Solution: Quiz 2

1. Let the set of real numbers $\mathbb{R}$ be the domain of x. Determine if the following statement is true or false:

$$x^2 > 4 \Rightarrow x > 2.$$

<u>Answer:</u>

Let T_h be the truth set of the predicate: "$x^2 > 4$"
and T_c be the truth set of the predicate: " $x > 2$"
For $x \in \mathbb{R}$, "$x^2 > 4$" is true when $x < -2$ and $x > 2$, or $x \in (-\infty, -2) \cup (2, \infty)$; and "$x > 2$" is true when $x \in (2, \infty)$.

That is, $T_h = (-\infty, -2) \cup (2, \infty)$ and $T_c = (2, \infty)$.
Since $T_h \nsubseteq T_c$, it is not true that every element in the truth set of "$x^2 > 4$" is an element in the truth set of " $x > 2$" for $x \in \mathbb{R}$. A counter example is when $x = -3 \in T_h$, but not in T_c. This implies that the statement is **false**.

2. Let $D = \{-1, 0, 1\}$ and $E = \{-1, 0, 2\}$. Consider the statement

$$\forall x \in D, \exists y \in E \text{ such that } x + y = 0.$$

(a) Write the negation for the above statement (without using negation symbol "$\sim$" in the final answer).

(b) Determine the truth value of the above statement. Explain your answer.

<u>Answer:</u>

(a)

$$\begin{aligned} \sim (\forall x \in D, \exists y \in E \text{ such that } x + y = 0) &\equiv \exists x \in D \sim (\exists y \in E \text{ such that } x + y = 0) \\ &\equiv \exists x \in D, \forall y \in E \text{ such that } \sim (x + y = 0) \\ &\equiv \exists x \in D, \forall y \in E \text{ such that } x + y \neq 0 \end{aligned}$$

(b) This statement is false because if we let $x = -1$, then $-1 + y \neq 0$ for all $y \in E = \{-1, 0, 2\}$. I.e.

$$\begin{aligned} y = -1, &\quad -1 + (-1) = -2 \neq 0 \\ y = 0, &\quad -1 + 0 = -1 \\ y = 2, &\quad -1 + 2 = 1. \end{aligned}$$

Or we can notice that for $x = -1$, $-1 + y = 0$ only when $y = 1$ which is not in E. So the statement is *false*.