Notes on Matrix Applications : Macroeconomic Models.

① Simple Keynesian Model.

Given $Y = C + I_0 + G_0 + X_0 - M$

$C = a + bY_d$

where $Y_d = Y - T$, $T = tY$ $\Rightarrow Y_d = Y - tY = (1-t)Y$.

$M = mY_d = m(1-t)Y$

2 endogenous variables are Y & C.

Rewrite the above system of equations to:

$$Y = C + I_0 + G_0 + X - m(1-t)Y \quad ①$$
$$C = a + b(1-t)Y \quad ②$$

$$① \Rightarrow [1 + m(1-t)]Y - C = I_0 + G_0 + X_0$$
$$② \Rightarrow -b(1-t)Y + C = a.$$

In matrix form, $\underbrace{\begin{bmatrix} 1+m(1-t) & -1 \\ -b(1-t) & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} Y \\ C \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} I_0 + G_0 + X_0 \\ a \end{bmatrix}}_{d}$.

For a unique solution to exist, we need $|A| \neq 0$;

ie. $|A| = 1 + m(1-t) - b(1-t) = 1 + (m-b)(1-t) \neq 0$.

Using Cramer's rule, $Y^* = \frac{1}{|A|} \cdot \begin{vmatrix} I_0 + G_0 + X_0 & -1 \\ a & 1 \end{vmatrix}$

$$\Rightarrow \boxed{Y^* = \frac{a + I_0 + G_0 + X_0}{1 + (m-b)(1-t)}}$$

Similarly, $C^* = \frac{1}{|A|} \begin{vmatrix} 1+m(1-t) & I_0 + G_0 + X_0 \\ -b(1-t) & a \end{vmatrix} = \frac{a[1 + m(1-t) + b(1-t)(I_0 + G_0 + X_0)]}{1 + (m-b)(1-t)}$.

② IS-LM model

Goods Mkt: $Y = C + I + G_0$
$C = a + bY_d$
$I = I_0 - ir$
$Y_d = Y - T, \; T = tY$
$\Rightarrow Y = a + b(1-t)Y + I_0 - ir + G_0$: IS

Money Mkt: $M^S = M_0$
$M^D = kY - hr$
$\Rightarrow M_0 = kY - hr$: LM

IS $\Rightarrow$ $[1-b(1-t)]Y + ir = a + I_0 + G_0$
LM $\Rightarrow$ $kY - hr = M_0$

In matrix form,
$$\underbrace{\begin{bmatrix} 1-b(1-t) & i \\ k & -h \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} Y \\ r \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} a + I_0 + G_0 \\ M_0 \end{bmatrix}}_{d}.$$

For a unique solution to exist, we need $|A| = -h[1-b(1-t)] - ik \neq 0$.

Using Cramer's rule,
$$Y^* = \frac{\begin{vmatrix} a + I_0 + G_0 & i \\ M_0 & -h \end{vmatrix}}{|A|} = \frac{-h[a+I_0+G_0) - iM_0}{-h[1-b(1-t) - ik}$$

$$\therefore \; Y^* = \frac{h(a+I_0+G_0) + iM_0}{h[1-b(1-t)] + ik} \quad \text{where } h[1-b(1-t)] + ik \neq 0.$$

$$r^* = \frac{\begin{vmatrix} 1-b(1-t) & a+I_0+G_0 \\ k & M_0 \end{vmatrix}}{|A|} = \frac{M_0[1-b(1-t)] - k(a+I_0+G_0)}{-h[1-b(1-t)] - ik}$$

$$\therefore \; r^* = \frac{k(a+I_0+G_0) - [1-b(1-t)]M_0}{h[1-b(1-t)] + ik} \quad \text{where } h[1-b(1-t)] + ik \neq 0.$$