

Chapter3 Differential calculus in Economic Theory

1. Find the derivatives $\left(\frac{dy}{dx}\right)$ of the following functions;
 - a) $Y=-283$
 - b) $Y=2x^3$
 - c) $Y=\sqrt{x}$
 - d) $Y=\frac{-2}{x^2}$
 - e) $Y=4x^4 - 2x^3 + x^2 - 3x + 20$
 - f) $Y=(4x^2-2)(3x+1)$
 - g) $Y=\frac{5x}{1+x^2}$
 - h) $Y=(x^2+3)x^{-1}$
2. Find $f'(x)$, $f''(x)$ and $f'''(x)$ of the following functions
 - a) $f(x) = 16x^5 - 2x^3 + 4x^2 - 3x - 32$
 - b) $f(x) = (3x+2)^3$
 - c) $f(x) = (6x-1)(2x-3)^2$
3. Find the first-order and second-order partial derivatives of the following functions
 - a) $Y = f(x,y) = 3x^2 - 2xy + 4y^2$
 - b) $Y = f(u,v) = (u+2)(2u+5v)$
4. Find the differential (dy) of the following functions
 - a) $Y = 4x^2 + 5x - 3$
 - b) $Y = 7x^4 - 2x^3 + 5x - 4$
 - c) $Y = \log(x+5)$
 - d) $Y = -x^2(x+3)$
 - e) $Y = (x-3)(2x+5)$
5. Find the total derivatives $\frac{dy}{dx}$ of the following functions
 - a) $Y=3z^2$ and $z=4x-1$
 - b) $Y=u^3+1$ and $u=5-x^2$
 - c) $Y=aw^2$ and $w=bx^2+cx$
 - d) $X^3 - 2x^2y + 3xy^2 - 22 = 0$
 - e) $X^2y^2 + e^x - e^y = 0$

6. Find $f'(1)$ and $f'(2)$ from the following functions;

a) $f(x)=18x$

b) $f(x)=-5x^{-2}$

c) $f(x)=\frac{3}{4}x^3$

d) $f(x)=-6x^{-1/3}$

7. If $z=x^2-8xy-y^2$

$$X=3t$$

$$Y=1-t$$

Find $\frac{dz}{dt}$

8. If $z=x^2+xy+y^2$

$$X=t+\frac{1}{t}$$

$$Y=t-\frac{1}{t}$$

Find $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$

9. Find the first three derivatives of the following function

$$Y=g(x)=\frac{x}{1+x}$$

10. Find marginal revenue functions of the following demand functions

a) $P=18-0.2Q$

b) $P=10$

11. Find the marginal cost functions and average cost functions of

a) $TC=0.02Q^3-3Q^2+12Q+300$

b) $TC=Q^{1/3}-0.6Q^2+30Q+120$

12. Find the marginal utility functions MU_1 and MU_2 of

a) $TU=U(Q_1, Q_2)=2Q_1^{0.5}Q_2^{0.5}$

b) $TU=U(Q_1, Q_2)=6Q_1^2+2Q_1Q_2+4Q_2^2$

13. Given $c=500+0.75Y^d$, find Marginal Propensity to consume (MPC) and marginal propensity to save (MPS).

14. Given the total cost function $C=Q^3-5Q^2+12Q+75$, write out a variable cost function (VC). Find the derivative of the VC function, and interpret the economic meaning of that derivative.

15. Given the demand function as $Q_d=10-0.5P$, find elasticity of demand (E_p^d) and identify whether it is inelastic, unitary elastic or relatively elastic

- a) Where $p=2$
- b) Where $p=10$
- c) Where $p=18$

16. Given a demand function of good x as $f(P_x, P_y, I)$

$$Q_x=100-2P_x+P_y+0.1I \quad \text{find}$$

- a) Price elasticity of demand of good x
- b) Cross elasticity of demand of good x to price of good y
- c) Income elasticity of demand of good x

At $P_x=10$, $P_y=12$, $I=1000$, and identify which type of good it is (Either substitute goods, complementary goods, luxury goods, necessary goods or inferior goods)

17. Given the production function as the cob-Douglas as

$$Q=10L^{0.5}K^{0.5}$$

Find MPL and MPK at $L=16$, $K=9$

18. Given the sale function of a company as $S=f(P, A, R)$

$S=(12000-900P)A^{0.5}R^{0.5}$ where P is price, A is advertising and R is representative, Find

- a) Elasticity of sale with respect to advertising
- b) Elasticity of sale with respect to price
- c) Elasticity of sale with respect to representative

At $P=6$, $R=49$, $A=8100$

19. Given $Q = 1.1 L^{0.6} K^{0.2}$ be the production function. Show that output elasticity of L and K are 0.6 and 0.2 respectively. Is this production function subjected to the decreasing return to scale?

20. The supply function of a certain commodity is $Q = a + bP^2 + R^{1/2}$ ($a < 0$, $b > 0$) that P is price and R is rainfall. Find the price elasticity of supply and the rainfall elasticity of supply.

21. The foreign demand for our exports X depends on the foreign income Y and our price level P, $X = Y^{1/2} + P^{-2}$. Find the partial elasticity of foreign demand for our exports with respect to our price level.

22. Given the national-income model as

$$C = a + bY_d \quad (a > 0, 0 < b < 1)$$

$$Y_d = Y - T$$

$$T = T_0 + tY \quad (T_0 > 0, 0 < t < 1)$$

$$I = I_0 + dY \quad (I_0 > 0, 0 < d < 1)$$

$$G = G_0 \quad (G_0 > 0)$$

$$X = X_0 \quad (X_0 > 0)$$

$$M = M_0 + mY \quad (M_0 > 0, 0 < m < 1)$$

Analyze the comparative statics of the model when I_0 , G_0 , T_0 or M_0 is changed

23. Given the market model as; $Q = 30 - 2P$ and $Q = 5P - 6$

Find the equilibrium of this market and analyze the comparative statics of the model when a new demand is given as $Q = 40 - 2P$

