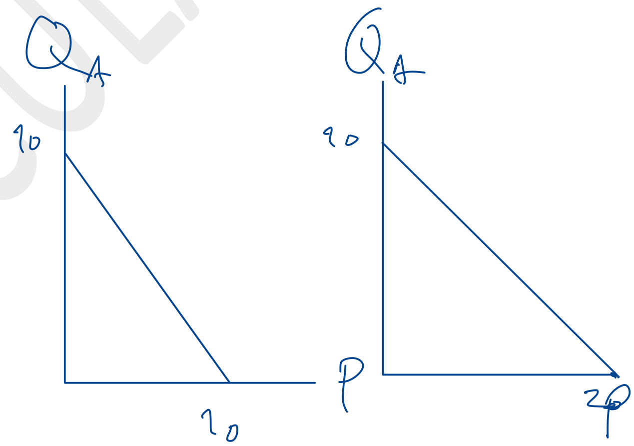
Example 3.G: Solve for the market equilibrium using the information in **Example 3.E** and **Example 3.F.** Justify your answer!

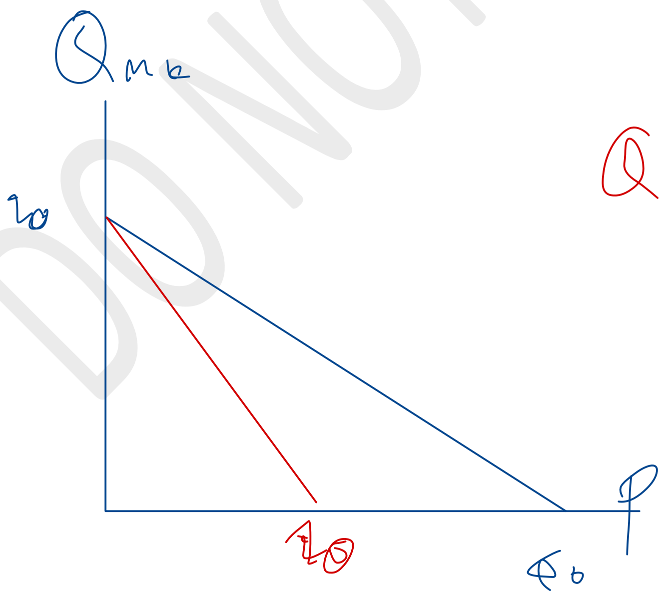
1) draw diagrams

2.) find equilibrium.

A : $Q_A = 20 - P$



B : $Q_B = 10 - \frac{1}{2} P$



$Q = 20 - \frac{3}{2} P$

Example 3.J: Excess burden *formula under linear model & Tax-Revenue-maxizing tax rate*

Demand: $p = a - bQ$; $a \geq 0, \quad b \leq 0.$

Supply : $p = c + dQ$; $d \geq 0.$

- Solve for quantity and prices equilibrium when the unit tax is imposed. Analyze the result

Before tax

Solve for P^*, Q^*

$P^* = f(a, b, c, d)$

$Q^* = f(a, b, c, d)$

$\begin{cases} p^s = p^d - t & \text{: tax on producer} \\ p^d = p^s + t & \text{: tax on consumer} \end{cases}$

Administrative structure of taxation will not affect the final outcome at the tax burden.

Eqn after tax → will be the same

$\rightarrow p^s + t = a - b \cdot Q^d \rightarrow p^s = a - t - b \cdot Q^d$

$p^s = c + dQ^s$

$Q^d = Q^s$ (Eqn^m)

$p^s = p^s \rightarrow a - t - b \cdot Q = c + d \cdot Q$

$a - t - c = (b + d) \cdot Q$

$Q = \dfrac{a - t - c}{b + d}$

$t = 0 \rightarrow Q = \dfrac{a - c}{b + d}$

$t > 0; \ Q = \dfrac{a - c}{b + d} - \dfrac{t}{b + d}$

$\dfrac{dQ}{dt} = \dfrac{\partial Q}{\partial t} = -\dfrac{1}{b + d} \rightarrow Q \downarrow$ when $\uparrow = \left\{ \dfrac{a - c}{b + d} - \dfrac{t}{b + d} \right\}$

$\Delta Q = Q_{\text{after tax}} - Q_{\text{before tax}} = \dfrac{-t}{b + t}$

After tax

Assume tax per unit $= t$

new S : $P = c + dQ + t$

$P^*_{tax} = f(a, b, c, d, t)$

$Q^*_{tax} = f(a, b, c, d, t)$

- Derive the excess burden formula for buyers and sellers

$t>0;\ Q=\frac{a-c}{b+d}-\frac{t}{b+d}$

$\frac{dQ}{dt}=\frac{\Delta Q}{\Delta t}=-\frac{1}{b+d} \rightarrow Q\downarrow$ when $t\uparrow=\frac{a-c}{b+d}-\frac{t}{b+d}$

$\Delta Q = Q_{\text{after tax}} - Q_{\text{before tax}} = \frac{-t}{b+d}$

$$\begin{aligned} p^s &= c+dQ^s \\ &= c+d\left[\frac{a-c}{b+d}-\frac{t}{b+d}\right] \\ &= c+\frac{d(a-c)}{b+d}-\frac{dt}{b+d} \\ &= \frac{cb+cd+da-dc}{b+d}-\frac{dt}{b+d}=0 \end{aligned}$$

$p^{s*}(t)\ \frac{cb+da}{b+d}-\frac{dt}{b+d}\rightarrow$

$p^d=p^s+t\rightarrow p^d=\frac{cb+da}{b+d}-\frac{dt}{b+d}+t$

$p^{dx}=\frac{cb+da}{b+d}+\frac{\cancel{d}}{b+d}\cdot t$ $\quad b=0$

$p^s : p^d$ depends on tax.

$$p^{s*}(t) \quad \frac{cb+da}{b+d} - \frac{dt}{b+d} \rightarrow$$

$$p^d = p^s + t \rightsquigarrow p^d = \frac{cb+da}{b+d} - \frac{dt}{b+d} + t$$

$$p^{d*} = \frac{cb+da}{b+d} + \frac{\cancel{d}^{\,b=0}}{b+d} \cdot t$$

$p^s : p^d$ depends on tax.

$$t=0 \rightsquigarrow t>0 \rightarrow |\Delta p^s| = \frac{dt}{b+d}$$

$$|\Delta p^d| = \frac{bt}{b+d}$$

$$\Big\} = \underline{t}$$

$$\overset{\text{old}}{t} > 0 \longrightarrow \overset{\text{new}}{t} > 0$$

$$\left|\frac{\Delta p^s}{\Delta t}\right| = \frac{d}{b+d} = \frac{\frac{\Delta P}{\Delta Q^s}}{\frac{\Delta P}{\Delta Q^d} + \frac{\Delta P}{\Delta Q^s}} = \frac{\frac{1}{\frac{\Delta Q^s}{\Delta P}} \cdot \frac{1}{\frac{P^*}{Q^*}}}{\frac{1}{\frac{\Delta Q^d}{\Delta P}} \cdot \frac{1}{\frac{P^*}{Q^*}} + \frac{1}{\frac{\Delta Q^s}{\Delta P}} \cdot \frac{1}{\frac{P^*}{Q^*}}}$$

$$\left|\frac{\Delta p^d}{\Delta t}\right| = \frac{b}{b+d}$$

"consumer" burden $= \dfrac{\varepsilon^s}{\varepsilon^s + \varepsilon^d}$

$$\frac{\Delta \text{ firm Burden}}{\Delta \varepsilon^s} = -\frac{\varepsilon^d}{(\varepsilon^s+\varepsilon^d)^2} = \frac{\frac{1}{\varepsilon^s}}{\frac{1}{\varepsilon^d}+\frac{1}{\varepsilon^s}} = \frac{\frac{1}{\varepsilon^s}}{\frac{\varepsilon^d+\varepsilon^d}{\varepsilon^d \cdot \varepsilon^s}} = \frac{\varepsilon^d}{\varepsilon^d+\varepsilon^s} \;\#$$

$\varepsilon^s \downarrow = \uparrow$ firm Burden

$$\frac{\Delta \text{ consumer Burden}}{\Delta \varepsilon^d} = -\frac{\varepsilon^s}{(\varepsilon^s+\varepsilon^d)}$$

$\varepsilon^d \uparrow \rightsquigarrow$ Burden fall $\downarrow$

inelastic $\varepsilon^d \downarrow \rightarrow$ Burden rise $\uparrow$

- Calculate the tax rate that maximizes the tax revenue of government.

$$T = t \cdot Q$$

$$t > 0;\ Q = \frac{a-c}{b+d} - \frac{t}{b+d} \qquad Q = \frac{a-t-c}{b+d}$$

HW

tax revenue $= t \times Q_{tax}$

$$\frac{d\ \text{tax revenue}}{dt} = 0$$

$$\rightarrow t^*$$

$$\frac{dQ}{dt} = \frac{\Delta Q}{\Delta t} = -\frac{1}{b+d} \rightarrow Q \downarrow \text{ when } t \uparrow = \left\{ \frac{a-c}{b-d} - \frac{t}{b+d} \right\}$$

$$\Delta Q = Q_{\text{after tax}} - Q_{\text{before tax}} = -\frac{t}{b+d}$$

$$T = t \cdot \left(\frac{a-c}{b+d} - \frac{t}{b+d} \right)$$

$$T(t) = \left(\frac{a-c}{b+d} \right) \cdot t - \frac{t^2}{b+d}$$

t^* maximize $T(t)$

$$\rightarrow t^* = \frac{-\left(\frac{a-c}{b+d}\right)}{2\left(-\frac{1}{b+d}\right)}$$

$$= \frac{a-c}{2}$$