CHAPTER 5: INTRODUCTION TO RISK, RETURN, AND THE HISTORICAL RECORD

PROBLEM SETS

6. a. The "Inflation-Plus" CD is the safer investment because it guarantees the purchasing power of the investment. Using the approximation that the real rate equals the nominal rate minus the inflation rate, the CD provides a real rate of 1.5% regardless of the inflation rate.

   b. The expected return depends on the expected rate of inflation over the next year. If the expected rate of inflation is less than 3.5% then the conventional CD offers a higher real return than the Inflation-Plus CD; if the expected rate of inflation is greater than 3.5%, then the opposite is true.

   c. If you expect the rate of inflation to be 3% over the next year, then the conventional CD offers you an expected real rate of return of 2%, which is 0.5% higher than the real rate on the inflation-protected CD. But unless you know that inflation will be 3% with certainty, the conventional CD is also riskier. The question of which is the better investment then depends on your attitude towards risk versus return. You might choose to diversify and invest part of your funds in each.

   d. No. We cannot assume that the entire difference between the risk-free nominal rate (on conventional CDs) of 5% and the real risk-free rate (on inflation-protected CDs) of 1.5% is the expected rate of inflation. Part of the difference is probably a risk premium associated with the uncertainty surrounding the real rate of return on the conventional CDs. This implies that the expected rate of inflation is less than 3.5% per year.

9. $E(q) = (0 \times 0.25) + (1 \times 0.25) + (2 \times 0.50) = 1.25$

   $\sigma_q = [0.25 \times (0 - 1.25)^2 + 0.25 \times (1 - 1.25)^2 + 0.50 \times (2 - 1.25)^2]^{1/2} = 0.8292$

10. (a) With probability 0.9544, the value of a normally distributed variable will fall within two standard deviations of the mean; that is, between –40% and 80%.

11. From Table 5.3 and Figure 5.6, the average risk premium for the period 1926-2009 was: (11.63% − 3.71%) = 7.92% per year

    Adding 7.92% to the 3% risk-free interest rate, the expected annual HPR for the S&P 500 stock portfolio is: 3.00% + 7.92% = 10.92%

CFA PROBLEMS

3. $E(r_X) = [0.2 \times (-20\%)] + [0.5 \times 18\%] + [0.3 \times 50\%] = 20\%$

   $E(r_Y) = [0.2 \times (-15\%)] + [0.5 \times 20\%] + [0.3 \times 10\%] = 10\%$

4. $\sigma_X^2 = [0.2 \times (-20 - 20)^2] + [0.5 \times (18 - 20)^2] + [0.3 \times (50 - 20)^2] = 592$

$\sigma_X = 24.33\%$

$\sigma_Y^2 = [0.2 \times (-15 - 10)^2] + [0.5 \times (20 - 10)^2] + [0.3 \times (10 - 10)^2] = 175$

$\sigma_Y = 13.23\%$

5. $E(r) = (0.9 \times 20\%) + (0.1 \times 10\%) = 19\% \rightarrow$ \$1,900 in returns

CHAPTER 7: OPTIMAL RISKY PORTFOLIOS

PROBLEM SETS

1. (a) and (e).

2. (a) and (c). After real estate is added to the portfolio, there are four asset classes in the portfolio: stocks, bonds, cash and real estate. Portfolio variance now includes a variance term for real estate returns and a covariance term for real estate returns with returns for each of the other three asset classes. Therefore, portfolio risk is affected by the variance (or standard deviation) of real estate returns and the correlation between real estate returns and returns for each of the other asset classes. (Note that the correlation between real estate returns and returns for cash is most likely zero.)

4. The parameters of the opportunity set are:

   $E(r_S) = 20\%$, $E(r_B) = 12\%$, $\sigma_S = 30\%$, $\sigma_B = 15\%$, $\rho = 0.10$

   From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \rho \times \sigma_S \times \sigma_B$]:

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

   The minimum-variance portfolio is computed as follows:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$$w_{Min}(B) = 1 - 0.1739 = 0.8261$$

   The minimum variance portfolio mean and standard deviation are:

$$E(r_{Min}) = (0.1739 \times .20) + (0.8261 \times .12) = .1339 = 13.39\%$$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{1/2}$$

$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$$= 13.92\%$$

13. False. If the borrowing and lending rates are not identical, then, depending on the tastes of the individuals (that is, the shape of their indifference curves), borrowers and lenders could have different optimal risky portfolios.

17. The correct choice is c. Intuitively, we note that since all stocks have the same expected rate of return and standard deviation, we choose the stock that will result in lowest risk. This is the stock that has the lowest correlation with Stock A.

More formally, we note that when all stocks have the same expected rate of return, the optimal portfolio for any risk-averse investor is the global minimum variance portfolio (G). When the portfolio is restricted to Stock A and one additional stock, the objective is to find G for any pair that includes Stock A, and then select the combination with the lowest variance. With two stocks, I and J, the formula for the weights in G is:

$$w_{Min}(I) = \frac{\sigma_J^2 - Cov(r_I, r_J)}{\sigma_I^2 + \sigma_J^2 - 2Cov(r_I, r_J)}$$

$$w_{Min}(J) = 1 - w_{Min}(I)$$

Since all standard deviations are equal to 20%:

$$Cov(r_I, r_J) = \rho\sigma_I\sigma_J = 400\rho \text{ and } w_{Min}(I) = w_{Min}(J) = 0.5$$

This intuitive result is an implication of a property of any efficient frontier, namely, that the covariances of the global minimum variance portfolio with all other assets on the frontier are identical and equal to its own variance. (Otherwise, additional diversification would further reduce the variance.) In this case, the standard deviation of G(I, J) reduces to:

$$\sigma_{Min}(G) = [200 \times (1 + \rho_{IJ})]^{1/2}$$

This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is Stock D. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is 17.03%.

18. No, the answer to Problem 17 would not change, at least as long as investors are not risk lovers. Risk neutral investors would not care which portfolio they held since all portfolios have an expected return of 8%.

19. Yes, the answers to Problems 17 and 18 would change. The efficient frontier of risky assets is horizontal at 8%, so the optimal CAL runs from the risk-free rate through G. This implies risk-averse investors will just hold Treasury Bills.

CFA PROBLEMS

5. c.
6. d.
7. b.
9. c.

CHAPTER 9: THE CAPITAL ASSET PRICING MODEL

PROBLEM SETS

2. If the security's correlation coefficient with the market portfolio doubles (with all other variables such as variances unchanged), then beta, and therefore the risk premium, will also double. The current risk premium is: 14% – 6% = 8%

   The new risk premium would be 16%, and the new discount rate for the security would be: 16% + 6% = 22%

   If the stock pays a constant perpetual dividend, then we know from the original data that the dividend (D) must satisfy the equation for the present value of a perpetuity:

   Price = Dividend/Discount rate

   $50 = D/0.14 \Rightarrow D = 50 \times 0.14 = \$7.00$

   At the new discount rate of 22%, the stock would be worth: $7/0.22 = $31.82

   The increase in stock risk has lowered its value by 36.36%.

4. The expected return is the return predicted by the CAPM for a given level of systematic risk.

6. The expected return of a stock with a $\beta = 1.0$ must, on average, be the same as the expected return of the market which also has a $\beta = 1.0$.

9. a. Call the aggressive stock A and the defensive stock D. Beta is the sensitivity of the stock's return to the market return, i.e., the change in the stock return per unit change in the market return. Therefore, we compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in the market return:

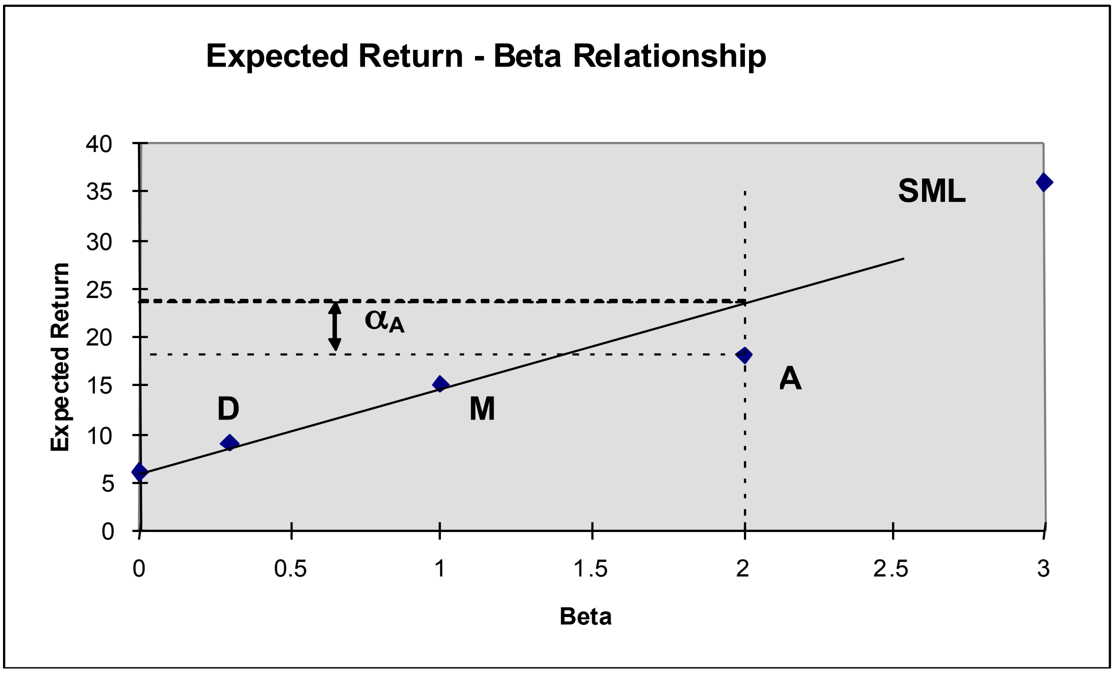
$$\beta_A = \frac{-.02 - .38}{.05 - .25} = 2.00 \qquad \beta_D = \frac{.06 - .12}{.05 - .25} = 0.30$$

   b. With the two scenarios equally likely, the expected return is an average of the two possible outcomes:

   $E(r_A) = 0.5 \times (-.02 + .38) = .18 = 18\%$

   $E(r_D) = 0.5 \times (.06 + .12) = .09 = 9\%$

   c. The SML is determined by the market expected return of [0.5 × (.25 + .05)] = 15%, with $\beta_M = 1$, and $r_f = 6\%$ (which has $\beta_f = 0$). See the following graph:



The equation for the security market line is:

$$E(r) = .06 + \beta \times (.15 - .06)$$

d. Based on its risk, the aggressive stock has a required expected return of:

$$E(r_A) = .06 + 2.0 \times (.15 - .06) = .24 = 24\%$$

The analyst's forecast of expected return is only 18%. Thus the stock's alpha is:

$$\alpha_A = \text{actually expected return} - \text{required return (given risk)}$$

$$= 18\% - 24\% = -6\%$$

Similarly, the required return for the defensive stock is:

$$E(r_D) = .06 + 0.3 \times (.15 - .06) = 8.7\%$$

The analyst's forecast of expected return for D is 9%, and hence, the stock has a positive alpha:

10. Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower than the expected return for Portfolio B. Thus, these two portfolios cannot exist in equilibrium.

11. Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk, represented by beta, rather than for the standard deviation, which includes nonsystematic risk. Thus, Portfolio A's lower rate of return can be paired with a higher standard deviation, as long as A's beta is less than B's.

12. Not possible. The reward-to-variability ratio for Portfolio A is better than that of the market. This scenario is impossible according to the CAPM because the CAPM predicts that the market is the most efficient portfolio. Using the numbers supplied:

$$S_A = \frac{.16 - .10}{.12} = 0.5 \qquad S_M = \frac{.18 - .10}{.24} = 0.33$$

Portfolio A provides a better risk-reward tradeoff than the market portfolio.

13. Not possible. Portfolio A clearly dominates the market portfolio. Portfolio A has both a lower standard deviation and a higher expected return.

14. Not possible. The SML for this scenario is: $E(r) = 10 + \beta \times (18 - 10)$

Portfolios with beta equal to 1.5 have an expected return equal to:

$$E(r) = 10 + [1.5 \times (18 - 10)] = 22\%$$

The expected return for Portfolio A is 16%; that is, Portfolio A plots below the SML ($\alpha_A = -6\%$), and hence, is an overpriced portfolio. This is inconsistent with the CAPM.

15. Not possible. The SML is the same as in Problem 14. Here, Portfolio A's required return is: $.10 + (0.9 \times .08) = 17.2\%$

This is greater than 16%. Portfolio A is overpriced with a negative alpha: $\alpha_A = -1.2\%$

16. Possible. The CML is the same as in Problem 12. Portfolio A plots below the CML, as any asset is expected to. This scenario is not inconsistent with the CAPM.