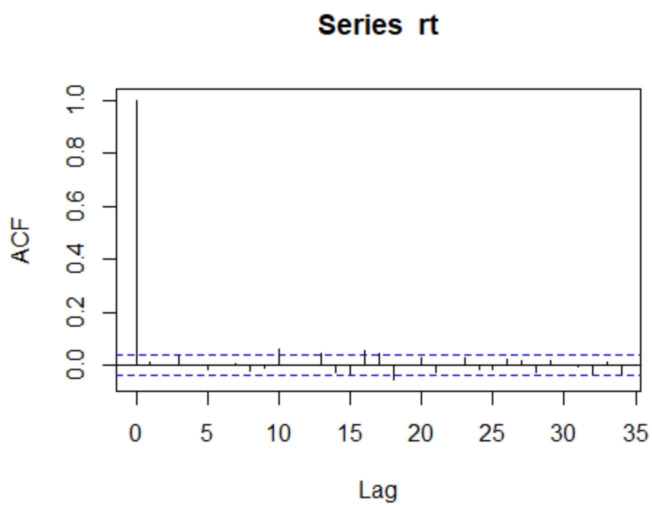
6104640542 Jak Aswadetmathakul

Question 1

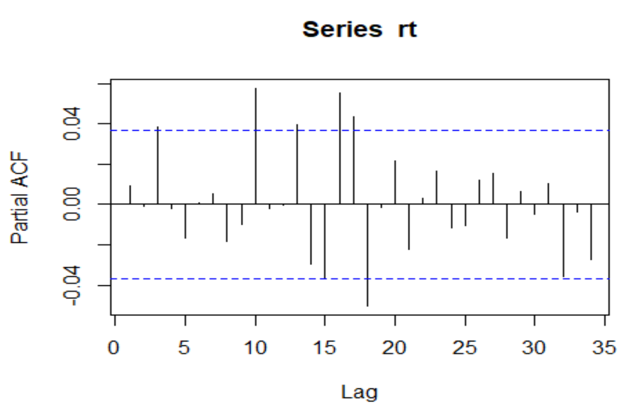
```
getSymbols("CAT",from="2006-01-03",to="2017-04-17")

## [1] "CAT"

rt <- diff(log(as.numeric(CAT[,6])))
```

1.a)

```
acf(rt)
```




```
pacf(rt)
```



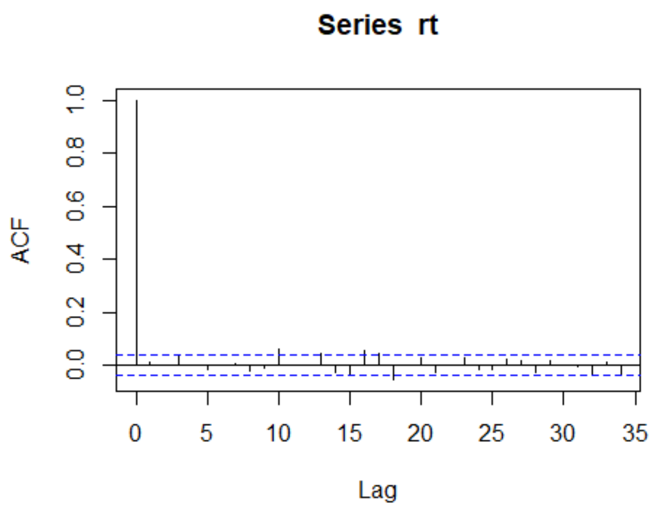

```
Box.test(rt, lag=10, type='Ljung')

##
## Box-Ljung test
##
## data:  rt
## X-squared = 16.229, df = 10, p-value = 0.09325
```

Question 1

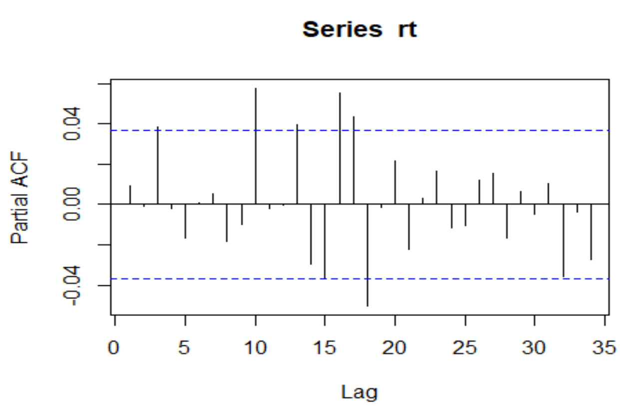
```
getSymbols("CAT",from="2006-01-03",to="2017-04-17")

## [1] "CAT"

rt <- diff(log(as.numeric(CAT[,6])))
```

1.a)

```
acf(rt)
```




```
pacf(rt)
```



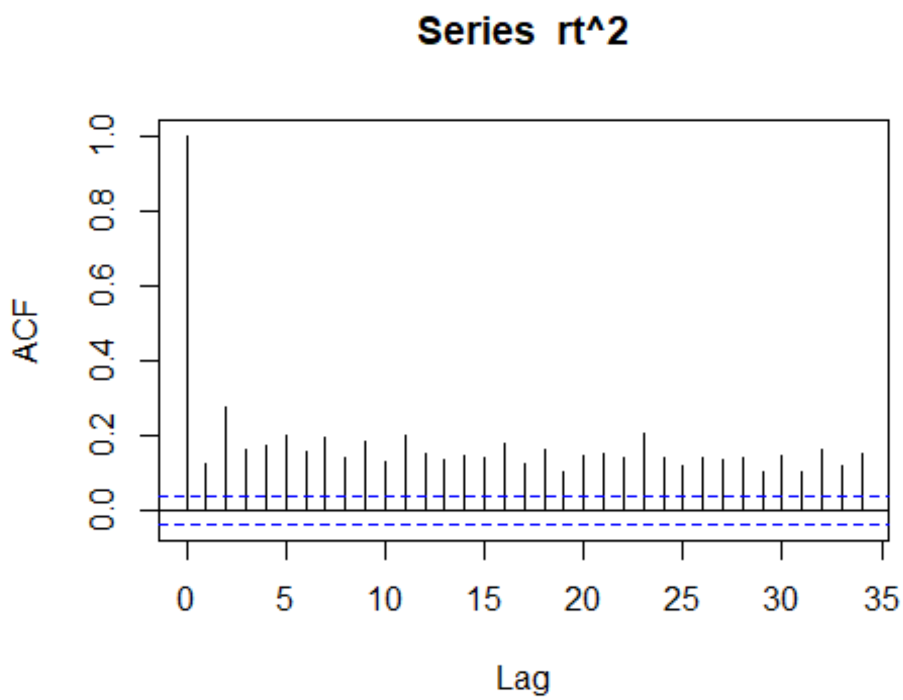

```
Box.test(rt, lag=10, type='Ljung')

##
## Box-Ljung test
##
## data:  rt
## X-squared = 16.229, df = 10, p-value = 0.09325
```

Ljung-Box Test : $H_0 : \rho_1 = \rho_2 = \dots = \rho_m = 0$ for r_t
(m=10) $H_1 : \exists \rho_i \neq 0$

ANS 1.a: There exists no serial correlation in log return rt with 95% CI, because the Ljung Box test of rt has p-value 0.09325 > 0.05 not rejecting null hypothesis of zero correlation among rt at 0.05 level of significance, implying efficient market. Also, the ACF and PACF indicates correlation value insignificantly differ from zero, showing no sign of serial correlation.

1.b)

```
acf(rt^2)
```




```
Box.test(rt^2, lag=10, type='Ljung')

##
## Box-Ljung test
##
## data:  rt^2
## X-squared = 917.58, df = 10, p-value < 2.2e-16
```

ANS 1.b: There exists ARCH effect in log return rt with 95% CI, because Ljung-Box test of rt^2 has p-value < 0.05, rejecting the null hypothesis of no non-linear dependent among rt at 0.05 level of significance. Also, ACF of rt^2 indicate pattern of values significantly differ from zero, confirming the non-linear dependent of log return rt (ARCH effect).

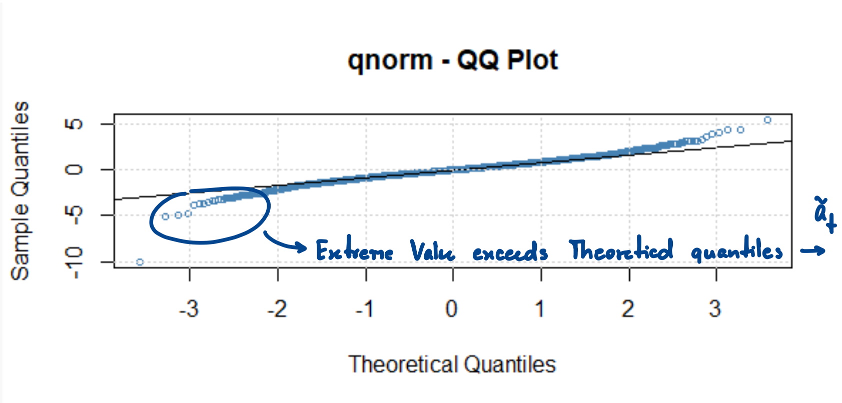
Test ARCH effect: $r_t^2 \sim a_t^2$

Ljung-Box . $H_0 : \rho_1 = \rho_2 = \dots = \rho_m$ of logko^2 / No ARCH effect
(m=10) $H_1 : \exists \rho_i \neq 0$ / There exists ARCH effect

1.c)

```
m1 <- garchFit(~ arma(1,0)+garch(1,1) ,data=rt ,trace=F)

summary(m1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt, trace = F)

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x000001d3e455d990>
 [data = rt]

Conditional Distribution:
 norm

Coefficient(s):
        mu         ar1       omega      alpha1       beta1
4.7490e-04  1.7677e-02  4.4860e-06  4.9755e-02  9.3861e-01

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     4.749e-04   3.075e-04    1.544 0.122480
ar1    1.768e-02   2.004e-02    0.882 0.377774
omega  4.486e-06   1.280e-06    3.503 0.000459 ***
```

```
alpha1 4.976e-02   8.200e-03    6.067  1.3e-09 ***
beta1  9.386e-01   1.032e-02   90.934  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 7381.067    normalized:  2.599883

Description:
 Tue Apr 27 20:33:15 2021 by user: ASUS


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  3293.722  0
 Shapiro-Wilk Test  R    W      0.9664334 0
 Ljung-Box Test     R    Q(10)  12.36538  0.2613468
 Ljung-Box Test     R    Q(15)  14.78355  0.4671183
 Ljung-Box Test     R    Q(20)  19.40031  0.4959589
 Ljung-Box Test     R^2  Q(10)  0.9807765 0.9998426
 Ljung-Box Test     R^2  Q(15)  3.684778  0.9986004
 Ljung-Box Test     R^2  Q(20)  6.924134  0.9969265
 LM Arch Test       R    TR^2   2.720581  0.9972158

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.196243 -5.185762 -5.196250 -5.192463
```



→ $\tilde{a}_t$ Series is Not normally distributed &

JB test : H_0 : series is normally dist^n
H_1 : series isn't normally dist^n

p-value < 0.05 ; H_0 is rejected at 0.05 level of sig.

That is, series not normally distributed with 95% CI

Ans 1.c) Model Checking on $\tilde{a}_t$: Standardized Residual

Ljung-Box Test	R	Q(10)	12.36538	0.2613468
Ljung-Box Test	R	Q(15)	14.78355	0.4671183
Ljung-Box Test	R	Q(20)	19.40031	0.4959589
Ljung-Box Test	R^2	Q(10)	0.9807765	0.9998426
Ljung-Box Test	R^2	Q(15)	3.684778	0.9986004
Ljung-Box Test	R^2	Q(20)	6.924134	0.9969265
LM Arch Test	R	TR^2	2.720581	0.9972158

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \ldots = \rho_m = 0$
$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t$ or Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$
H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = \ldots = \rho_m = 0$ for $\tilde{a}_t^2$
H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$ & LM Arch Test p-value > 0.05
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect
variance equation is appropriate with 95% CI

The model yields adequate mean & variance equation, but the distribution is not normal → requiring another dist^n

The fitted model →

Mean equation : $\hat{r}_t = 4.749e\text{-}04[1 - 0.01768] + 0.01768\, r_{t-1}$
(3.073e-4) (0.02004)

→ Coefficient of mean equation is insignificant

Variance equation : $\sigma_t^2 = 4.486e\text{-}06^{***} + 0.04976^{***}\, a_{t-1} + 0.9386^{***}\, \sigma_{t-1}^2$
(1.28e-06) (0.0082) (0.01032)

1.d)

```
m2 <- garchFit(~garch(1,1), data=rt, cond.dist = "std", trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of le
ngth > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m2)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001dd4f208>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 5.9008e-04  4.2138e-06  7.2389e-02  9.2028e-01  5.1049e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.901e-04   2.703e-04    2.183  0.02904 *
## omega  4.214e-06   1.574e-06    2.678  0.00741 **
## alpha1 7.239e-02   1.375e-02    5.265  1.4e-07 ***
## beta1  9.203e-01   1.474e-02   62.448  < 2e-16 ***
## shape  5.105e+00   4.839e-01   10.549  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7509.292    normalized:  2.645048
##
## Description:
##  Tue Apr 27 20:08:46 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
```

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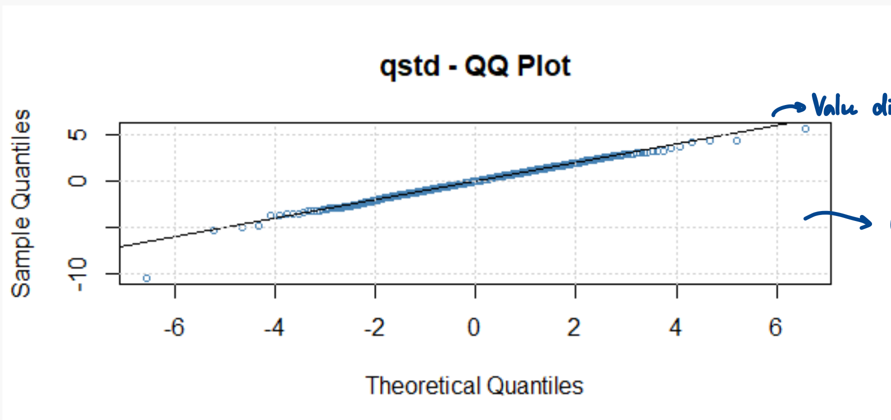
```
##  Jarque-Bera Test   R    Chi^2  4047.363  0
##  Shapiro-Wilk Test  R    W      0.9639883 0
##  Ljung-Box Test     R    Q(10)  14.87685  0.1366167
##  Ljung-Box Test     R    Q(15)  16.83467  0.3288433
##  Ljung-Box Test     R    Q(20)  20.67317  0.4165887
##  Ljung-Box Test     R^2  Q(10)  2.955969  0.9824413
##  Ljung-Box Test     R^2  Q(15)  5.488358  0.9871219
##  Ljung-Box Test     R^2  Q(20)  9.459063  0.9769545
##  LM Arch Test       R    TR^2   4.275692  0.9779321
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.286574 -5.276093 -5.286580 -5.282793
```



→ Value align in Theoretical quantile

→ $\tilde{a}_t \sim$ t-distribution

Ans 1.d) Model Checking on $\tilde{a}_t$: Standardized Residual

```
##  Ljung-Box Test     R    Q(10)  14.87685  0.1366167
##  Ljung-Box Test     R    Q(15)  16.83467  0.3288433
##  Ljung-Box Test     R    Q(20)  20.67317  0.4165887
##  Ljung-Box Test     R^2  Q(10)  2.955969  0.9824413
##  Ljung-Box Test     R^2  Q(15)  5.488358  0.9871219
##  Ljung-Box Test     R^2  Q(20)  9.459063  0.9769545
##  LM Arch Test       R    TR^2   4.275692  0.9779321
```

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$

$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t$ or

Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$

H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = \dots = \rho_m = 0$ for $\tilde{a}_t^2$

H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20 & LM Arch Test p-value > 0.05

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect

variance equation is appropriate with 95% CI

∴ Model is adequate for mean & variance equation

$a_t \sim t_{5.105}$

Ans 1.e) Mean equation : $\hat{r}_t = \underset{(0.0002703)}{0.0005901^{*}}$

Variance equation : $\sigma_t^2 = \underset{(1.574e-06)}{4.214e-06^{**}} + \underset{(0.01535)}{0.07239^{***}} a_{t-1}^2 + \underset{(0.01474)}{0.9203^{***}} \sigma_{t-1}^2$ $\quad a_t \sim t_{5.105}$

1.f)

```
predict(m2,5)

##   meanForecast  meanError standardDeviation
## 1 0.0005900796 0.01557857        0.01557857
## 2 0.0005900796 0.01565654        0.01565654
## 3 0.0005900796 0.01573356        0.01573356
## 4 0.0005900796 0.01580965        0.01580965
## 5 0.0005900796 0.01588481        0.01588481
```

Ans 1.f $\hat{r}_h(1) = \hat{r}_h(2) = \hat{r}_h(3) = \hat{r}_h(4) = \hat{r}_h(5) = 0.00059$

$\sigma_h(1) = 0.01557857 = \sqrt{var(e_h(1)|\cdot)}$

$\sigma_h(2) = 0.01565654 = \sqrt{var(e_h(2)|\cdot)}$

$\sigma_h(3) = 0.01573356 = \sqrt{var(e_h(3)|\cdot)}$

$\sigma_h(4) = 0.01580965 = \sqrt{var(e_h(4)|\cdot)}$

$\sigma_h(5) = 0.01588481 = \sqrt{var(e_h(5)|\cdot)}$

Ans 1.g :

1-step : $0.00059 \pm 1.96(0.01558) = [-0.0299, 0.0311]$ 95% CI

2-step : $0.00059 \pm 1.96(0.01566) = [-0.0301, 0.0313]$ 95% CI

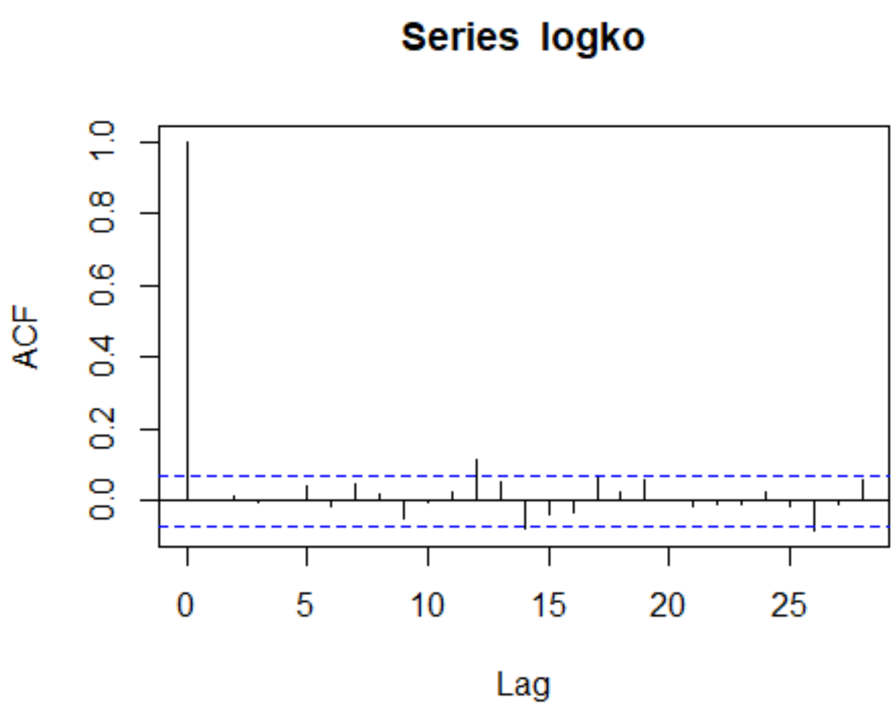
3-step : $0.00059 \pm 1.96(0.01573) = [-0.0302, 0.0314]$ 95% CI

4-step : $0.00059 \pm 1.96(0.01581) = [-0.0304, 0.0316]$ 95% CI

5-step : $0.00059 \pm 1.96(0.01588) = [-0.0305, 0.0317]$ 95% CI

2)

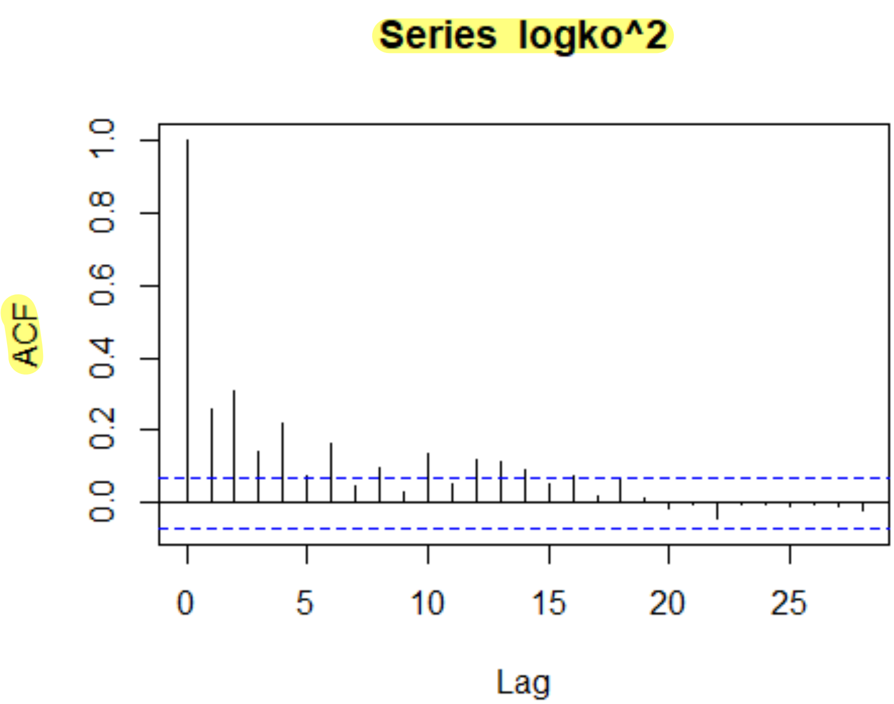
```
m.kovw.5116 <- read.csv("C:/Users/ASUS/Downloads/m-kovw-5116.txt", sep="")
ko <- m.kovw.5116[,3]
logko <- log(1+ko)

t.test(logko)

##
##  One Sample t-test
##
## data:  logko
## t = 4.9853, df = 779, p-value = 7.628e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.00625636 0.01438347
## sample estimates:
##  mean of x
## 0.01031992

acf(logko)
```




```
Box.test(logko, lag=10, type='Ljung')

##
##  Box-Ljung test
##
## data:  logko
## X-squared = 5.9201, df = 10, p-value = 0.8219
```

> 0.05

```
acf(logko^2)
```




```
Box.test(logko^2, lag=10, type='Ljung')

##
##  Box-Ljung test
##
## data:  logko^2
## X-squared = 228.23, df = 10, p-value < 2.2e-16
```

< 0.05

Ans 2a. T-test : $H_0: \mu$ of log return KO (logko) $= 0$

$H_1: \mu \neq 0$

The computed p-value : 7.623e-07 < 0.05. H_0 is rejected at 0.05 level of significance

That is, $\mu \neq 0$ with 95% CI

Ljung-Box Test (m=10): $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$ for logko

$H_1: \exists \rho_i \neq 0$

There exists no serial correlation in log return KO with 95% CI, because the Ljung Box test of logko has p-value 0.08219 > 0.05 not rejecting null hypothesis of zero correlation among rt at 0.05 level of significance, implying efficient market. Also, the ACF indicates correlation value insignificantly differ from zero, showing no sign of serial correlation.

Test ARCH effect: $logko^2 \sim a_t^2$

Ljung-Box (m=10). $H_0: \rho_1 = \rho_2 = \dots = \rho_m$ of $logko^2$ / No ARCH effect

$H_1: \exists \rho_i \neq 0$ / There exists ARCH effect

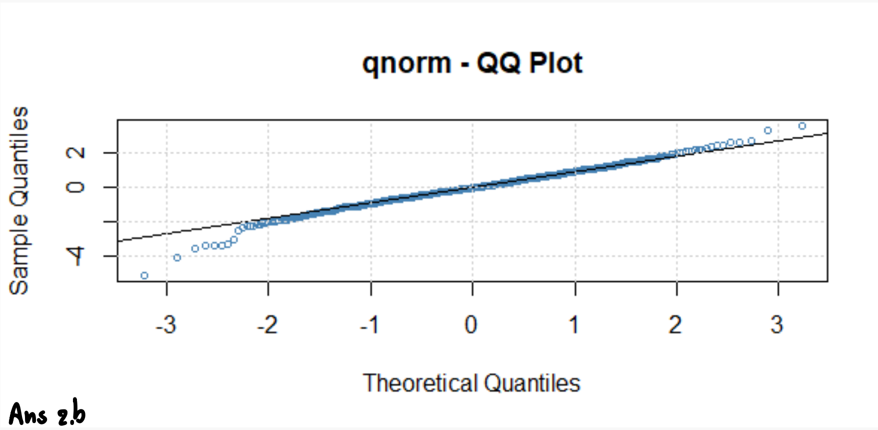
There exists ARCH effect in log return ko with 95% CI, because Ljung-Box test of logko^2 has p-value < 0.05, rejecting the null hypothesis of no non-linear dependent among logko at 0.05 level of significance. Also, ACF of logko^2 indicate pattern of values significantly differ from zero, confirming the non-linear dependent of log return ko (ARCH effect).

2.b)

```
m1 <- garchFit(~ arma(1,0)+garch(1,1) ,data=logko ,trace=F)

summary(m1)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = logko, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001f7ed880>
##  [data = logko]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124544  -0.02633742   0.00018112   0.09535029   0.84861593
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.125e-02   1.897e-03    5.929 3.05e-09 ***
## ar1    -2.634e-02   3.881e-02   -0.679  0.49740
## omega   1.811e-04   5.852e-05    3.095  0.00197 **
## alpha1  9.535e-02   1.915e-02    4.978 6.42e-07 ***
## beta1   8.486e-01   2.766e-02   30.675  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.664    normalized:  1.500852
##
## Description:
##  Tue Apr 27 20:42:30 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  92.91946  0            < 0.05
##  Shapiro-Wilk Test  R    W      0.9857081 6.655604e-07
##  Ljung-Box Test     R    Q(10)  9.306169  0.5033144  }
##  Ljung-Box Test     R    Q(15)  22.9901   0.0843502  }  > 0.05
##  Ljung-Box Test     R    Q(20)  27.44814  0.1231201  }
```

```
##  Ljung-Box Test     R^2  Q(10)  12.63377  0.2448749
##  Ljung-Box Test     R^2  Q(15)  13.62088  0.5544545
##  Ljung-Box Test     R^2  Q(20)  15.19817  0.7649584
##  LM Arch Test       R    TR^2   10.65102  0.5590389
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.988883 -2.959016 -2.988965 -2.977396
```

} > 0.05



JB test : H_0: series is normally dist^n
H_1 : series isn't normally dist^n
p-value < 0.05 ; H_0 is rejected at 0.05 level of sig.
That is, series not normally distributed with 95% CI

Ans 2.b

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1=\rho_2=\dots=\rho_m=0$
$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20
Ho is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t$ or
Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$
H_0 : there is no ARCH effect or $\rho_1=\rho_2=\dots=\rho_m=0$ for $\tilde{a}_t^2$
H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20 & LM Arch Test p-value > 0.05
Ho is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect
variance equation is appropriate with 95% CI

∴ Mean & Var. Model is adequate but Distribution is not Normal

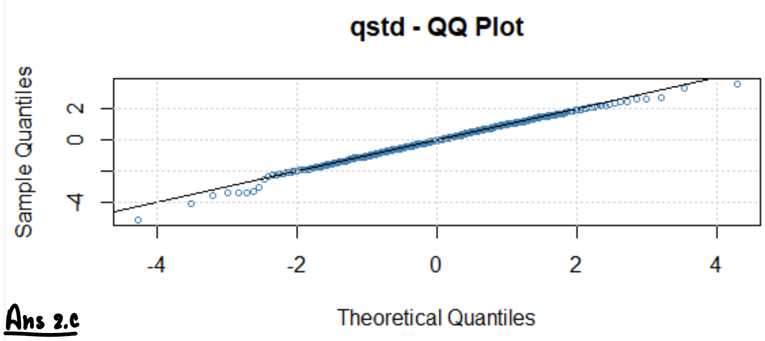
M1: Fitted equation → Mean : $\hat{r}_t = \underset{(0.0019)}{0.0125^{***}}(1+0.02634) - \underset{(0.02881)}{0.02634}\, r_{t-1}$ → insignificant coefficient of r_{t-1}
ARMA(1,0) + GARCH(1,1)

Variance : $\sigma_t^2 = \underset{(5.352e-05)}{0.0003811^{**}} + \underset{(0.01913)}{0.09535^{***}}\, a_{t-1}^2 + \underset{(0.02766)}{0.8486^{***}}\, \sigma_{t-1}^2$

2.c)

```
m2 <- garchFit(~arma(1,0)+garch(1,1), data=logko, cond.dist = "std", trace=F)

summary(m2)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = logko, cond.dist = "
std",
##     trace = F)
##
## Mean and Variance Equation:
```

```
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x0000000020efbb28>
##  [data = logko]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1          sha
pe
##  0.01124020  -0.01887601   0.00017395   0.09642927   0.85044151   7.478777
80
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.124e-02   1.810e-03    6.211 5.27e-10 ***
## ar1    -1.888e-02   3.691e-02   -0.511  0.60904
## omega   1.739e-04   6.596e-05    2.637  0.00836 **
## alpha1  9.643e-02   2.338e-02    4.124 3.72e-05 ***
## beta1   8.504e-01   3.267e-02   26.028  < 2e-16 ***
## shape   7.479e+00   1.840e+00    4.066 4.79e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.863    normalized:  1.519055
##
## Description:
##  Tue Apr 27 20:42:30 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  93.6433   0
##  Shapiro-Wilk Test  R    W      0.9857385 6.832848e-07
##  Ljung-Box Test     R    Q(10)  8.966733  0.5352637    } >0.05
##  Ljung-Box Test     R    Q(15)  22.44818  0.09657967   }
##  Ljung-Box Test     R    Q(20)  26.86769  0.1390276    }
##  Ljung-Box Test     R^2  Q(10)  12.48941  0.2536355    } >0.05
##  Ljung-Box Test     R^2  Q(15)  13.37442  0.5734021    }
##  Ljung-Box Test     R^2  Q(20)  14.90709  0.7816987    }
##  LM Arch Test       R    TR^2   10.48089  0.5738501    }
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.022725 -2.986885 -3.022843 -3.008941
```



Ans 2.c

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$

$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t$ or

Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$

H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = \dots = \rho_m = 0$ for $\tilde{a}_t^2$

H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20 & LM Arch Test p-value > 0.05

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect

variance equation is appropriate with 95% CI

∴ Mean & variance equation is adequate

QQ plot align with t-dist$^{\underline{n}}$

M2 Fitted eq. : ARMA(1,0) + GARCH(1,1), $a_t \sim$ t-dist.

Mean : $\hat{r}_t = \underset{(0.00181)}{\overset{***}{0.0124}}(1 + 0.01888) - 0.01888\, r_{t-1}$ → insignificant coefficient of r_{t-1}

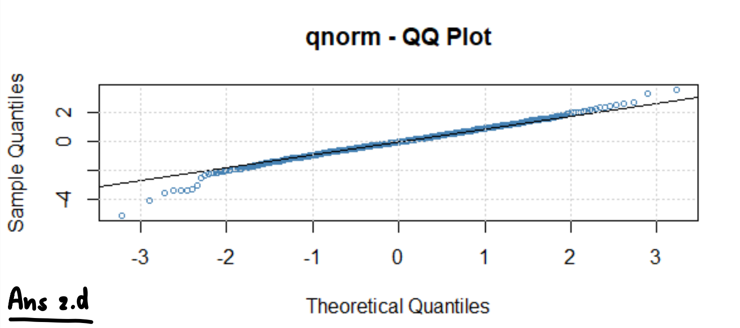
Variance : $\sigma_t^2 = \underset{(6.596e-05)}{\overset{**}{1.739e-04}} + \underset{(0.02338)}{\overset{***}{0.09643}}\, a_{t-1}^2 + \underset{(0.03217)}{\overset{***}{0.08504}}\, \sigma_{t-1}^2$

$a_t \sim t_{7.479}$

2.d)

```
m3 <- garchFit(~garch(1,1) ,data=logko ,trace=F)

summary(m3)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = logko, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x0000000020992f68>
##  [data = logko]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 0.01098417  0.00018497  0.09479925  0.84780406
##
## Std. Errors:
```

```
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.098e-02   1.846e-03    5.950 2.68e-09 ***
## omega  1.850e-04   5.899e-05    3.135  0.00172 **
## alpha1 9.480e-02   1.912e-02    4.958 7.11e-07 ***
## beta1  8.478e-01   2.787e-02   30.416  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.393    normalized:  1.500504
##
## Description:
##  Tue Apr 27 20:42:30 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.07163  0        < 0.05
##  Shapiro-Wilk Test  R    W      0.9856773 6.481596e-07
##  Ljung-Box Test     R    Q(10)  8.125181  0.6166108  }
##  Ljung-Box Test     R    Q(15)  21.27199  0.128362   } > 0.05
##  Ljung-Box Test     R    Q(20)  25.62765  0.1784646  }
##  Ljung-Box Test     R^2  Q(10)  12.90586  0.228983   }
##  Ljung-Box Test     R^2  Q(15)  13.87463  0.5350581  } > 0.05
##  Ljung-Box Test     R^2  Q(20)  15.35522  0.755734   }
##  LM Arch Test       R    TR^2   10.96004  0.532346   }
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.990752 -2.966858 -2.990804 -2.981562
```



Ans 2.d

JB test : H_0 : series is normally dist^n
H_1 : series isn't normally dist^n
p-value < 0.05 ; H_0 is rejected at 0.05 level of sig.
That is, series not normally distributed with 95% CI

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = .. = \rho_m = 0$
$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t$ or
Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$
H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = .. = \rho_m = 0$ for $\tilde{a}_t^2$
H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20 & LM Arch Test p-value > 0.05
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect
variance equation is appropriate with 95% CI

∴ Mean & Var. Model is adequate but Distribution is not Normal

M3 fitted eq : Mean eq. : $\hat{r}_t = \underset{(0.001846)}{0.01098^{***}}$

GARCH (1,1)

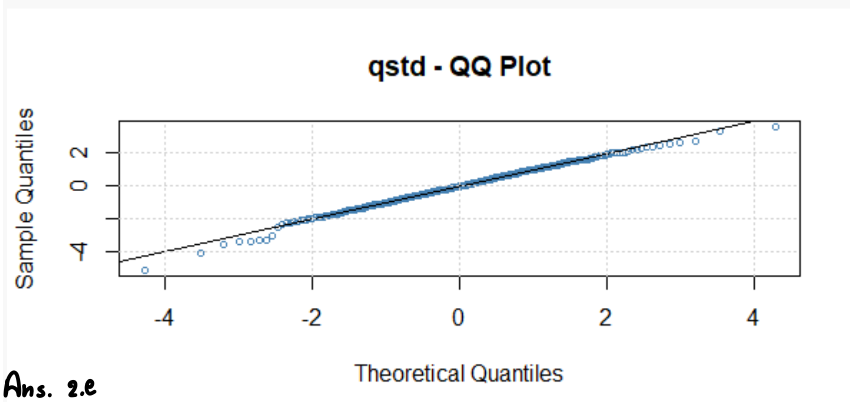
Variance eq : $\sigma_t^2 = \underset{(5.899e-05)}{1.85e-04^{**}} + \underset{(0.01912)}{0.0948^{***}} a_{t-1}^2 + \underset{(0.0287)}{0.8478^{***}} \sigma_{t-1}^2$

2.e)

```
m4 <- garchFit(~garch(1,1), data=logko, cond.dist = "std", trace=F)

summary(m4)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = logko, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001d95e758>
##  [data = logko]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
```

```
## Description:
##  Tue Apr 27 20:42:30 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
##  Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
##  LM Arch Test       R    TR^2   10.62989  0.560875
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

(Handwritten beside Ljung-Box R rows: > 0.05; beside Ljung-Box R^2 rows and LM Arch Test: > 0.05)



Ans. 2.e

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1=\rho_2=\dots=\rho_m=0$

$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$

Ho is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t$ or

Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$

H_0 : there is no ARCH effect or $\rho_1=\rho_2=\dots=\rho_m=0$ for $\tilde{a}_t^2$

H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$ & LM Arch Test p-value > 0.05

Ho is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect

variance equation is appropriate with 95% CI

∴ Mean & Var. Model is adequate

& QQ plot align with t-dist^n

M4 : fitted eq → Mean : $\hat{r}_t = \underset{(0.001757)}{0.0105^{***}}$

$a_t \sim t_{7.486}$

Variance : $\sigma_t^2 = \underset{(6.627e-05)}{1.753e-04^{**}} + \underset{(0.02337)}{0.09633^{***}}\, a_{t-1}^2 + \underset{(0.0327)}{0.8501^{***}}\, \sigma_{t-1}^2$

6104640542 Jak Aswadetmathakul

Ans. 2.f

Model m4 from 2.e) is to be selected from its
GARCH (1,1) w/ t-dist^{n}

1. Model Adequacy for both mean & variance
2. Significant coefficient estimates
3. Lowest AIC & BIC value among m1, m2, m3, m4

A5_Q3.R

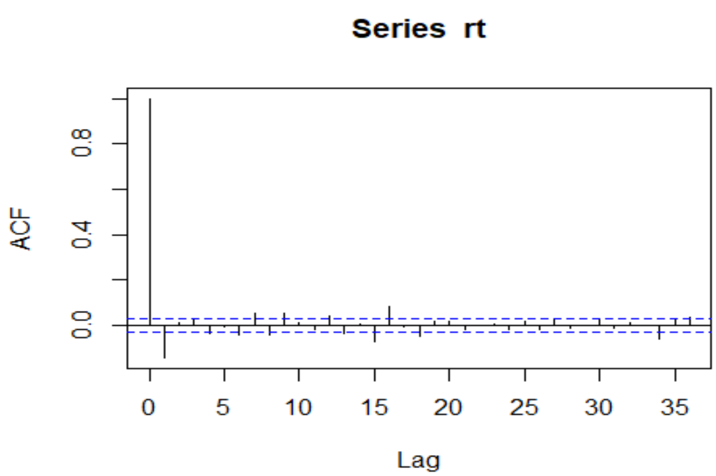
ASUS

2021-04-28

```
getSymbols("^GSPC",from="2005-01-02",to="2021-03-31")

## [1] "^GSPC"

rt <- diff(log(as.numeric(GSPC[,6])))*100
```

3.Q1

```
t.test(rt)

##
##  One Sample t-test
##
## data:  rt
## t = 1.4961, df = 4086, p-value = 0.1347
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.009051939  0.067374643
## sample estimates:
##  mean of x
## 0.02916135

acf(rt)
```




```
pacf(rt)
```

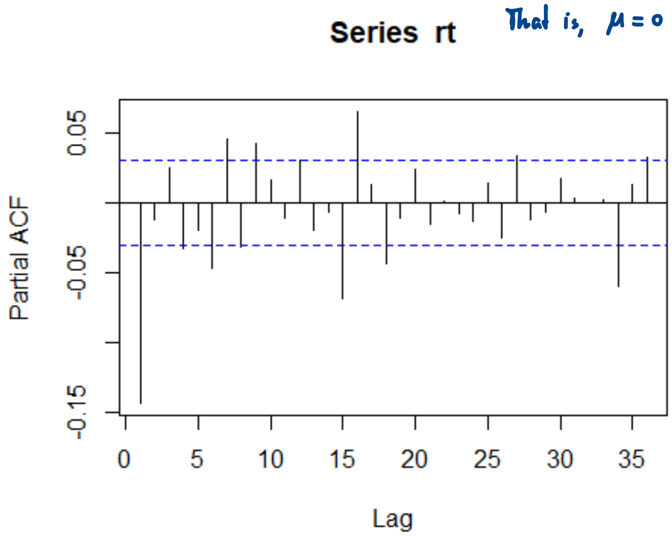
Ans 3.a

T-test : $H_0: \mu$ of log return $(r_t) = 0$ / expected value of $r_t = 0$

$H_1: \mu \neq 0$

The computed p-value : $0.1347 > 0.05$. H_0 not rejected at 0.05 level of significance

That is, $\mu = 0$ with 95% CI



Ljung-Box Test : $H_0: \rho_1 = \rho_2 = .. = \rho_m = 0$ for r_t $(m=10)$

$H_1: \exists \rho_i \neq 0$

There exists serial correlation in log return rt with 95% CI, because the Ljung Box test of rt has p-value 2.2e-16<0.05 rejecting null hypothesis of zero correlation among rt at 0.05 level of significance, implying serial correlation in rt. Also, the ACF indicates correlation value significantly differ from zero, showing sign of serial correlation.

```
Box.test(rt, lag=10, type='Ljung')

##
##  Box-Ljung test
##
## data:  rt
## X-squared = 131.85, df = 10, p-value < 2.2e-16  < 0.05
```

3.b)

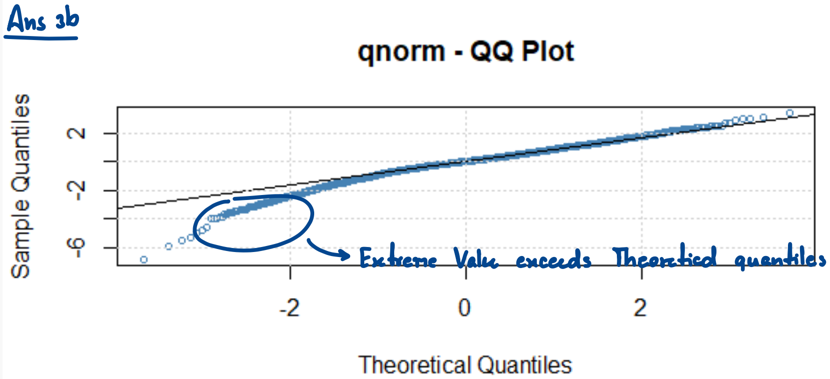
```
auto.arima(rt)

## Series: rt
## ARIMA(2,0,1) with non-zero mean
##
## Coefficients:
##           ar1      ar2     ma1    mean
##       -0.9468  -0.1434  0.8010  0.0292
## s.e.   0.0673   0.0164  0.0662  0.0166
##
## sigma^2 estimated as 1.518:  log likelihood=-6650.61
## AIC=13311.21   AICc=13311.23   BIC=13342.79

m1 <- garchFit(~arma(2,1)+garch(1,1), data=rt, trace=F)

## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.639559 2.650376 2.639553 2.643389

m1 <- garchFit(~arma(2,1)+garch(1,2), data=rt, trace=F)

## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.640235 2.652597 2.640228 2.644613

m1 <- garchFit(~arma(2,1)+garch(2,2), data=rt, trace=F)
```

```
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC 
## 2.637229 2.651136 2.637219 2.642153

m1 <- garchFit(~arma(2,1)+garch(2,1), data=rt, trace=F)

summary(m1)

## 
## Title:
##  GARCH Modelling 
## 
## Call:
##  garchFit(formula = ~arma(2, 1) + garch(2, 1), data = rt, trace = F) 
## 
## Mean and Variance Equation:
##  data ~ arma(2, 1) + garch(2, 1)
## <environment: 0x000000001e92d148>
##  [data = rt]
## 
## Conditional Distribution:
##  norm 
## 
## Coefficient(s):
##         mu         ar1         ar2         ma1       omega      alpha1  
##  0.0050029   0.8804915   0.0487681  -0.9563119   0.0330500   0.0928626  
##     alpha2       beta1  
##  0.0714233   0.8099714  
## 
## Std. Errors:
##  based on Hessian 
## 
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)    
## mu      0.005003    0.002720    1.839 0.065861 .  
## ar1     0.880491    0.033761   26.080  < 2e-16 ***
## ar2     0.048768    0.017464    2.792 0.005231 ** 
## ma1    -0.956312    0.027934  -34.235  < 2e-16 ***
## omega   0.033050    0.004608    7.172 7.39e-13 ***
## alpha1  0.092863    0.016794    5.530 3.21e-08 ***
## alpha2  0.071423    0.021414    3.335 0.000852 ***
## beta1   0.809971    0.016262   49.806  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Log Likelihood:
##  -5382.088    normalized:  -1.31688 
## 
## Description:
```

```
##  Wed Apr 28 20:12:46 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1195.199  0
##  Shapiro-Wilk Test  R    W      0.9701498 0
##  Ljung-Box Test     R    Q(10)  14.64738  0.1454563
##  Ljung-Box Test     R    Q(15)  19.46584  0.1933948
##  Ljung-Box Test     R    Q(20)  29.29348  0.08213946
##  Ljung-Box Test     R^2  Q(10)  15.57838  0.1123545
##  Ljung-Box Test     R^2  Q(15)  17.55699  0.2866712
##  Ljung-Box Test     R^2  Q(20)  18.89051  0.5289531
##  LM Arch Test       R    TR^2   16.21643  0.1815229
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.637674 2.650037 2.637667 2.642052
```

(Handwritten notes beside the output: Jarque-Bera p-value 0 < 0.05; Ljung-Box R, Q(10)–Q(20): > 0.05; Ljung-Box R^2 Q(10)–Q(20) and LM Arch: > 0.05)

Ans 3b



Extreme Value exceeds Theoretical quantiles → $\tilde{a}_t$ Series is Not normally distributed

Model : ARMA(2,1) + GARCH (2,1) is the most appropriate because of its lowest AIC, BIC and adequacy of mean and variance equation compared to other models detected with some inadequacy in Box test

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$

$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t$ or

Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$

H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = \dots = \rho_m = 0$ for $\tilde{a}_t^2$

H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$ & LM Arch Test p-value > 0.05

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect

variance equation is appropriate with 95% CI

Still, the distribution isn't Normal :

JB test : H_0 : series is normally dist^n

H_1 : series isn't normally dist^n

p-value < 0.05 ; H_0 is rejected at 0.05 level of sig.

That is, series not normally distributed with 95% CI

Fitted equation → Mean : $\hat{r}_t = \underset{(0.00272)}{0.005003}[1-0.88-0.0487] + \underset{(0.033761)}{0.880491}^{***} r_{t-1} + \underset{(0.017464)}{0.048768}^{**} r_{t-1} - \underset{(0.0279)}{0.956312}^{***} a_{t-1}$

Variance : $\sigma_t^2 = \underset{(0.0046)}{0.03305}^{***} + \underset{(0.016794)}{0.092863}^{***} a_{t-1}^2 + \underset{(0.0214)}{0.071423}^{***} a_{t-2}^2 + \underset{(0.01626)}{0.809971}^{***} \sigma_{t-1}^2$

3C)

```
m2 <- garchFit(~arma(2,1)+garch(2,1), data=rt, cond.dist = "std", trace=F)

summary(m2)

## Mean and Variance Equation:
##  data ~ arma(2, 1) + garch(2, 1)
## <environment: 0x000000001ef9aa30>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu         ar1         ar2         ma1       omega      alpha1     alpha
2
##  0.014320   0.810295   0.027506  -0.885775   0.024294   0.064072   0.11425
1
##     beta1      shape
##  0.817398   4.980315
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.014320    0.004954    2.891  0.00384 **
## ar1     0.810295    0.046448   17.445  < 2e-16 ***
## ar2     0.027506    0.017408    1.580  0.11409
## ma1    -0.885775    0.043800  -20.223  < 2e-16 ***
## omega   0.024294    0.005140    4.726 2.29e-06 ***
## alpha1  0.064072    0.019848    3.228  0.00125 **
## alpha2  0.114251    0.027763    4.115 3.87e-05 ***
## beta1   0.817398    0.018815   43.444  < 2e-16 ***
## shape   4.980315    0.410321   12.138  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -5240.369    normalized:  -1.282204
##
## Description:
##  Wed Apr 28 20:12:49 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1607.046  0
##  Shapiro-Wilk Test  R    W      0.9663432 0
##  Ljung-Box Test     R    Q(10)  15.74732  0.1070976
##  Ljung-Box Test     R    Q(15)  21.39762  0.1245991
```

```
##  Ljung-Box Test     R    Q(20)  27.69856  0.1167292
##  Ljung-Box Test     R^2  Q(10)  19.59868  0.03328531
##  Ljung-Box Test     R^2  Q(15)  23.64604  0.07135342
##  Ljung-Box Test     R^2  Q(20)  26.16611  0.1603958
##  LM Arch Test       R    TR^2   20.62897  0.0560848
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.568813 2.582721 2.568803 2.573737
```

```
m2 <- garchFit(~arma(2,1)+garch(1,1), data=rt, cond.dist = "std", trace=F)
```

```
summary(m2)
```

```
## Mean and Variance Equation:
##  data ~ arma(2, 1) + garch(1, 1)
## <environment: 0x0000000022a4c358>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu         ar1         ar2         ma1       omega      alpha1      beta
1
##  0.014227   0.810015   0.028283  -0.885918   0.016755   0.140909   0.85727
7
##     shape
##  5.015345
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.014227    0.004866    2.924  0.00346 **
## ar1     0.810015    0.046076   17.580  < 2e-16 ***
## ar2     0.028283    0.017750    1.593  0.11108
## ma1    -0.885918    0.043097  -20.556  < 2e-16 ***
## omega   0.016755    0.003588    4.670 3.01e-06 ***
## alpha1  0.140909    0.014562    9.676  < 2e-16 ***
## beta1   0.857277    0.012751   67.235  < 2e-16 ***
## shape   5.015345    0.416421   12.044  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -5248.092    normalized:  -1.284094
##
```

```
## Description:
##  Wed Apr 28 20:35:10 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1523.243  0
##  Shapiro-Wilk Test  R    W      0.9674506 0
##  Ljung-Box Test     R    Q(10)  16.18298  0.09451293
##  Ljung-Box Test     R    Q(15)  21.71156  0.1155933
##  Ljung-Box Test     R    Q(20)  27.93394  0.1109725
##  Ljung-Box Test     R^2  Q(10)  14.81597  0.1389166
##  Ljung-Box Test     R^2  Q(15)  19.22039  0.20392
##  Ljung-Box Test     R^2  Q(20)  22.51641  0.3131583
##  LM Arch Test       R    TR^2   16.17086  0.183535
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.572103 2.584465 2.572095 2.576480
```

(Ljung-Box R: } > 0.05; Ljung-Box R^2 and LM Arch: } > 0.05)

Ans 3c

t-distribution

Model: ARMA(2,1) + GARCH(1,1) is the most appropriate. Though it doesn't contain lowest AIC/BIC, it has adequacy of mean and variance equation compared to other models detected with some inadequacy in Box test

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$

$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$

Ho is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t$ or

Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$

H_0: there is no ARCH effect or $\rho_1 = \rho_2 = \dots = \rho_m = 0$ for $\tilde{a}_t^2$

H_1: there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$ & LM Arch Test p-value > 0.05

Ho is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect

variance equation is appropriate with 95% CI

Fitted equation → Mean: $\hat{r}_t = \underset{(0.0049)}{0.014227^{***}}[1 - 0.81 - 0.0283] + \underset{(0.046)}{0.81^{***}} r_{t-1} + \underset{(0.01773)}{0.028} r_{t-2} - \underset{(0.043)}{0.886^{***}} a_{t-1}$

Variance: $\sigma_t^2 = \underset{(0.00359)}{0.016733^{***}} + \underset{(0.01456)}{0.1409^{***}} a_{t-1}^2 + \underset{(0.012731)}{0.857277^{***}} \sigma_{t-1}^2$ $\quad a_t \sim t_{5.015}$

Ans 3d

```
predict(m2,5)
```

from ARMA (2,1) + GARCH (1,1) ~ t-dist.

```
##   meanForecast meanError standardDeviation
## 1   0.09405845 0.8896828         0.8896828
## 2   0.08147010 0.9007860         0.8982512
## 3   0.08287907 0.9097626         0.9067232
## 4   0.08366431 0.9184822         0.9151017
## 5   0.08434023 0.9270045         0.9233892
```

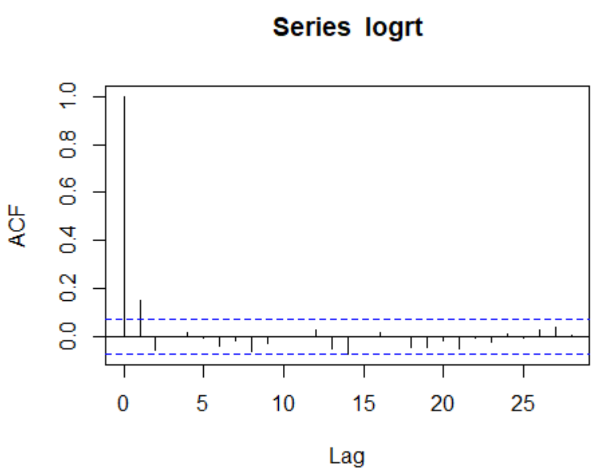
$\hat{r}_h(1) = 0.09405845$
$\hat{r}_h(2) = 0.08147010$
$\hat{r}_h(3) = 0.08287907$
$\hat{r}_h(4) = 0.08366431$
$\hat{r}_h(5) = 0.08434023$

$\sigma_h(1) = 0.8896828$
$\sigma_h(2) = 0.8982512$
$\sigma_h(3) = 0.9067232$
$\sigma_h(4) = 0.9151017$
$\sigma_h(5) = 0.9233892$

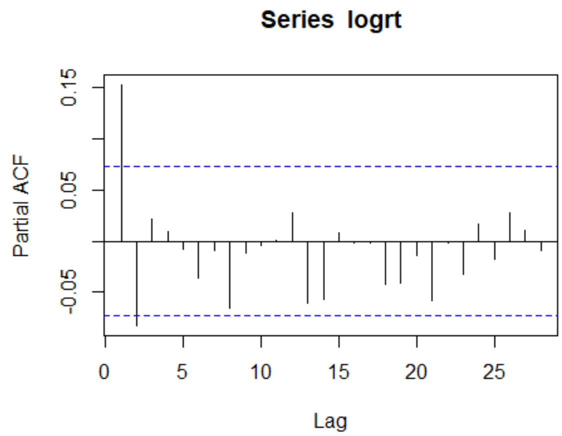
```
m.deciles <- read.csv("C:/Users/ASUS/Downloads/m-deciles.txt", sep="")
View(m.deciles)

logrt <- log(as.numeric(m.deciles[,10])+1)
```

4.a)

```
t.test(logrt)

##
##  One Sample t-test
##
## data:  logrt
## t = 5.1808, df = 719, p-value = 2.873e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.005946562 0.013203545
## sample estimates:
##   mean of x
## 0.009575054

acf(logrt)
```




```
pacf(logrt)
```




```
Box.test(logrt, lag=10, type='Ljung')

##
##  Box-Ljung test
##
## data:  logrt
## X-squared = 24.257, df = 10, p-value = 0.006946

auto.arima(logrt)

## Series: logrt
## ARIMA(0,0,2) with non-zero mean
##
## Coefficients:
##          ma1      ma2    mean
##       0.1660  -0.0558  0.0096
## s.e.  0.0373   0.0370  0.0020
##
## sigma^2 estimated as 0.002392:  log likelihood=1152.66
## AIC=-2297.32   AICc=-2297.26   BIC=-2279

m1 <- arima(logrt, order=c(0,0,2))
m1

##
## Call:
## arima(x = logrt, order = c(0, 0, 2))
##
## Coefficients:
##          ma1      ma2  intercept
##       0.1660  -0.0558     0.0096
## s.e.  0.0373   0.0370     0.0020
##
## sigma^2 estimated as 0.002382:  log likelihood = 1152.66,  aic = -2297.32
```

```
Box.test(m1$residuals, lag=10, type='Ljung')

##
##  Box-Ljung test
##
## data:  m1$residuals
## X-squared = 4.3881, df = 10, p-value = 0.9281
```

Ans 4.a) T-test : $H_0 : \mu$ of log return (logrt) = 0 / expected value of logrt = 0

$H_1 : \mu \neq 0$

The computed p-value 2.87e-07 < 0.05 . H_0 is rejected at 0.05 level of significance

That is, $\mu \neq 0$ with 95% CI

Ljung-Box Test : $H_0 : e_1 = e_2 = .. = e_m = 0$ for logrt
($m = 10$) $H_1 : \exists e_i \neq 0$

There exists serial correlation in log return rt with 95% CI, because the Ljung B ox test of rt has p-value 0.0065 < 0.05 rejecting null hypothesis of zero correlation among logrt at 0.05 level of significance, implying serial correlation in logrt. Also, the ACF indicates correlation value significantly differ from zero, showing sign of serial correlation.

Also from ACF & PACF show 2-lag (finite memory)

Auto.arma → generate MA(2) model for logrt

$\hat{r}_t = 0.0096 + 0.166 a_{t-1} - 0.0558 a_{t-2}$

checking the serial correlation in residual : $\hat{a}_t$ of MA(2)

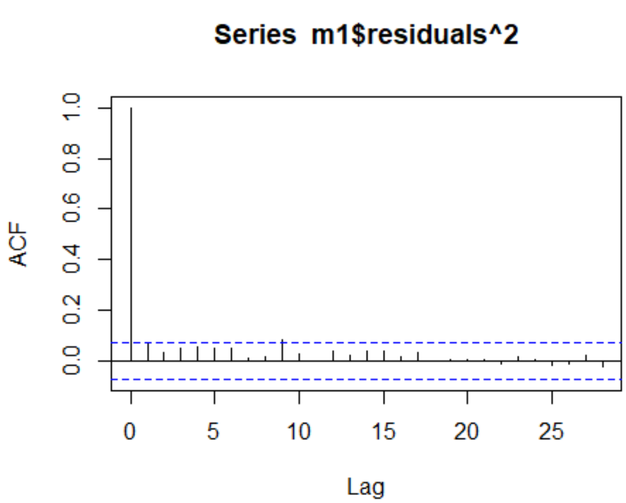
Ljung-Box Test : $H_0 : e_1 = e_2 = .. = e_m = 0$ for a_t
($m = 10$) $H_1 : \exists e_i \neq 0$

The computed p-value ≈ 0.92 > 0.05. H_0 is not rejected at 0.05 level of sig.

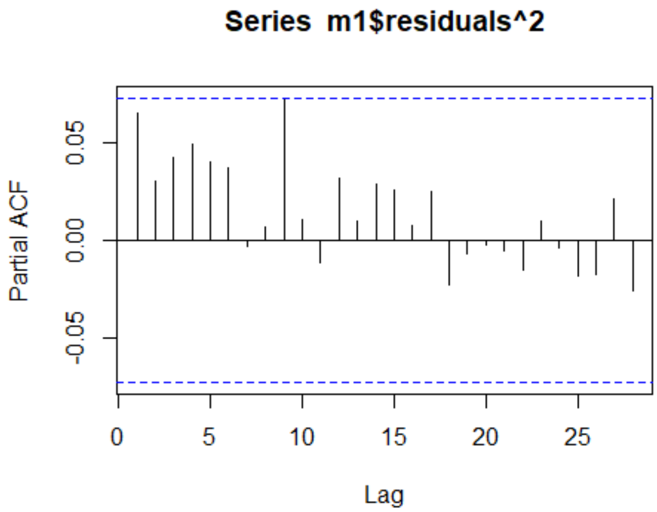
That is, model is adequate : no serial correlation in a_t

4.b)

```
acf(m1$residuals^2)
```



```
pacf(m1$residuals^2)
```




```
Box.test(m1$residuals^2, lag=10, type='Ljung')

##
##  Box-Ljung test
##
## data:  m1$residuals^2
## X-squared = 16.677, df = 10, p-value = 0.08183
```

Ans 4.b

Box test

H_0: there is no ARCH effect in r_t (log) / $\rho_1 = \rho_2 = \dots = \rho_n$ in $\hat{a}_t^2$ from MA(2)

H_1: there is ARCH effect

the computed p-value 0.081 > 0.05. H_0 is not rejected at .05 level of significance.

That is, there is no ARCH effect w/ 95% CI.

Also, ACF & PACF of $\hat{a}_t^2$ show values differ insignificantly from zero, indicating no non-linear dependent in $\hat{a}_t$ or no ARCH effect.

4.C

```
m2 <- garchFit(~arma(1,0)+garch(1,0), data=logrt, trace=F)

summary(m2)

## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x000000001fc38378>
##  [data = logrt]
##
## Conditional Distribution:
```

```
##  norm
##
## Coefficient(s):
##      mu      ar1    omega   alpha1
## 0.01053  0.14707  0.00200  0.18152
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0105300   0.0019275    5.463 4.68e-08 ***
## ar1    0.1470670   0.0424301    3.466 0.000528 ***
## omega  0.0020000   0.0001482   13.493  < 2e-16 ***
## alpha1 0.1815182   0.0651142    2.788 0.005309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1158.628    normalized:  1.609206
##
## Description:
##  Wed Apr 28 20:46:00 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  647.6357  0          < 0.05
##  Shapiro-Wilk Test  R    W      0.962804  1.502752e-12
##  Ljung-Box Test     R    Q(10)  8.582995  0.572082  }
##  Ljung-Box Test     R    Q(15)  12.95811  0.6055337 } > 0.05
##  Ljung-Box Test     R    Q(20)  15.77362  0.7305655 }
##  Ljung-Box Test     R^2  Q(10)  8.153476  0.6138486 }
##  Ljung-Box Test     R^2  Q(15)  12.29206  0.6568012 } > 0.05
##  Ljung-Box Test     R^2  Q(20)  13.57787  0.851238  }
##  LM Arch Test       R    TR^2   8.24593   0.7656286 }
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.207301 -3.181861 -3.207363 -3.197480
```

JB test : H_0: series is normally dist^{n}
H_1 : series isn't normally dist^{n}
p-value < 0.05 ; H_0 is rejected at 0.05 level of sig.
That is, series not normally distributed with 95% CI

Ans 4.c

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = .. = \rho_m = 0$
$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t$ or
Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$
H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = .. = \rho_m = 0$ for $\tilde{a}_t^2$
H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20 & LM Arch Test p-value > 0.05
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect
variance equation is appropriate with 95% CI

Model has adequate Mean & Variance equation but distribution is not normal

Fitted eq → Mean : $\hat{r}_t = \underset{(0.00193)}{0.01053^{***}} + \underset{(0.042)}{0.147^{***}} r_{t-1}$

Variance : $\sigma_t^2 = \underset{(0.0001)}{0.002^{***}} + \underset{(0.065)}{0.1815^{***}} a_{t-1}^2$

4.d)

```
m3 <- garchFit(~arma(1,0)+garch(1,0), data=logrt, cond.dist = "std", trace=F)

summary(m3)

## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x00000000212a4928>
##  [data = logrt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu        ar1      omega     alpha1      shape
## 0.0116189  0.1076982  0.0019203  0.1900830  6.4225253
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0116189   0.0017710    6.561 5.35e-11 ***
## ar1    0.1076982   0.0403169    2.671  0.00756 **
## omega  0.0019203   0.0001818   10.564  < 2e-16 ***
## alpha1 0.1900830   0.0713692    2.663  0.00774 **
## shape  6.4225253   1.3115905    4.897 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1187.516    normalized:  1.649328
##
## Description:
##  Wed Apr 28 20:46:00 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  680.4834  0
##  Shapiro-Wilk Test  R    W      0.9612945 7.49198e-13
##  Ljung-Box Test     R    Q(10)  9.486455  0.4866411  } >0.05
##  Ljung-Box Test     R    Q(15)  13.8214   0.5391158  }
##  Ljung-Box Test     R    Q(20)  17.0087   0.6524086  }
##  Ljung-Box Test     R^2  Q(10)  7.567444  0.671006   } >0.05
##  Ljung-Box Test     R^2  Q(15)  11.42176  0.7221637  }
##  Ljung-Box Test     R^2  Q(20)  12.79422  0.8860373  }
##  LM Arch Test       R    TR^2   7.723697  0.8063327  }
##
## Information Criterion Statistics:
```

Ans 4.d

```
##        AIC       BIC       SIC      HQIC
## -3.284768 -3.252967 -3.284863 -3.272491
```

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$

$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20

Ho is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t$ or

Mean equation is appropriate with 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$

H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = \dots = \rho_m = 0$ for $\tilde{a}_t^2$

H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20 & LM Arch Test p-value > 0.05

Ho is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect

Variance equation is appropriate with 95% CI

Model has adequate Mean & Variance equation with t-distribution

Fitted eq. → Mean : $\hat{r}_t = \underset{(0.00177)}{0.01162^{***}} + \underset{(0.040317)}{0.1076982^{**}}\, r_{t-1}$

$a_t \sim t_{6.42}$

Variance: $\sigma_t^2 = \underset{(0.00013)}{0.00192^{***}} + \underset{(0.0714)}{0.19008^{**}}\, a_{t-1}^2$

```r
m4 <- garchFit(~garch(1,0), data=logrt, trace=F)
summary(m4)
```

```
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x0000000021589ec8>
##  [data = logrt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu     omega    alpha1
## 0.012346  0.002016  0.194126
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.012346    0.001923    6.421 1.35e-10 ***
## omega   0.002016    0.000145   13.900  < 2e-16 ***
## alpha1  0.194126    0.062209    3.121  0.00181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Log Likelihood:
##  1152.289    normalized:  1.600401
##
## Description:
##  Wed Apr 28 20:46:00 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8024  0
##  Shapiro-Wilk Test  R    W      0.9570702 1.171061e-13
##  Ljung-Box Test     R    Q(10)  18.72223  0.04393591
##  Ljung-Box Test     R    Q(15)  23.27998  0.07837365
##  Ljung-Box Test     R    Q(20)  27.61004  0.1189566
##  Ljung-Box Test     R^2  Q(10)  7.012055  0.7243064
##  Ljung-Box Test     R^2  Q(15)  10.3604   0.7964784
##  Ljung-Box Test     R^2  Q(20)  11.97825  0.9168225
##  LM Arch Test       R    TR^2   7.047101  0.8544853
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.192468 -3.173388 -3.192503 -3.185102
```

(Handwritten annotations beside the output: Jarque-Bera p-value "< 0.05"; Ljung-Box R Q(10), Q(15) "< 0.05"; Ljung-Box R Q(20) "> 0.05"; Ljung-Box R^2 Q(10)–Q(20) and LM Arch Test "> 0.05")

JB test : H_0 : series is normally dist^n
H_1 : series isn't normally dist^n
p-value < 0.05 ; H_0 is rejected at 0.05 level of sig.
That is, series not normally distributed with 95% CI

Ans 4.e

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$
$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for only $m = 10, 15$
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t$.
But for $m = 20$, H_0 is rejected at .05 level of sig.
That is, serial correlation in $\tilde{a}_t$ for $m = 20$
Mean equation may not be appropriate w/ 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$
H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = \dots = \rho_m = 0$ for $\tilde{a}_t^2$
H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all $m = 10, 15, 20$ & LM Arch Test p-value > 0.05
H_0 is not rejected at 0.05 level of significance
That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect
Variance equation is appropriate with 95% CI

∴ Variance equation is adequate while mean equation is not fully adequate, and not Normally distributed

Fitted eq → Mean : $\hat{r}_t = \underset{(0.0019)}{0.01235}^{***}$

Variance : $\sigma_t^2 = \underset{(0.000141)}{0.002016}^{***} + \underset{(0.0622)}{0.194126}^{**}$

```r
m5 <- garchFit(~garch(1,0), data=logrt, cond.dist = "std", trace=F)
summary(m5)
```

```
##
## Title:
##  GARCH Modelling
```

```
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = logrt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x000000001f12ad00>
##  [data = logrt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##       mu     omega    alpha1     shape
## 0.013356  0.001928  0.204163  6.220222
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.013356    0.001685    7.927 2.22e-15 ***
## omega   0.001928    0.000185   10.421  < 2e-16 ***
## alpha1  0.204163    0.072375    2.821  0.00479 **
## shape   6.220223    1.236608    5.030 4.90e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1183.349    normalized:  1.64354
##
## Description:
##  Wed Apr 28 20:46:00 2021 by user: ASUS
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.3878  0
##  Shapiro-Wilk Test  R    W      0.9574271 1.363165e-13
##  Ljung-Box Test     R    Q(10)  18.51406  0.04688705   <0.05
##  Ljung-Box Test     R    Q(15)  22.99926  0.08415548   } >0.05
##  Ljung-Box Test     R    Q(20)  27.3947   0.1245202    }
##  Ljung-Box Test     R^2  Q(10)  6.667871  0.7563837    } >0.05
##  Ljung-Box Test     R^2  Q(15)  10.0157   0.8187514    }
##  Ljung-Box Test     R^2  Q(20)  11.63085  0.928196     }
##  LM Arch Test       R    TR^2   6.819315  0.8693193    }
##
## Information Criterion Statistics:
```

```
##        AIC       BIC       SIC      HQIC
## -3.275969 -3.250529 -3.276031 -3.266148
```

Ans 4.8

Test linear dependent in $\tilde{a}_t$: $H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$

$H_1: \exists \rho_i \neq 0$

Ljung-Box test yield p-value > 0.05 for only m = 15, 20

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t$.

But for m = 10, H_0 is rejected at .05 level of sig.

That is, serial correlation in $\tilde{a}_t$ for m = 10

Mean equation may not be appropriate w/ 95% CI

Test linear dependent in $\tilde{a}_t^2$ or non-linear dependent in $\tilde{a}_t$

H_0 : there is no ARCH effect or $\rho_1 = \rho_2 = \dots = \rho_m = 0$ for $\tilde{a}_t^2$

H_1 : there exists ARCH effect

Ljung-Box test yield p-value > 0.05 for all m = 10, 15, 20 & LM Arch Test p-value > 0.05

H_0 is not rejected at 0.05 level of significance

That is, no serial correlation in $\tilde{a}_t^2$ or No ARCH effect

Variance equation is appropriate with 95% CI

∴ Variance equation is adequate while mean equation is not fully adequate.

Fitted eq. → Mean : $\hat{r}_t = \underset{(0.001686)}{0.013356^{***}}$

$a_t \sim t_{6.22}$

Variance : $\sigma_t^2 = \underset{(0.000185)}{0.001928^{***}} + \underset{(0.0724)}{0.204163^{**}} a_{t-1}^2$

Ans 4.9 The most appropriate model is m3 : AR(1) - ARCH(1) with t-distribution because
1. Adequate Mean & Variance equation
2. Lowest AIC/BIC
3. All coefficient estimates are significant