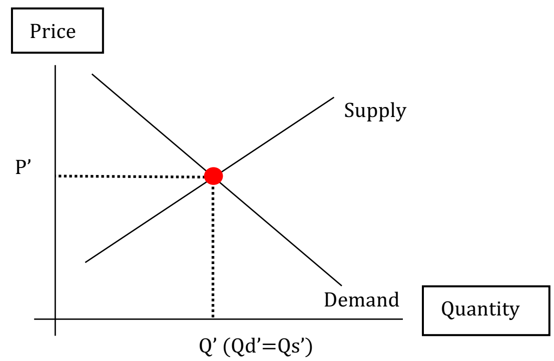
Question 1

$$Qd = a - bP$$
$$Qs = -c + dP$$

$Equilibrium\ Point\ occurs\ when\ Qd = Qs$



$Qd = Qs$

$a - bP \quad = -c + dP$

$a + c \quad\ = dP + bP$

$P' = \frac{a+c}{d+b}$ (1)

Put the equilibrium **price (P') into Qd or Qs function**, we will obtain equilibrium quantity.

$$Qd' = a - b\left(\frac{a+c}{d+b}\right)$$

$$Qd' = \frac{a(d+b) - b(a+c)}{d+b}$$

$$Qd' = \frac{ad + ab - ba - bc}{d+b}$$

$$\boldsymbol{Qd'} = \frac{\boldsymbol{ad - bc}}{\boldsymbol{d + b}}\ (\mathbf{2})$$

We would like to show you that you could put P' into Qd or Qs function because the answer is the same.

$$Qs' = -c + d\left(\frac{a+c}{d+b}\right)$$

$$Qs' = \frac{-c(d+b) + d(a+c)}{d+b}$$

$$Qs' = \frac{-cd - cb + da + dc}{d+b}$$

$$Qs' = \frac{-cb + da}{d+b}$$

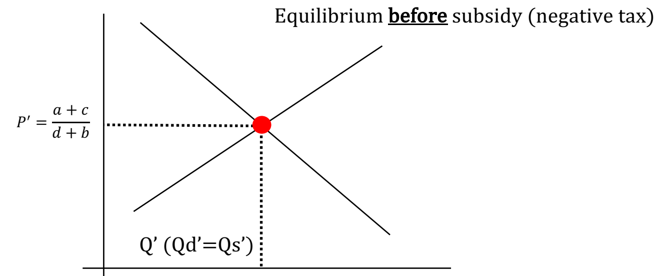
$$\boldsymbol{Qs'} = \frac{\boldsymbol{ad - bc}}{\boldsymbol{d + b}}\ (3)$$

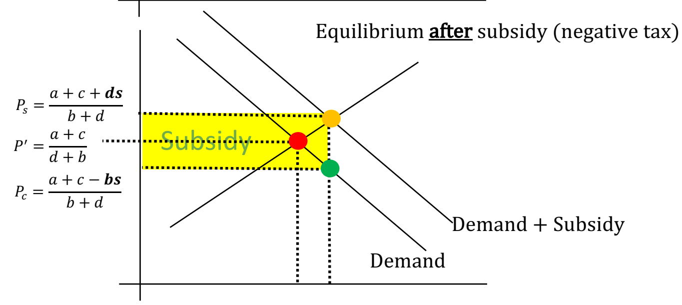
Therefore, the equilibrium price is $P' = \frac{a+c}{d+b}$ and quantity is $\boldsymbol{Qd'} = \boldsymbol{Qs'} = \frac{\boldsymbol{ad-bc}}{\boldsymbol{d+b}}$

Question 2

The government subsidizes(negative tax) consumers by \$s **per unit**, this means that for every single unit (\$P/unit) consumers have to pay only \$(P-s) per unit, for example, if Steve would like to buy a book that costs \$10 and the government subsidizes \$5 dollar for each book, Steve pays only \$5 dollar for a book. In addition, the sellers receive \$5 from Steve and \$5 from the government. As a result, the sellers receive full amount of price that they are willing to sell.

So, the next question is **what is the new equilibrium?**





Due to the subsidy \$s per unit,

Price that consumers pay (P_c)

Price that sellers receive (P_s)

P_s-P_c = s (Subsidy)

P_c = P_s - s (Re-write)

Therefore, the new condition is

$$Qd(P_c) = Qs(P_s)$$

$$Qd(P_s - s) = Qs(P_s)$$

$$a - b(P_s - s) = -c + d(P_s)$$

$$a - bP_s + bs = -c + dP_s$$

$$a + c + bs \quad = bP_s + dP_s$$

$$\boldsymbol{P_s{}' = \frac{a+c+bs}{b+d}}$$

$$Recall \;\; \mathrm{Pc} = \mathrm{Ps} - \mathrm{s}$$

$$P'_c{}' = \frac{a+c+bs}{b+d} - s$$

$$P_c{}' = \frac{a+c+bs-s(b+d)}{b+d}$$

$$\boldsymbol{P_c{}' = \frac{a+c-ds}{b+d}}$$

We could find the equilibrium price by plugging **<u>Pc' into the new Qd function</u>** or **<u>Ps' into Qs</u>** function respectively**,** we will obtain equilibrium quantity. However, since we don't know exactly what is the new demand function.

We have to use **<u>Ps' into Qs</u>**

$$Qs' = -c + d\left(\frac{a + c + \boldsymbol{bs}}{d + b}\right)$$

$$Qs' = \frac{-c(d + b) + d(a + c + \boldsymbol{bs})}{d + b}$$

$$Qs' = \frac{-cd - cb + da + dc + bds}{d + b}$$

$$Qs' = \frac{-cb + da + bds}{d + b}$$

$$\boldsymbol{Qs'} = \frac{\boldsymbol{ad} - \boldsymbol{bc} + bds}{\boldsymbol{d} + \boldsymbol{b}} \ (3)$$

Therefore, the equilibrium price for consumer is $\boldsymbol{P_c}' = \frac{\boldsymbol{a+c-ds}}{\boldsymbol{b+d}}$, for sellers $\boldsymbol{P_s}' = \frac{\boldsymbol{a+c+bs}}{\boldsymbol{b+d}}$ and quantity is $\boldsymbol{Qs}' = \frac{\boldsymbol{ad-bc}+bds}{\boldsymbol{d+b}}$

On the other hand, if the government subsidizes on producer

Due to the subsidy \$s per unit,

Price that consumers pay (P_c)

Price that sellers receive (P_s)

$P_s = P_C + s$

Therefore, the new condition is

$$Qd(Pc) = Qs(Pc + s)$$

$$a - b(P_c) = -c + d(P_c + s)$$

$$a - bP_c = -c + d(P_s + s)$$

$$a + c - ds \quad = bP_c + dP_c$$

$$\boldsymbol{P_c{}' = \frac{a + c - ds}{b + d}}$$

$$Recall \;\; \mathrm{Ps} = \mathrm{Pc} + \mathrm{s}$$

$$P_s = \frac{a + c - ds}{b + d} + s$$

$$P_s = \frac{a + c - ds + s(b + d)}{b + d}$$

$$\boldsymbol{P_s{}' = \frac{a + c + bs}{b + d}}$$

We could find the equilibrium price by plugging **Pc' into the Qd function** or **Ps' into the new Qs function** function respectively, we will obtain equilibrium quantity. However, since we don't know exactly what is the new supply function.

We have to use **Pc' into Qd**

$$Qd' = a - b\left(\frac{a + c + \boldsymbol{ds}}{d + b}\right)$$

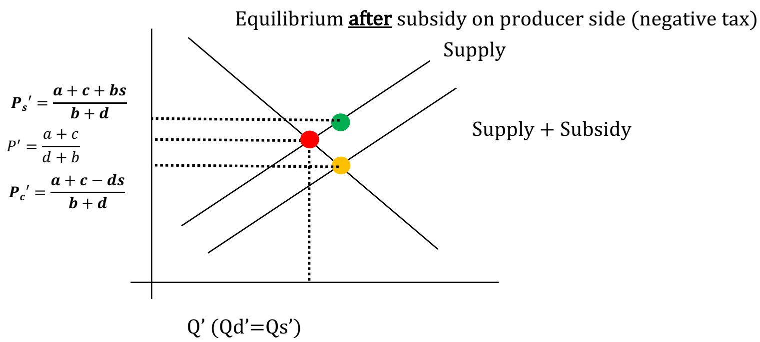
$$Qs' = \frac{a(d + b) - b(a + c + \boldsymbol{ds})}{d + b}$$

$$Qs' = \frac{ad + ab - ab - ac + bds}{d + b}$$

$$Qs' = \frac{ad - ac + bds}{d + b}$$

$$\boldsymbol{Qs'} = \frac{\boldsymbol{ad} - \boldsymbol{bc} + bds}{\boldsymbol{d} + \boldsymbol{b}} \quad (3)$$

Therefore, the equilibrium price for consumer is $\boldsymbol{P_c}' = \frac{\boldsymbol{a+c-ds}}{\boldsymbol{b+d}}$, for sellers $\boldsymbol{P_s}' = \frac{\boldsymbol{a+c+bs}}{\boldsymbol{b+d}}$ and quantity is $\boldsymbol{Qs'} = \frac{\boldsymbol{ad-bc}+bds}{\boldsymbol{d+b}}$



Price that consumer has to pay when the government subsidy on consumer and producer

1.2) $\boldsymbol{P_c}' = \frac{\boldsymbol{a+c-ds}}{\boldsymbol{b+d}}$ (subsidy on consumer) $=$ $\boldsymbol{P_c}' = \frac{\boldsymbol{a+c-ds}}{\boldsymbol{b+d}}$ (subsidy on producer)

1.3) Subsidy expenditure function

$$s\ (Q^*);\ \ s(Q^*) = s^*\left(\frac{\boldsymbol{ad-bc}+bds}{\boldsymbol{d+b}}\right)$$