Costs

EE211

Costs in the Short Run

Term	Definition	Mathematical Description
Fixed costs	Costs that do not vary with the quantity of output produced	FC
Variable costs	Costs that vary with the quantity of output produced	VC
Total cost	The market value of all the inputs that a firm uses in production	$TC = FC + VC$
Average fixed cost	Fixed cost divided by the quantity of output	$AFC = FC/Q$
Average variable cost	Variable cost divided by the quantity of output	$AVC = VC/Q$
Average total cost	Total cost divided by the quantity of output	$ATC = TC/Q$
Marginal cost	The increase in total cost that arises from an extra unit of production	$MC = \Delta TC/\Delta Q$

Costs in the Long Run

Costs in the Long Run

- In the long run all inputs are variable by definition.
- If the manager of the firm wishes to produce a given level of output at the lowest possible cost and is free to choose any input combination she pleases, which one should she choose?
- The answer to this question depends on the relative prices of capital and labor.

Isocost line

- A set of input bundles each of which costs the same amount
- Total cost C of producing any particular output is given by the sum of the firm's labor cost (wL) and its capital cost (rK):

$$C = wL + rK$$

If we rewrite the total cost equation as an equation for a straight line, we get

$$K = \frac{C}{r} - \left(\frac{W}{r}\right) L$$

The isocost line has a scope of $\frac{\Delta K}{\Delta L} = -\left(\frac{W}{r}\right)$

- It tells us that if the firm gave up a unit of labor (and recovered w dollars in cost) to buy $\frac{w}{r}$ units of capital at a cost of r dollars per unit, its total cost of production would remain the same

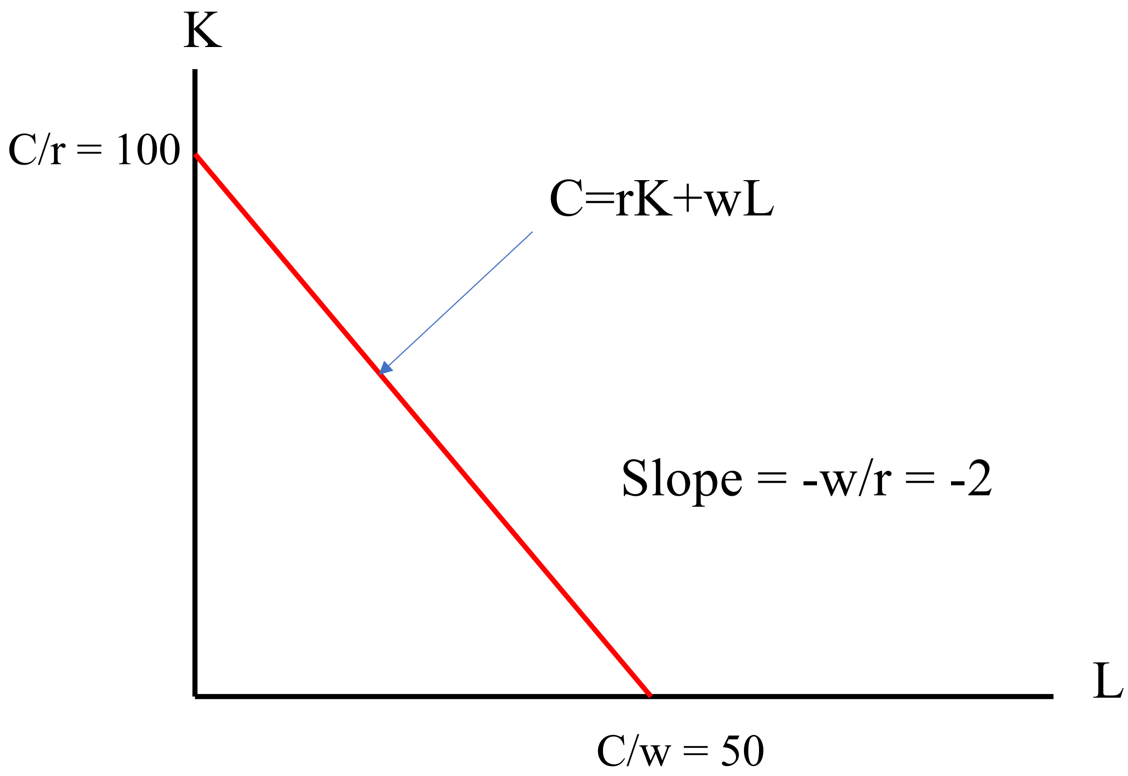
- E.g. if the wage rate were $10 and the rental cost of capital $5, the firm could replace one unit of labor with two units of capital with no change in total cost

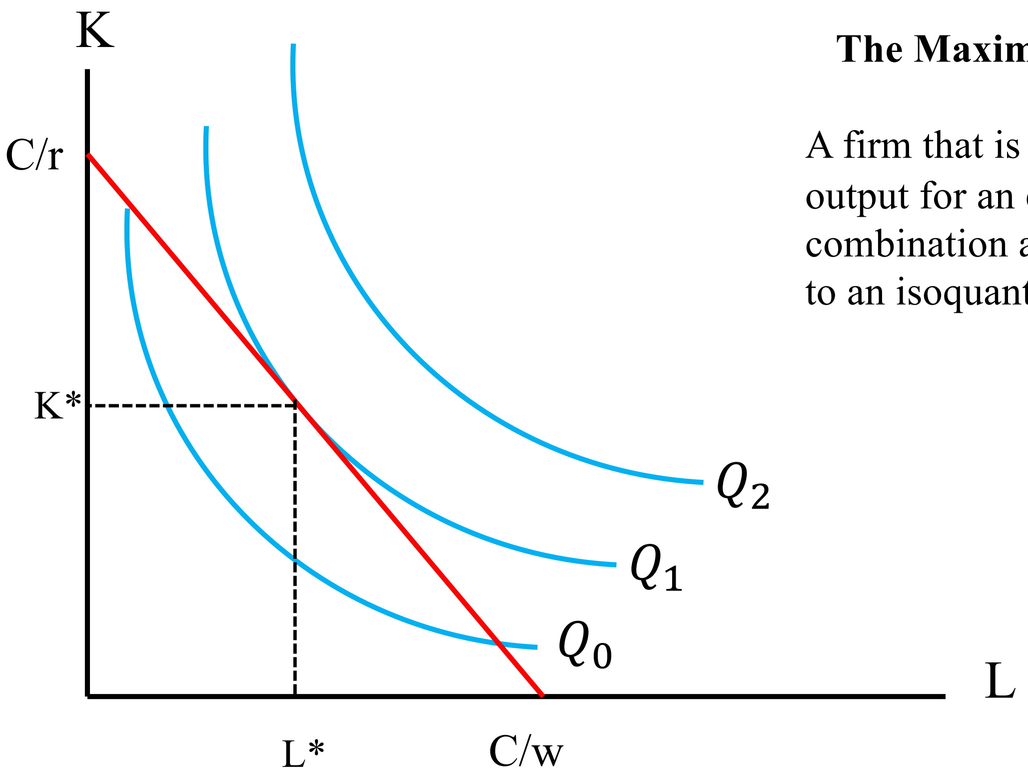
Choosing the optimal input combination

Let us begin with the case of a firm that wants to maximize output from a given level of expenditure

Suppose it uses only two inputs, capital (K) and labor (L), whose prices measured in dollars per unit of input per day, are $r=2$ and $w=4$, respectively.

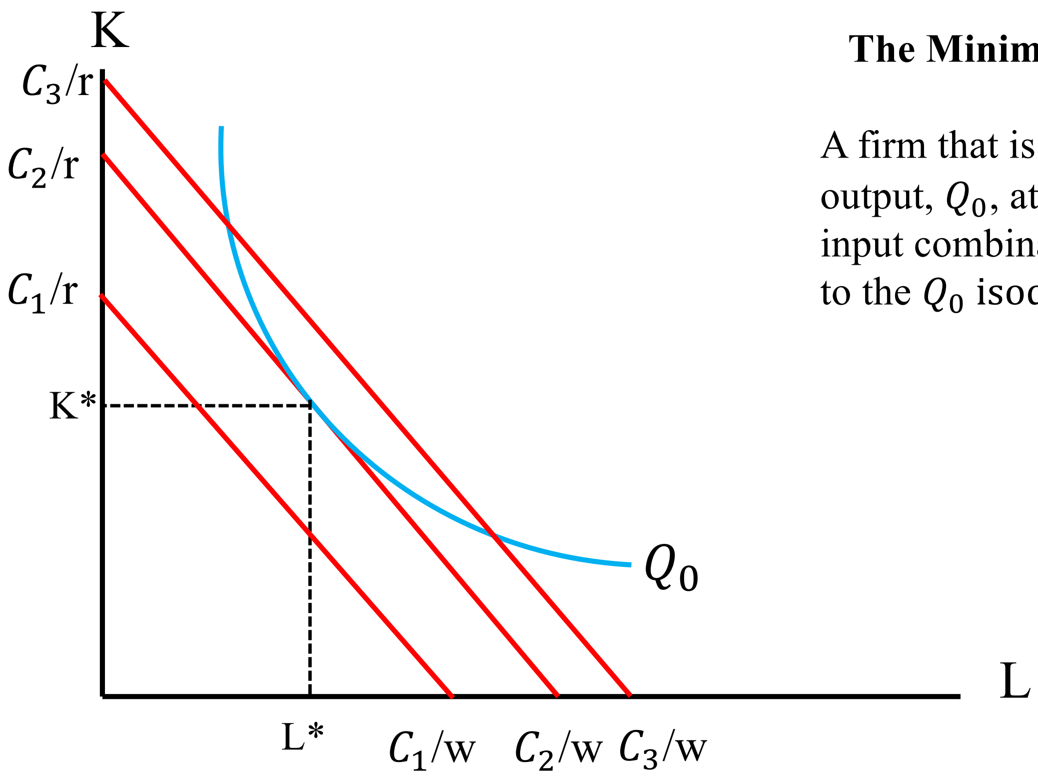
What different combinations of inputs can this firm purchase for a total expenditure of $C = \$200$/ day?


K
C/r = 100
C=rK+wL
Slope = -w/r = -2
L
C/w = 50



The Maximum output for a Given Expenditure

A firm that is trying to produce the largest possible output for an expenditure of C will select the input combination at which the isocost line for C is tangent to an isoquant



The Minimum cost for a Given Level of Output

A firm that is trying to produce a given level of output, Q_0, at the lowest possible cost will select the input combination at which an isocost line is tangent to the Q_0 isoquant.

- The slope of isoquant at any point is equal to $\frac{-MP_L}{MP_K}$, the negative of the ratio of the marginal product of labor to the marginal product of capital at that point. (The absolute value of this ratio is called the marginal rate of technical substitution)

- Combining with the result that minimum cost occurs at a point of tangency with the isocost line (whose slope is $-w/r$), it follows that

$$\frac{MP_{L*}}{MP_{K*}} = \frac{w}{r}$$

Where K* and L* again denote the minimum cost values of K and L. Cross-multiplying, we have

$$\frac{MP_{L*}}{w} = \frac{MP_{K*}}{r}$$

$$\frac{MP_{L*}}{w} = \frac{MP_{K*}}{r}$$

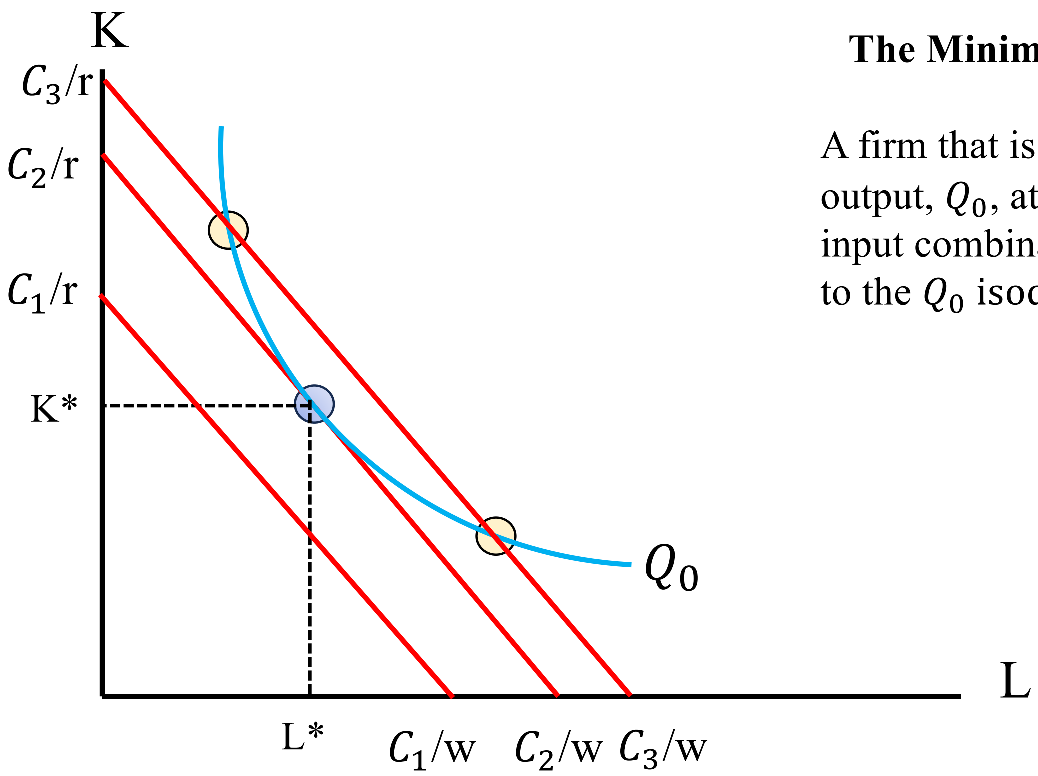
MP_{L*} is simply the extra output obtained from an extra unit of L at the cost-minimizing point.

w is the cost, in dollars, of an extra unit of L.

The ratio $\frac{MP_{L*}}{w}$ is the extra output we get from the last dollar spent on L.

The ratio $\frac{MP_{K*}}{r}$ is the extra output we get from the last dollar spent on K.

When costs are at a minimum, the extra output we get from the last dollar spent on an input must be the same for all inputs.



The Minimum cost for a Given Level of Output

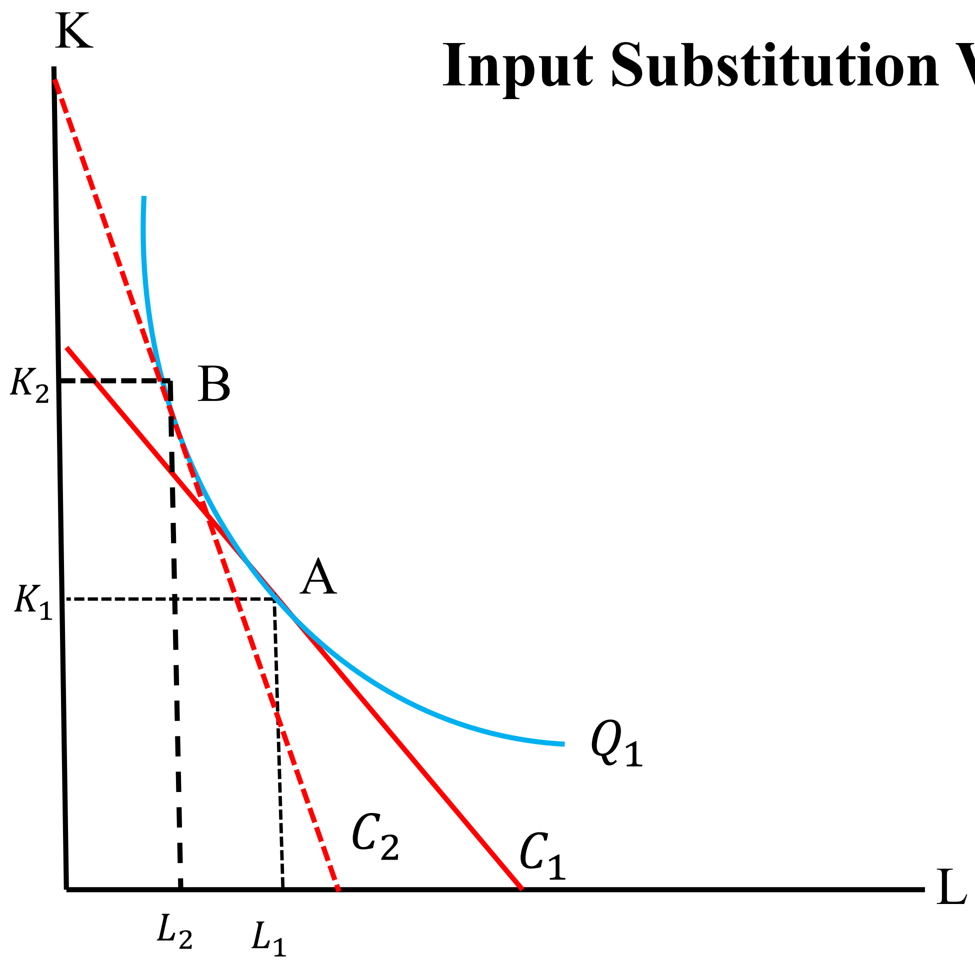
A firm that is trying to produce a given level of output, Q_0, at the lowest possible cost will select the input combination at which an isocost line is tangent to the Q_0 isoquant.

Input Substitution When an Input Price Changes

- Suppose that the price of one of the inputs, such as labor, were to increase.
- The slope of the isocost line $-(w/r)$ would increase in magnitude and the isocost line would become steeper.
- The isocost line is C_1, and the firm minimizes its costs of producing output q_1 at A by using L_1 units of labor and K_1 units of capital
- When the price of labor increases, the isocost line becomes steeper. The isocost line is C_2 reflects the higher price of labor.

- Facing the higher price of labor, the firm minimizes its cost of producing output q_1 by producing at B, using L_2 units of labor and K_2 units of capital
- The firm has responded to the higher price of labor by substituting capital for labor in the production process

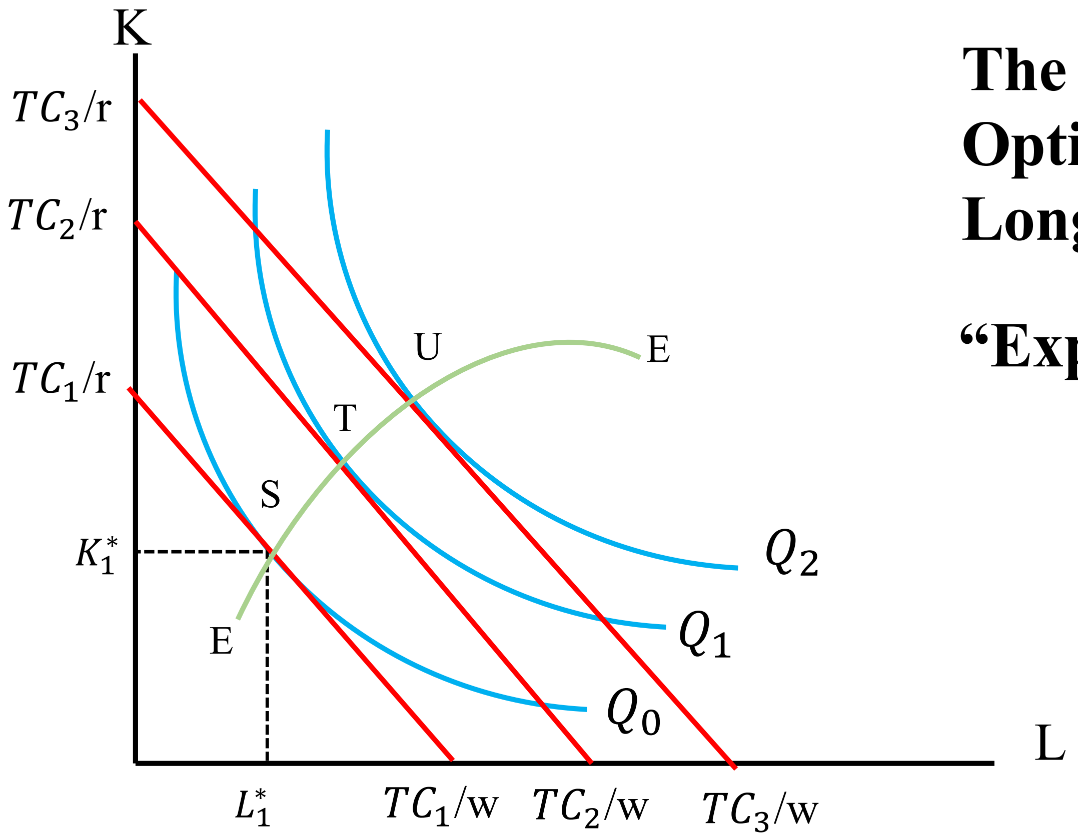
Input Substitution When an Input Price Changes



Facing an isocost curve C_1, the firm produces output q_1 at A by using L_1 units of labor and K_1 units of capital. When the price of labor increases, the isocost curves become steeper. Output q_1 is now produced at point B on isocost curve C_2 by using L_2 units of labor and K_2 units of capital.

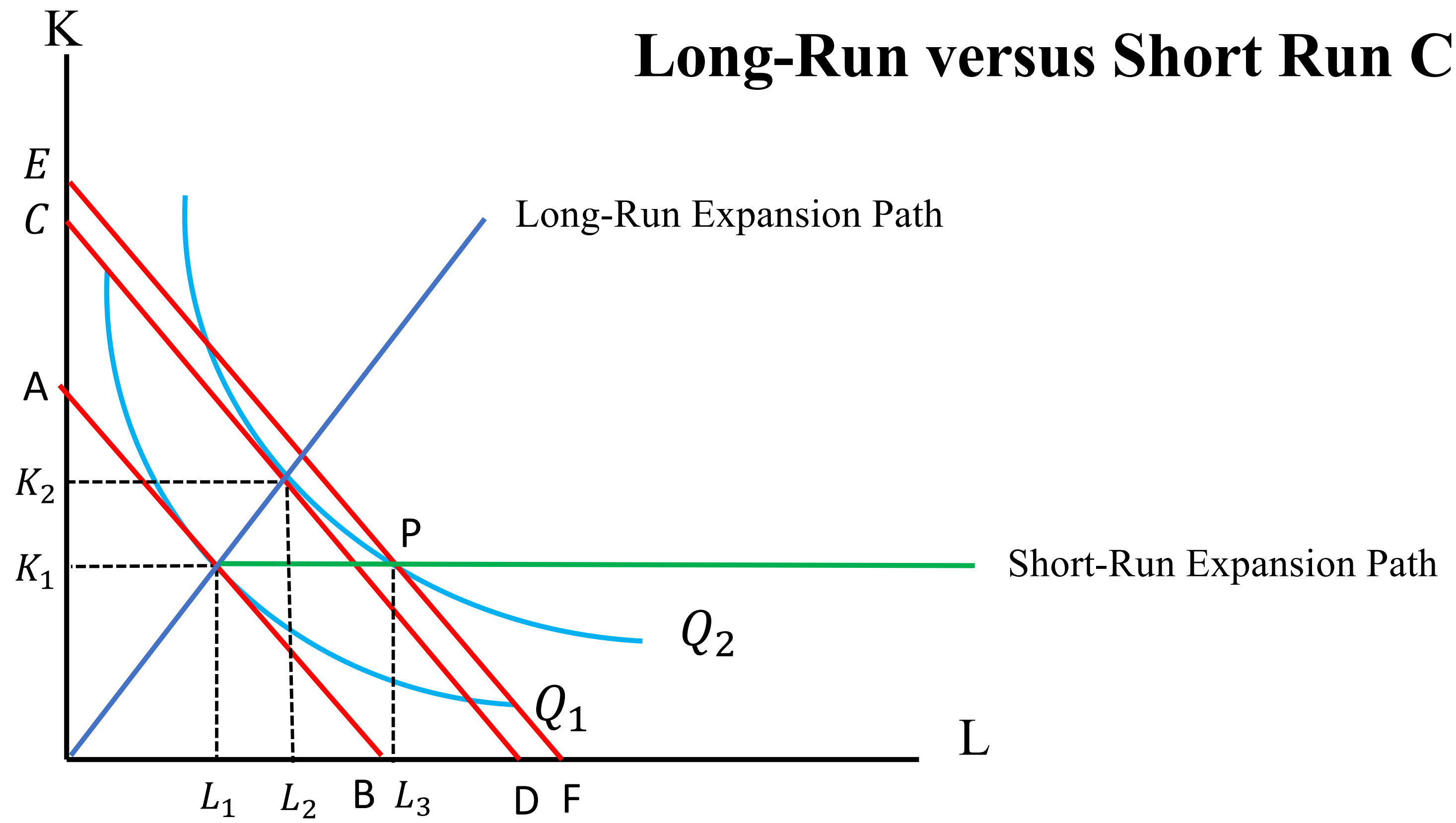
The relationship between Optimal Input Choice and Long-Run Costs

"Expansion path"



The firm can always buy the cost-minimizing input bundle that corresponds to any particular output level and relative input prices.
With fixed input prices r and w, bundles S,T,U, and others along the locus EE represent the least costly ways of producing the corresponding levels of output.

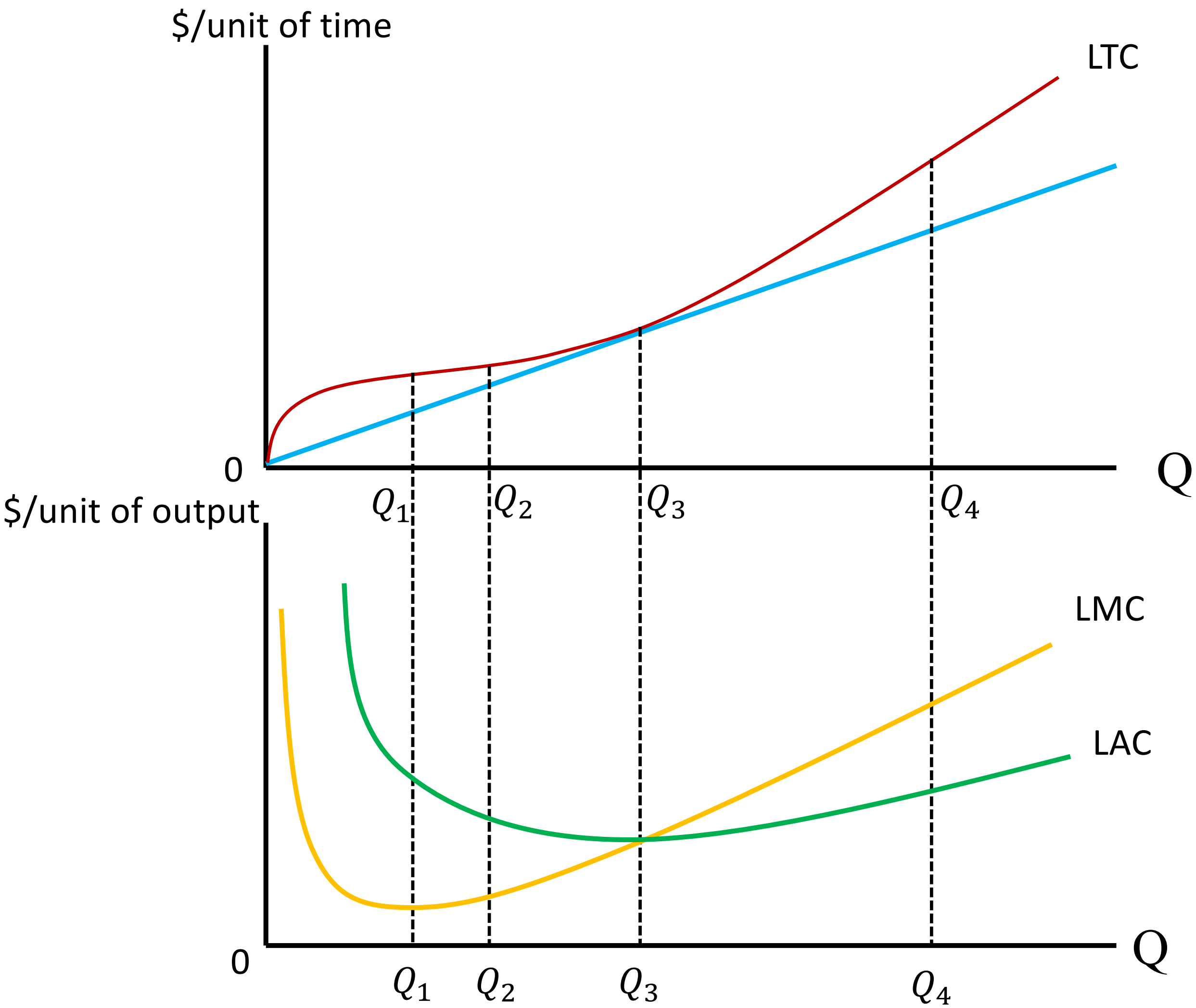
Long-Run versus Short Run Cost Curves



The inflexibility of Short-Run Production

- When a firm operates in the short-run, its cost of production may not be minimized because of inflexibility in th euse of capital inputs
- Output is initially at level Q_1. In the short-run, output Q_2 can be produced only by increasing labor from L_1 to L_3 because capital is fixed at K_1
- In the long run, the same output can be produced more cheaply by increasing labor from L_1 to L_2 and capital from K_1 to K_2

The Long-Run Total, Average, and Marginal Cost Curves



In the long run, the firm always has the option of ceasing operations and ridding itself of all its inputs.

The long-run total cost curve will always pass through the origin.

The long-run average and long-run marginal cost curves are derived from the long-run total cost curve in a manner completely analogous to the short-run case.

The Long-Run Total, Average, and Marginal Cost Curves

- In the long run, the firm always has the option of ceasing operations and ridding itself of all its inputs.
- The long-run total cost curve will always pass through the origin because in the long run the firm can liquidate all of its inputs.
- If the firm elects to produce no output, it need not retain, or pay for, the services of any of its inputs.
- The long-run average and long-run marginal cost curves are derived from the long-run total cost curve in a manner completely analogous to the short-run case.

- Long run marginal cost (LMC) is the slope of the long-run total cost curve

$$LMC = \frac{\Delta LTC}{\Delta Q}$$

LMC is the cost to the firm, in the long run, of expanding its output by 1 unit.

- Long run average cost (LAC) is the ratio of the long-run total cost to output

$$LAC = \frac{\Delta LTC}{Q}$$

The slope of the LTC curve is diminishing up to the output level Q_1 and increasing thereafter, which means that the LMC curve takes its minimum value at Q_1.

The slope of LTC and the slope of the rate to LTC are the same at Q_3, which means that LAC and LMC intersect at that level of output.

The traditional average-marginal relationship holds: LAC is declinging whenever LMC lies below it, and rising whenever LMC lies above it

Long-Run Average Cost

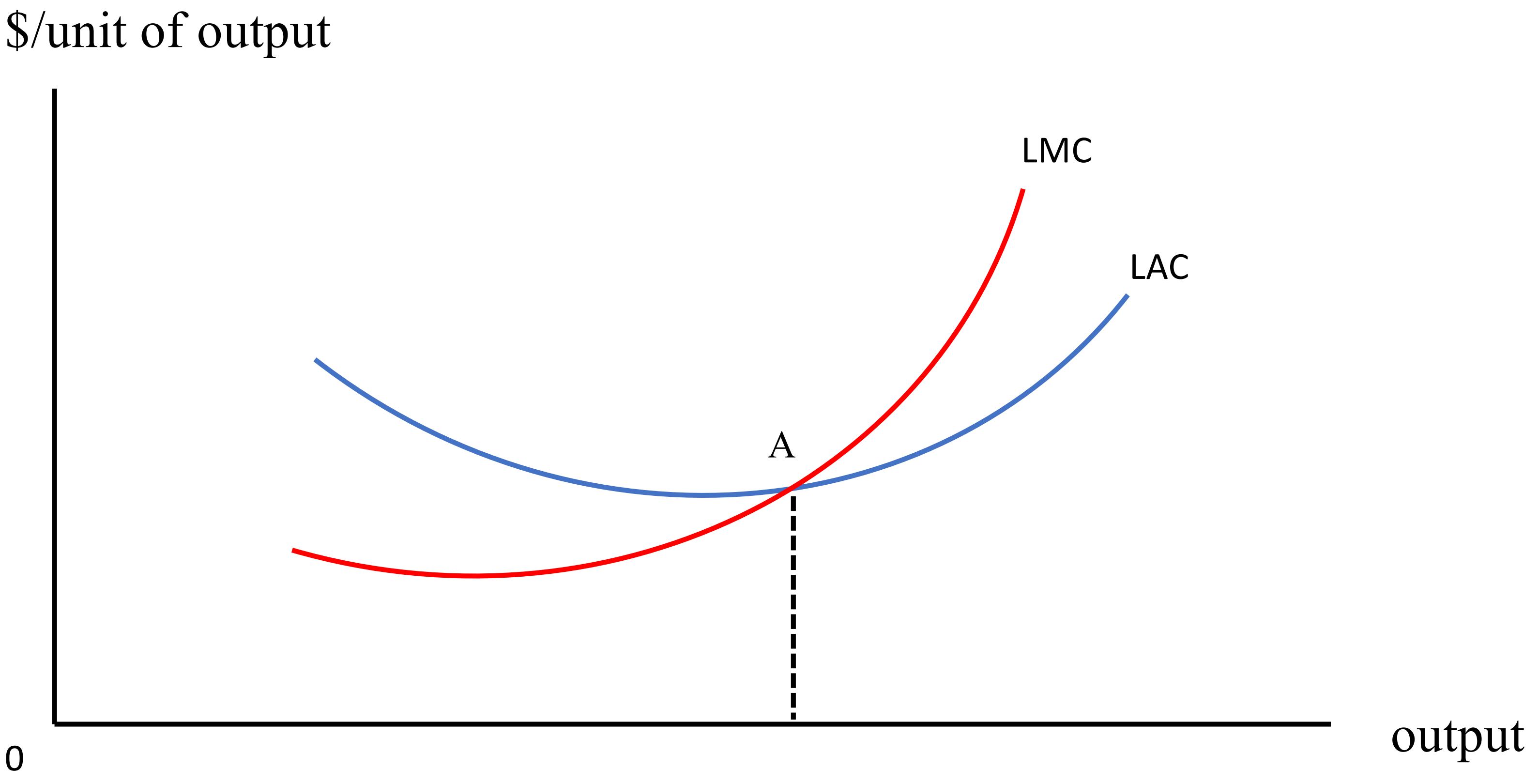
- The most important determinant of the shape of the long-run average and marginal cost curves is **the relationship between the scale of the firm's operation and the inputs that are required to minimize its costs**.

- Suppose, that the firm's production process exhibits constant returns to scale at all input levels. In this case, a doubling of inputs leads to a doubling of output. Because input prices remain unchanged as output increases, the average cost of production must be the same for all levels of output

- Suppose the firm's production process is subject to **increasing return to scale**: A doubling of inputs leads to more than a doubling or output. The average cost of production falls with output because a doubling of costs if associated with a more than twofold increase in output.

- The average cost of production falls with output because a doubling of costs is associated with a more than twofold increase in output. When there are **decreasing returns to scale**, the average cost of production must be increasing with output

Long-Run Average and Marginal Cost



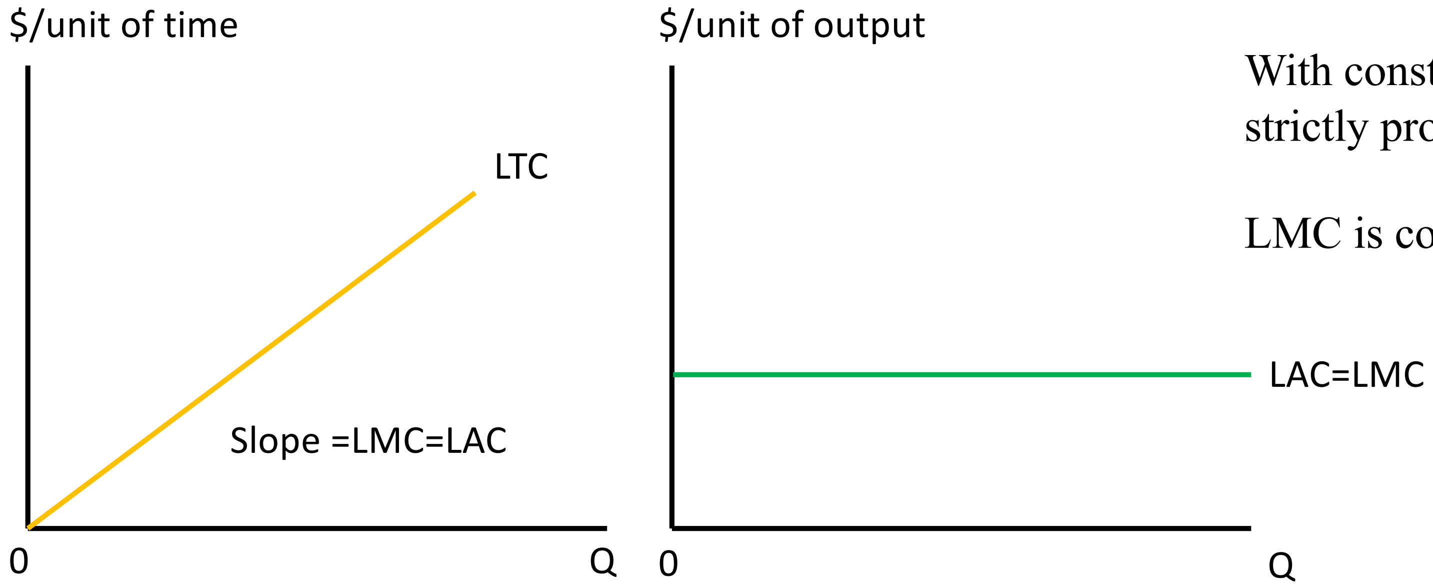
Long-Run Average and Marginal Cost

- When a firm is producing at an output at which the long-run average cost LAC is falling, the long-run marginal cost LMC is less than LAC.

- When LAC is increasing, LMC is greater than LAC.

- The two curves intersect at A, where the LAC curve achieves its minimum.

Figure A: The LTC, LMC, and LAC curves with Constant Returns to Scale

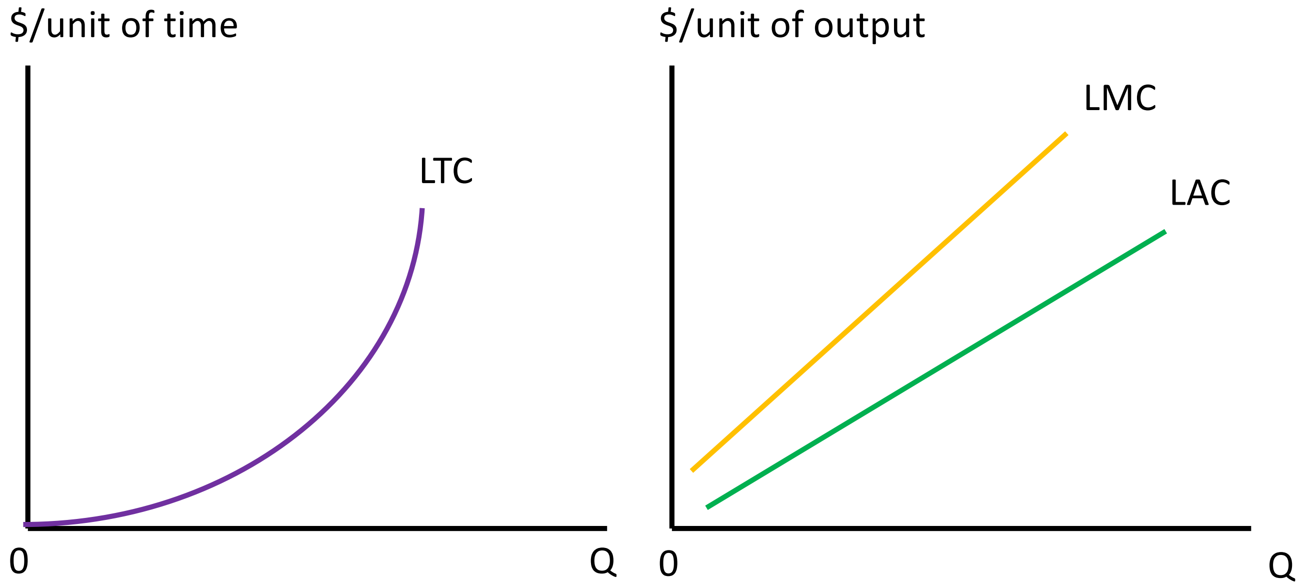


With constant returns, long-run total cost is strictly proportional to output.

LMC is constant and equal to LAC

For constant returns to scale production function, doubling output exactly doubles costs. The LTC curve for a production function with constant returns to scale is a straight line through the origin. Because the slope of LTC is constant, the associated LMC curve is a horizontal line, and is exactly the same as the LAC curve.

Figure B: The LTC, LMC, and LAC curves for a production process with Decreasing Returns to Scale



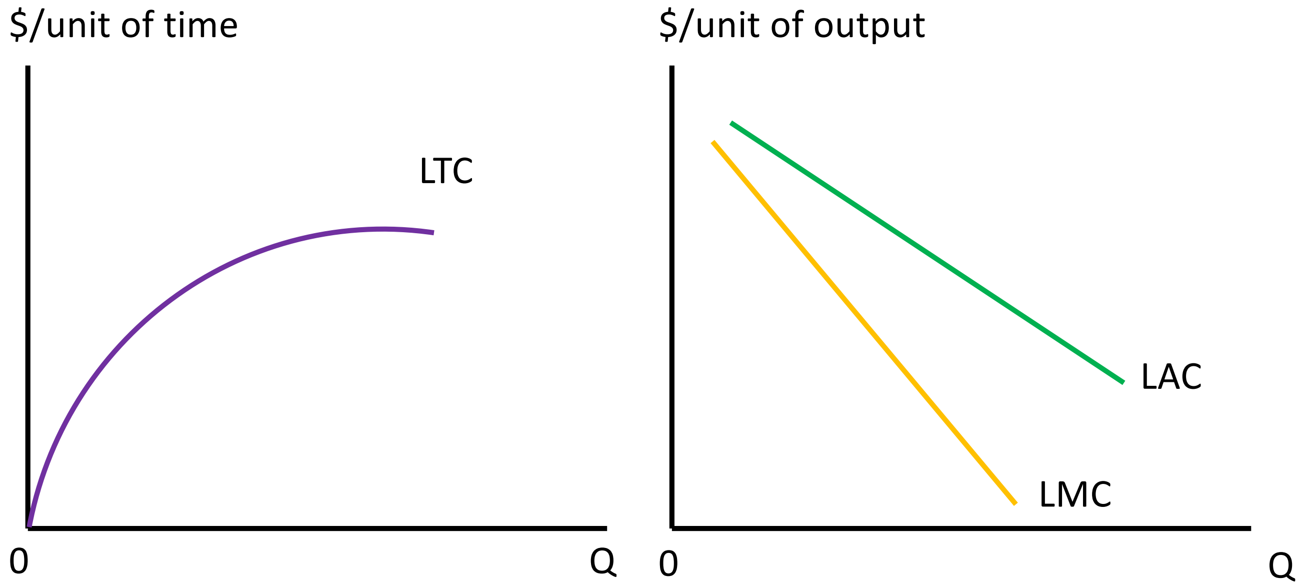
Under decreasing returns, output grows less than in proportion to the growth in inputs, which means that total cost grows more than in proportion to growth in output.

When the proportion function has decreasing returns to scale, a given proportional increase in output requires a greater proportional increase in all inputs and hence a greater proportional increase in costs.

The general property of the decreasing returns case is that it gives rise to an upward-sloping LTC curve and upward-sloping LAC and LMC curves.

LMC exceeds LAC ensures that LAC must rise with output.

Figure C: The LTC, LMC, and LAC curves for a production process with Increasing Returns to Scale



With increasing returns, the large-scale firm has lower average and marginal costs than the smaller-scale firm.

The case of increasing returns to scale- output grows more than in proportion to the increase in inputs. In consequence, long-run total cost rises less than in proportion to increases in output. The distinguishing feature of the LAC and LMC curves under increasing returns to scale is not the linear form shown in particular example, but in fact that they are downward sloping.

- The production processes whose long-run cost curves are pictured in figures A,B, and C are “pure cases,” exhibiting constant, decreasing, and increasing returns to scale, respectively, over their entire ranges of output.
- The degree of returns to scale of a production function need not be the same over the whole range of output.

Long-Run Costs and The Structure of Industry

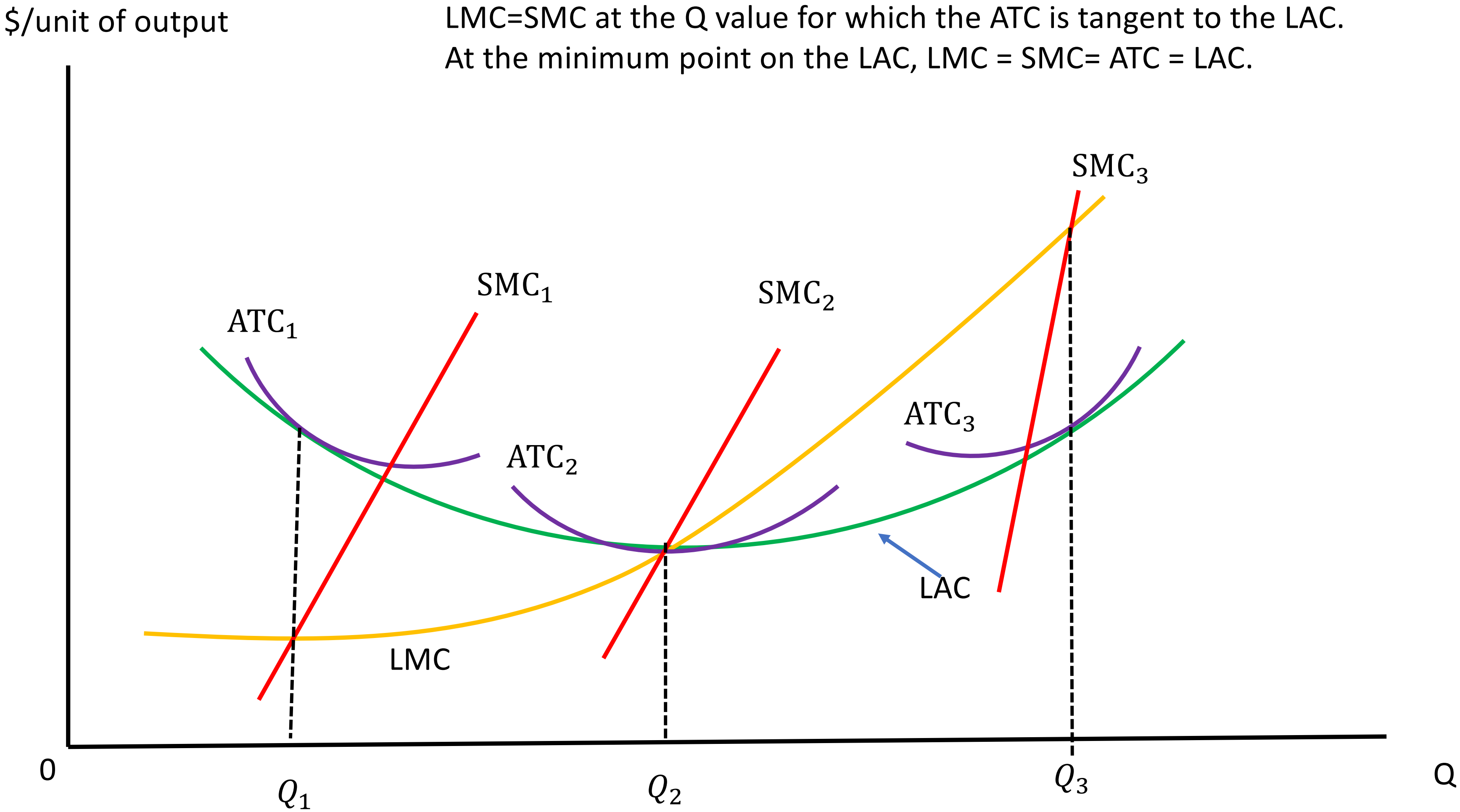
The Relationship between Long-Run and Short-Run Cost Curves

The Relationship between Long-Run and Short-Run Cost Curves

- One way of thinking of the LAC curve is as an envelope of all the short-run average total cost (ATC) curves.
- The output level at which a given ATC is tangent to the LAC, the long run marginal cost (LMC) of producing that level of output is the same as the short-run marginal cost (SMC). Thus $LMC\ (Q_1) = SMC\ (Q_1)$, $LMC\ (Q_2) = SMC\ (Q_2)$ and $LMC\ (Q_3) = SMC\ (Q_3)$.
- Each point along a given ATC curve, except for the tangency point, lies above the corresponding point on the LAC curve.
- At the minimum point on the LAC curve ($Q = Q_2$), the long run and short run marginal and average costs all take exactly the same value.

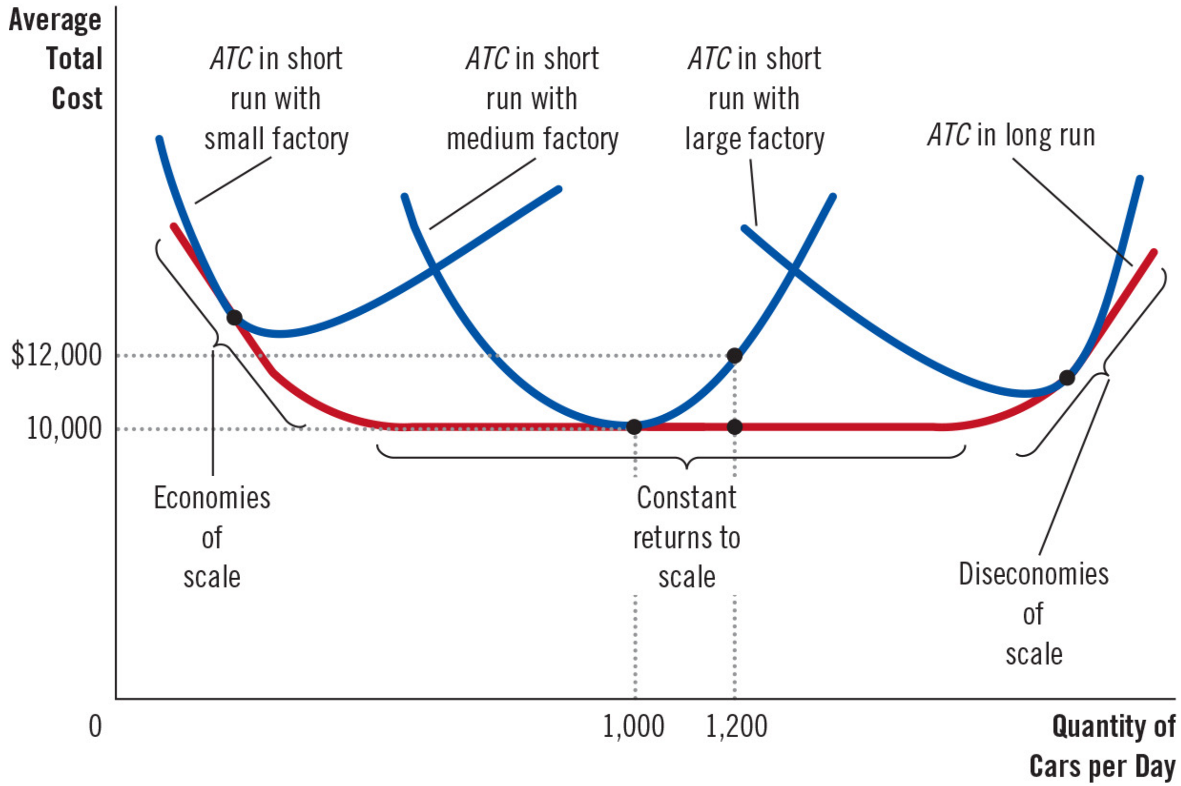
- Some intuition about ATC-LAC relationship for a given ATC curve is afforded by noting that to the left of the ATC-LAC tangency, the firm has “too much” capital, with the result that its fixed costs are higher than necessary;
- That to the right of the ATC-LAC tangency, the firm has “too little” capital, so that diminishing returns to labor drive its costs up.
- Only at the tangency point does the firm have the optimal quantities of both labor and capital for producing the correponding level of output.

The LAC curve is the outer envelope of the ATC curves.
LMC=SMC at the Q value for which the ATC is tangent to the LAC.
At the minimum point on the LAC, LMC = SMC= ATC = LAC.



Average Total Cost in the Short and Long Runs

- Because fixed costs are variable in the long run, the average-total-cost curve in the short run differs from the average-total-cost curve in the long run.



Economies and Diseconomies of Scale

- **Economies of scale**
  - Long-run average total cost falls as the quantity of output increases
- **Constant returns to scale**
  - Long-run average total cost stays the same as the quantity of output changes
- **Diseconomies of scale**
  - Long-run average total cost rises as the quantity of output increases

Economies of Scale

As output increases, the firm’s average cost of producing that output is likely to decline, at least to a point. This can happen for the following reasons:

- If the firm operates on a larger scale, workers can specialize in the activities at which they are most productive
- Scale can provide flexiblity. By varying the combination of inputs utilized to produce the firm’s output, managers can organize the production process more effectively

Diseconomies of scale

The average cost of production will begin to increase with output

- At least in the short run, factory space and machinery may make it more difficult for workers to do their job effectively
- Managing a larger firm may become more complex and inefficient as the number of tasks increases
- The advantages of buying in bulk may have disappeared once certain quantities are reached. At some point, available supplies of key inputs may be limited, pushing their costs up.

References

- Lipsey, Regan, and Storer (2008)
- Frank, R.H. (2010)
- Mankiw, N.G., (2023)
- Pindyck and Rubinfeld (2018)