FOUNDATIONS OF FINANCE: EXPECTED UTILITY THEORY

EE 434 Behavioral Finance, SEM1/2022

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The foundation of finance

The foundations of modern finance are based on rational decision-making.

- Individuals should maximize utility subjecting to the constraints they face and all available information.

- Under risk, individuals should maximize expected utility subjecting to the constraints they face and all available information.

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01

RATIONAL PREFERENCES

Rational preferences

Suppose a person is confronted with the choice between two outcomes, x and y.

- The relation $x \succ y$ means that x is always the preferred choice when x and y are offered to some individual.
- The relation $x \sim y$ means that the person values the two outcomes the same.
- The relation $x \succcurlyeq y$ means that the person prefers x or is indifferent between x and y .

Rational preferences: Completeness

People's preferences are **complete.**

This means that a person can compare all possible choices and assess preference or indifference.

For any pair of choices x and y :

- $x \succcurlyeq y$ or
- $y \succcurlyeq x$ or
- both (which mean $x \sim y$)

People know what they like and what they do not like.

Rational preferences: Transitivity

People's preferences are **transitive.**

Suppose now that a person is confronted with a choice among three outcomes: $x, y, and\ z$.

If $x \succ y$ and $y \succ z$, then $x \succ z$.

If I prefer vanilla ice cream to chocolate, and chocolate to strawberry, I should also prefer vanilla to strawberry.

02

UTILITY MAXIMIZATION

A utility function

A utility function $u(\cdot)$ assigns numbers to possible outcomes so that preferred choices receive higher numbers. That is, a preference relation $\succcurlyeq$ can be represented by $u(\cdot)$.

If $x \succcurlyeq y$, then $U(x) > U(y)$.

Utility over goods:

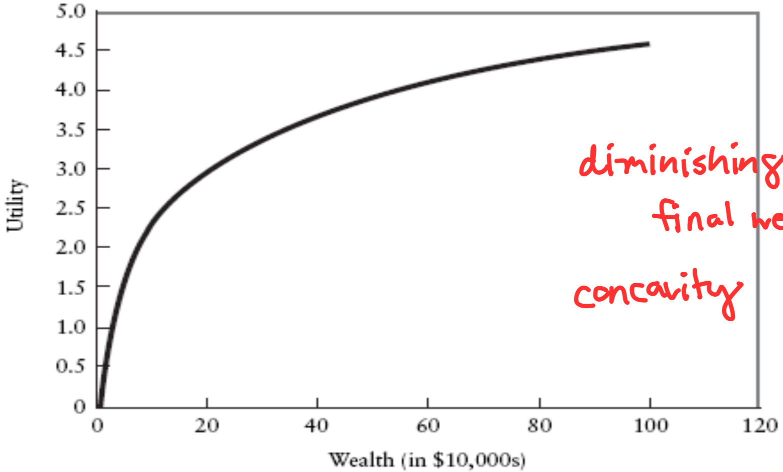
$$U(2\ bread, 1\ water) > U(1\ bread, 2\ water)$$

Utility over money:

$$U(W_2) > U(W_1)\ if\ W_2 > W_1$$

A utility function over wealth

$$U(w) = \ln(w)$$



diminishing mu of final wealth

concavity

Utility maximization

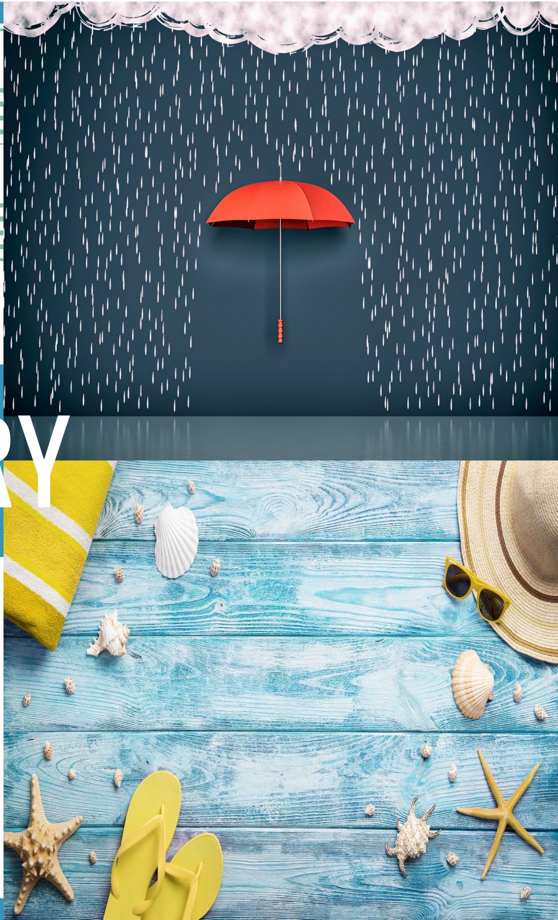
To arrive at her optimal choice, an individual considers all possible bundles of goods that satisfy her budget constraint (based on wealth or income), and then chooses the bundle that maximizes her utility.

Neoclassical economics assumes that people maximize their utility using full information of the choice set.

03

EXPECTED UTILITY THEORY

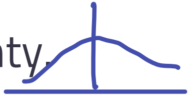
In financial decision-making,
there is clearly a great deal of uncertainty about outcomes.



Expected Utility Theory

"rationality"

- says that individuals *should* act when confronted with decision-making under risk in a certain way.
- The theory is "normative," which means that it describes how people should rationally behave
- Theory is really set up to deal with risk, not uncertainty.



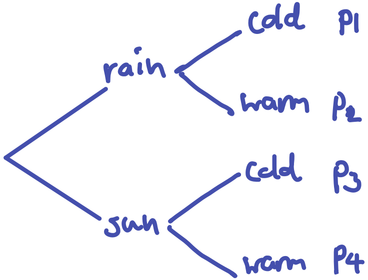
- **Risk** is when you know what the outcomes could be, and can assign probabilities. That is, risk is measurable using probability.
- **Uncertainty** is when you can't assign probabilities; or you can't come up with a list of possible outcomes

States of the world & wealth outcomes

Say there are a given number of states of the world:

A. rain or sun

B. cold or warm



Leading to 4 states: rain and cold, rain and warm, sun and cold, sun and warm

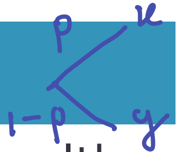
States of the world & wealth outcomes

And individuals can assign probabilities to each of these states. For example, probability of rain+cold is .1, etc.

Say income (or wealth) level can be assigned to each state of world. Think of an ice cream vendor:

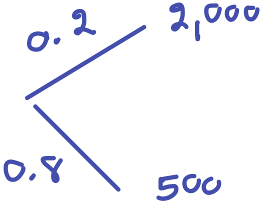
a. Rain+cold: Baht100/day
b. Sun+warm: Baht 500/day

Prospect $X = (x, p; y, 1 - p)$



A prospect is defined as a series of wealth or income levels and associated probabilities.

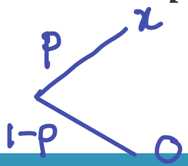
Example:



500 baht with probability .8
2,000 baht with probability .2

P1=(500, 0.8; 2,000, 0.2)

When 2nd option is zero $X = (x, p; 0, 1 - p), write\ X = (x, p)$



P2=(500, 0.8)

Prospect with n possible outcomes

$$X = (x_1, p_1; x_2, p_2; \ldots; x_n, p_n)$$

Payoffs: $(x_1, x_2, \ldots, x_n)$

Probabilities: $(p_1, p_2, \ldots, p_n)$,

where $p_1 + p_2 + \cdots + p_n = 1$

Expected Utility theory

Based on assumptions completeness and transitivity (and others), it can be shown that when such choices over risky prospects are to be made, people should act *as if* they are maximizing expected utility:

$$EU(X) = E[U(X)] = \sum_{i=1}^{n} p_i U(x_i)$$

Example 1: Choosing prospect

$$U(w) = w^{0.5}$$

Prospect: P3=(1,024, 0.5; 400, 0.5)

Prospect: P4=(1,225, 0.6; 256, 0.4)

Based on EUT, which prospect should you prefer?

Example 1: Answer

$$U(w) = w^{0.5}$$

↑final wealth

P3=(1,024, 0.5; 400, 0.5) vs. P4=(1,225, 0.6; 256, 0.4)

EU(P3)= $0.5U(1{,}024) + 0.5U(400)$
EU(P3)= $0.5(1{,}024)^{0.5} + 0.5(400)^{0.5} = 0.5\times 32 + 0.5\times 20 = 26$

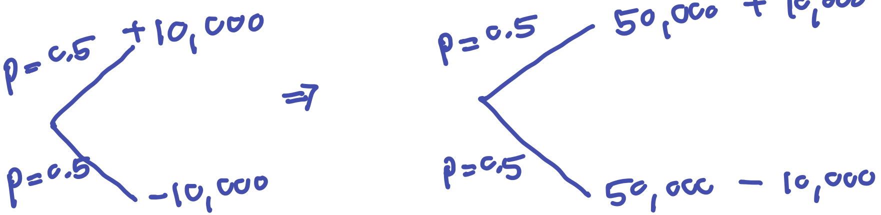
EU(P4)= $0.6U(1{,}225) + 0.4U(256)$
EU(P4)= $0.6(1{,}225)^{0.5} + 0.4(256)^{0.5} = 0.6\times 35 + 0.4\times 16 = 27.4$

$$EU(P4) > EU(P3) \;\therefore P4 \succ P3$$

Example 2: Accepting a bet?

Let's say that you now have 50,000 baht in income this month to spend.

If you are faced with a choice of accepting a coin toss which **pays you** 10,000 if you win (heads with p=.5 odds) or **you pay** 10,000 if you lose (tails with 1-p=.5 odds), should you accept the bet?



Expected value of accepting the bet =

$= 50{,}000 + 0.5(10{,}000) + 0.5(-10{,}000)$

$= 50{,}000$

Example 2: Answer

Based on EUT and let $U(w) = \ln(w)$

Expected value theory ?

Expected utility is the probability-weighted sum of utilities:

EU = .5*ln(50k + 10k) + .5*ln(50k – 10k) = .5*(11) + .5*(10.6)

EU = 10.80 expected utils.

Which compares to a utility of refusing the bet of:

EU = 1.0*ln(50k) = 10.82 certain utils.

$EU_{reject} > EU_{accept}$

$\Rightarrow$ reject the bet

Clearly, a person who would like to be happier would refuse the bet since 10.82 utils is better than 10.80 utils.

Expected utility on a graph

Consider prospect P5= (50,000, 0.4; 1,000,000, 0.6),

Let $U(w) = \ln(w)$

Expected utility from the prospect:

$$E[U(W)] = 0.4U(50{,}000) + 0.6U(1{,}000{,}000)$$

In 10,000 baht:

$$E[U(W)] = 0.4U(5) + 0.6U(100)$$

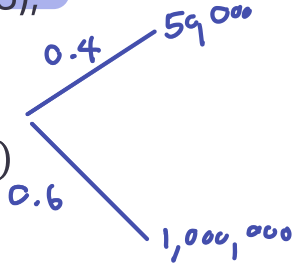
$$E[U(W)] = 0.4\times 1.6094 + 0.6\times 4.6052 = 3.41$$

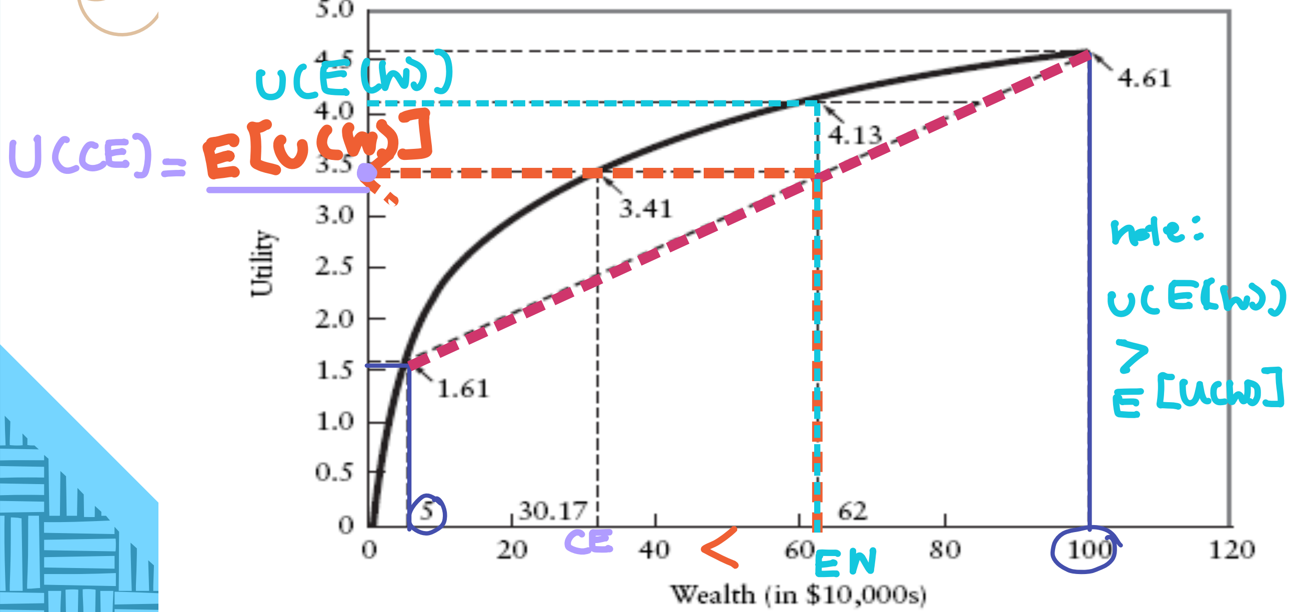
Expected value of the prospect is:

$$E[W] = 0.4\times 5 + 0.6\times 100 = 62$$

Utility of expected value of prospect:

$$U\big(E(W)\big) = \ln(62) = 4.13$$





5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
Utility
4.61
4.13
3.41
1.61
5
30.17
62
0
20
40
60
80
100
120
Wealth (in $10,000s)
U(E(W))
U(CE) = E[U(W)]
CE
<
EW
note:
U(E[W])
>
E[U(W)]

Certainty equivalent

Certainty equivalent is defined as the wealth level which leads decision-maker to be indifferent between a particular prospect and that certain wealth level.

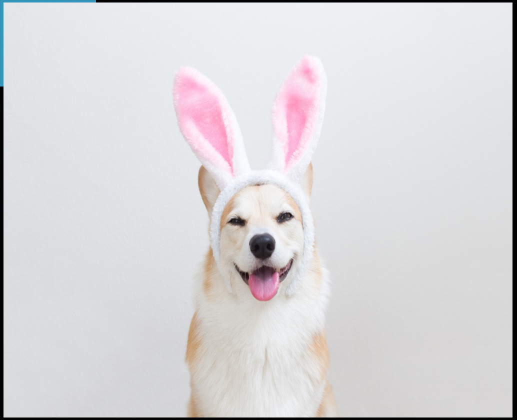
That is,

$$U(ce) = E[U(W)]$$

Thus, for P5, $U(ce) = E[U(W)]$ =3.41.
Hence, $ce = 30.17$

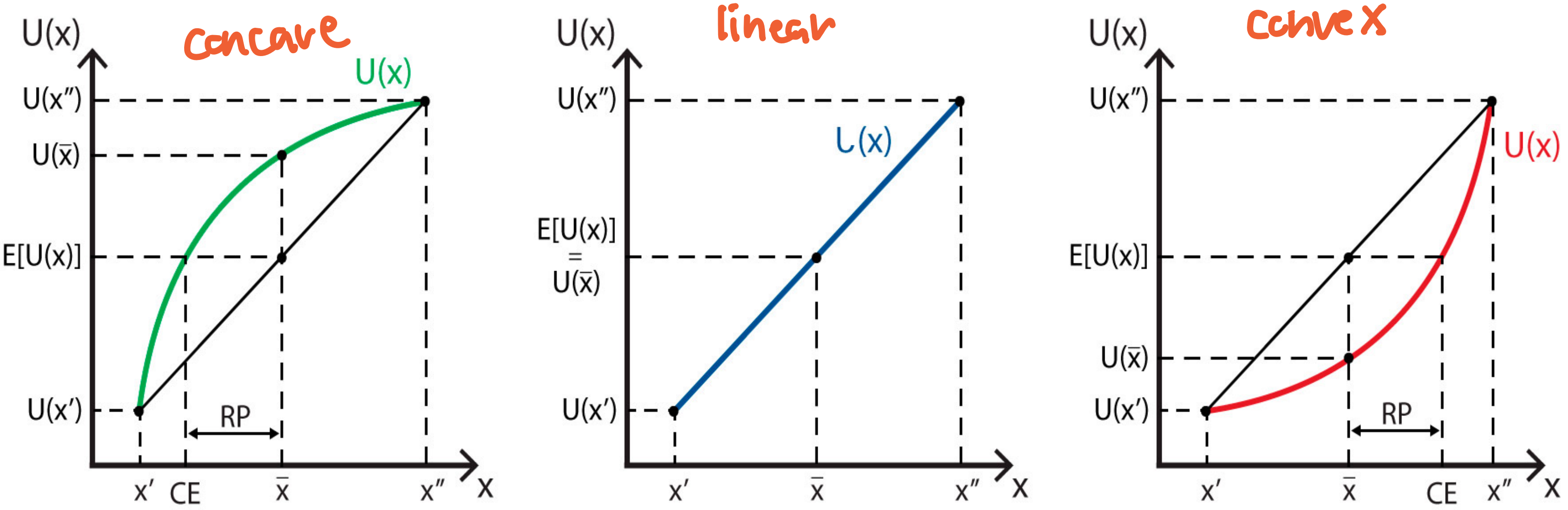
04

RISK ATTITUDE



Risk attitudes&Certainty equivalent $\boxed{U(ce) = E[U(X)]}$ $X = (x', p; x'', 1-p),\ E(X) = \bar{x}$

The **certainty equivalent** is an amount of money that provides equal utility to the random payoff of the gamble. The certainty equivalent is labeled *CE* in the figure. Note that *CE* is less than the expected outcome, if the person is risk averse. This is because risk-averse individuals prefer the expected outcome to the risky outcome.



Risk averse individual | Risk neutral individual | Risk loving individual

$E[U(x)] < U(\bar{x})$ | $E[U(x)] = U(\bar{x})$ | $E[U(x)] > U(\bar{x})$

$CE < \bar{x}$ | $CE = \bar{x}$ | $CE > \bar{x}$

Source: 1, 2

Risk aversion assumption

This comes from frequent observation that most people most of the time are not willing to accept a fair gamble:

Would you be willing to bet me $100 that you can predict a coin flip?

- Most would say no.
- And if one of you says yes, I will say no, since I am risk averse.

Risk aversion implies concavity.

Risk aversion

If an investor is risk-averse, when choosing between two stocks with the same expected return, he would invest in the one with the lower risk.

If he is going to take on a riskier investment, he will demand a higher return to compensate for the risk.

DANKE!