_2 and c_3 are negative. We can tell that good 1 and good 2 are substitutes, and good 1 and good 3 are complements. However, we cannot tell the relationship between good 2 and good 3.

b. Consider the following system of equations:

$$Q_{d1} = 20 - P_1 + 2P_2 \; ; \qquad Q_{s1} = 2P_1 - 2$$
$$Q_{d2} = 18 + 3P_1 - 2P_2 \; ; \qquad Q_{s2} = 2 + 4P_2$$

Find the equilibrium price and quantity for the two goods.

$P_1 = 41/3; \; P_2 = 19/2$

$Q_1 = 76/3; \; Q_2 = 40$

3. Suppose the market demand function for an eco car for a group of consumers is given by: $Q = 30 + 0.1Y - 2P$, where P is the price of an eco car, and Y is the average income of these consumer (in \$1,000). Suppose also that the market supply function is: $P = 0.5Q + 15$.

a. (2 points) Is the car considered as normal or inferior goods to this group of consumers. Explain.

Ans. Normal goods.

b. (4 points) Derive the income elasticity of demand and the price elasticity of demand for this eco car.

Ans. $\varepsilon_d = -2P/Q$ and $\varepsilon_Y = 0.1Y/Q$

c. (4 points) Suppose that the average income is $15,000. Determine the equilibrium price and equilibrium quantity in this market.

Ans. (Q*,P*) = (750, 390)

4. Consider the following IS-LM model:

Commodity market:

$$Y = C + I + G_0$$
$$C = C_0 + bY, \qquad (C_0 > 0,\ 0 < b < 1)$$
$$I = I_0 - ar + iY, \qquad (I_0 > 0,\ a > 0,\ i > 0)$$

Money market:

$$M_S = M_0$$
$$M_D = mY - hr, \qquad (m > 0,\ h > 0)$$

a. (4 points) Write out the explicit IS-LM system of equations, and determine the equilibrium national income (Y) and equilibrium interest rate (r).

Ans. IS: $Y = \frac{C_o + I_0 + G_0 - ar}{(1 - b + i)}$

LM: $Y = \frac{M_0 + hr}{m}$

$Y^* = \frac{h(C_o + I_0 + G_0) + aM_0}{am + (1 - b + i)h}$ and $r^* = \frac{m(C_o + I_0 + G_0) - (1 - b + i)M_0}{am + (1 - b + i)h}$

b. (2 points) Find the impact of an exogenous increase in money supply (M_0) on the equilibrium national income found in part (a). Assume everything else remains constant.

Ans. $\frac{\Delta Y^*}{\Delta M_0} = \frac{a}{am + (1 - b + i)h}$

c. (2 points) Suppose that $C_0 = 150$, $I_0 = 100$, $G_0 = 50$, $b = 0.6$, $a = 400$, $h = 800$, $M_0 = 100$, $i = 0.1$ and $m = 0.4$. Find the equilibrium national income and interest rate.

Ans. $Y^* = 560$ and $r^* = 0.05$

d. (2 points) Based on the information in part (c), if the autonomous consumption (C_0) change by the amount λ ($\lambda > 0$), all else constant, what is the *change* in the equilibrium national income?

Ans. $\frac{\Delta Y}{\Delta C_0} = \frac{h}{am+(1-b+i)h} = 1.6$

If $\Delta C_0 = \lambda$, then $\Delta Y^* = 1.6\lambda$.

5. The demand and supply curves in the market for wine are given by the following equations:

Demand: $P = 40 - 0.25Q_d$

Supply: $Q_s = 2P + 4$

where Q is the quantity of wine in bottles and P is the price per bottle of wine.

Answer the following questions:

a. (1 point) Find the pre-tax equilibrium price and quantity, i.e. (P*,Q*).

Ans. (P*,Q*) = (26, 56)

b. (3 point) Calculate the price elasticities of demand and supply at the equilibrium. Based on the values of calculated elasticities, what would the total revenue change if the market price can be increased by 1%?

Ans. Elasticity of demand = (-4)*(26/56) = -13/7 = -1.86

Elasticity of supply = 2*(26/56) = 13/14 = 0.93

If the price increased by 1%, the quantity demanded will *decrease* by 1.86%. Since TR = PxQ, the 1% increase in price and the 1.86% decrease in quantity will result in a reduction in total revenue.

Now suppose that the government in this economy has levied an excise tax of $6 on the *producers* of wine.

c. (2 points) Find the post-tax equilibrium prices and quantity.

Ans. Set Ps = Pd-6 ⇒ 160-4Pd = 2(Pd-6) + 4.

⇒ After-tax equilibrium quantity = 48 units; after-tax equilibrium price for consumer = $28; after-tax equilibrium price for producer = $22.

d. (2 point) How much total tax revenue can government collect?

Ans. Tax revenue =t*Q = $6*48 = $288

e. (2 point) In terms of economic incidence, what percentages of the tax are borne by consumers and producers?

Ans.

Per-unit tax burden for consumer = (28-26)/6 = ⅓ or 33.33%

Per-unit tax burden for producer = (26-22)/6 = ⅔ or 66.67%

Since producer's supply is relatively more inelastic than consumer's demand, producer would bear more tax burden.

6. AIS, TRUE and DTAC all want to purchase some Samsung-S7 phones from Samsung inc. and hold these phones in their stock so that they can promote their new contract with the Samsung-S7. Their respective demand equations for Samsung-S7 phones are as follows:

AIS:	P = 30 - Qa
TRUE:	P = 100 - 5Qt
DTAC:	Qd = 40 - 2P

where Qa is the quantity of Samsung-S7 phones demanded by AIS, Qt is the quantity of Samsung-S7 phones demanded by TRUE, Qd is the quantity of Samsung-S7 phones demanded by DTAC, and P is the price per Samsung-S7 phone

6.1) (1 point) Justify the domain set of prices that allow all the three providers staying active in the market.

AIS: max-price = $30

DTAC: max-price = $40

TRUE: max-price = \$20

Ranking types: DTAC > AIS > TRUE

All three remain active only when P is lower than \$20.

6.2) (3 points) Derive the equation for market demand curve and sketch the demand curve. Locate all the important points.

$$Q = \begin{cases} 0 & ; P \geq 40 \quad \text{(no one)} \\ 20 - 0.5P & ; 30 \leq P < 40 \quad \text{(DTAC only)} \\ 50 - 1.5P & ; 20 \leq P < 30 \quad \text{(DTAC + AIS)} \\ 90 - 3.5P & ; 0 \leq P < 20 \quad \text{(DTAC + AIS + TRUE)} \end{cases}$$

Now continue with a new piece of information given. On the supply side in this market suppose Samsung-S7, the manufacturer or provider of these phones, outsources their production of the Samsung-S7 phones to two firms, Firm A and Firm B. These firms' supply curves are given below where Qa is the quantity of Samsung-S7 phones supplied by Firm A, Qb is the quantity of Samsung-S7 phones supplied by Firm B, and P is the price per Samsung-S7 phone:

Firm A: $P = 15 + Qa$

Firm B: $P = 10 + 2Qb$

6.3) (3 points) Derive the market supply equation and sketch the supply curve. Locate all the important points.

Firm A: min-price = \$15

Firm B: min-price = \$10

In terms of cost competitiveness, B is better than A as it needs only \$10 to stay in the market.

Rewrite the two equations in Q-equal form:

$Q_a = P - 15$;

$Q_b = 0.5P - 5$;

$$Q = \begin{cases} 0 & ; \ 0 \leq P < 10 \quad \text{(no firm)} \\ 0.5P - 5 & ; \ 10 \leq P < 15 \quad \text{(B only)} \\ 1.5P - 20 & ; \ P \geq 15 \quad \text{(B and then A)} \end{cases}$$

6.4) (3 points) Based on the market demand equation in question (6.1), what is the equilibrium price and quantity in this market? Explain how you found your answer and how you decided which segments of the demand curve and the supply curve were the relevant segments to consider.

Case 1: Two firms occur only when $P > 15$. One consumer (when $30 \leq P < 40$)

- $1.5P - 20 = 20 - 0.5P \Rightarrow P = 20$. But, when $P = 20$, there are two consumers. Contradict!

Case 2: Two firms occur only when $P > 15$. Two consumers (when $20 \leq P < 30$)

- $1.5P - 20 = 50 - 1.5P \Rightarrow P = 70/3 = 23.33$. Check if Q is positive. For $P = 23.33 \Rightarrow Q = 15$ units. This is the equilibrium.

Case 3: Two firms occur only when $P > 15$. Three consumers (when $0 \leq P < 20$)

- $1.5P - 20 = 90 - 3.5P \Rightarrow P = 110/5 = 22 \Rightarrow$ BUt at this price, we would have only 2 consumers. Contradict!

Case 4: One firms occur only when $10 < P \leq 15$. One consumer (when $30 \leq P < 40$)

- $0.5P - 5 = 20 - 0.5P \Rightarrow P = 15$. But if this is the equilibrium price, there must be three consumers in the market. Contradict!

Case 5: One firms occur only when $10 < P \leq 15$. Two consumers (when $20 \leq P < 30$)

- $0.5P - 5 = 50 - 1.5P \Rightarrow P = 45/2 = 22.5$, but if $P = 22.5$, there must be two firms. Contradict!

Case 6: One firms occur only when $10 < P \leq 15$. Three consumers (when $0 \leq P < 20$)

- $0.5P - 5 = 90 - 3.5P \Rightarrow P = 85/4 = 21.25$. But if price is 21.25, there would be only 2 consumers in the market. Contradict!

Case 2 is the equilibrium where $P = 70/3$ and $Q = 15$. There are two consumers in the market; meanwhile, both firms stay active in the market. Can you figure out how much each firm produce, and how much does each consumer buy?