# EE320 (2/2012)

## INTRODUCTORY MATHEMATICAL ECONOMICS

DERIVATIVES OF MORE-THAN-ONE INDEPENDENT VARIABLE FUNCTION

(Part 2)

# Topics

- Derivatives of multiple-independent-variable function: Economic applications
  - Partial market equilibrium
  - Multipliers in macroeconomic models
  - Utility function
  - Production function
  - Elasticity

# Partial Market Equilibrium (1)

- Recall the one-commodity market model:

| | | |
|---|---|---|
| Demand: | $Q = a - bP$ | $(a, b > 0)$ |
| Supply: | $Q = -c + dP$ | $(c, d > 0)$ |

- Solutions:

- *Comparative-static derivatives* are the partial derivatives of $P^*$ and $Q^*$ with respect to parameters ($a$, $b$, $c$, and $d$):
  - $\partial P^*/\partial a$, $\partial P^*/\partial b$, $\partial P^*/\partial c$, $\partial P^*/\partial d$
  - $\partial Q^*/\partial a$, $\partial Q^*/\partial b$, $\partial Q^*/\partial c$, $\partial Q^*/\partial d$

# Partial Market Equilibrium (2)

- Partial derivatives of *P** with respect to parameters *a, b, c,* and *d* are:

$$\frac{\partial P^*}{\partial a} =$$

$$\frac{\partial P^*}{\partial b} =$$

$$\frac{\partial P^*}{\partial c} =$$

$$\frac{\partial P^*}{\partial d} =$$

# Partial Market Equilibrium (3)

- Graphical illustration of a change in each parameter

1. Increase in *a*
2. Increase in *b*
3. Increase in *c*
4. Increase in *d*

# Partial Market Equilibrium (4)

- Partial derivatives of $Q^*$ with respect to parameters *a, b, c*, and *d* are:

$$\frac{\partial Q^*}{\partial a} =$$

$$\frac{\partial Q^*}{\partial b} =$$

$$\frac{\partial Q^*}{\partial c} =$$

$$\frac{\partial Q^*}{\partial d} =$$

# Partial Market Equilibrium (5)

- Suppose now $Q_d$ is a function of P as well as $Y_0$.

$$Q_d = Q_s$$

$$Q_d = D(P, Y_0) \qquad (\partial D/\partial P < 0;\ \partial D/\partial Y_0 > 0)$$

$$Q_s = S(P) \qquad (\partial S/\partial P > 0)$$

- Equilibrium condition:

$$D(P, Y_0) - S(P) = 0.$$

- Equilibrium price:

$$P^* = P^*(Y_0).$$

- Equilibrium identity:

# Partial Market Equilibrium (6)

- Comparative-static derivatives:
  - $$\frac{\partial P^*}{\partial Y_0} = -\frac{\partial F / \partial Y_0}{\partial F / \partial P^*} =$$
  - $$\frac{\partial Q^*}{\partial Y_0} = ?$$

# Multipliers in Macroeconomic Models (1)

- Consider a national-income model

$$Y = C + I_0 + G_0$$

$$C = \alpha + \beta(Y - T) \qquad (\alpha > 0;\ 0 < \beta < 1)$$

$$T = \gamma + \delta Y \qquad (\gamma > 0;\ 0 < \delta < 1)$$

➢ Equilibrium national income (in reduced form):

# Multipliers in Macroeconomic Models (2)

- Comparative-static derivatives:
  - $\frac{\partial Y^*}{\partial \alpha}; \frac{\partial Y^*}{\partial \beta}; \frac{\partial Y^*}{\partial I_0}; \frac{\partial Y^*}{\partial G_0}; \frac{\partial Y^*}{\partial \gamma}; \frac{\partial Y^*}{\partial \delta}$
- Comparative-static derivatives with special *policy significance*:
  - Government-expenditure multiplier:
  - Nonincome-tax multiplier:
  - Partial derivatives of Y* w.r.t. the income tax rate (δ):

# Utility Function

- Consider a utility function

$$U = Ax^a y^b,$$

- Marginal utility of x:
- Marginal utility of y:
- Marginal rate of substitution (MRS) as the slope of the indifference curve:

# Production Function (1)

- Consider an agricultural production function

$$Q = F(K, L, T) = AK^{\alpha}L^{\beta}T^{\gamma} \qquad (A > 0; \alpha > 0, \beta > 0, \gamma > 0)$$

Where K = capital, L = labor, and T = land.

- Marginal product of capital is:
- Marginal product of labor is:
- Marginal product of land is:
- Second-order partial derivatives:
- Cross partial derivatives:

# Production Function (2)

- **Example**: Given a production function

$$Q = 36KL - 2K^2 - 3L^2$$

- Marginal product of capital is:
- Marginal product of labor is:
- If the marginal revenue (MR) at Q = 2 and L = 3 is $5, the marginal revenue product (MRP) for the *third* unit of L is:

# Production Function (3)

- **Example** : Given the equation for a production isoquant

$$F(K,L) = 16K^{0.25}L^{0.75} = 2144,$$

Use the implicit function rule to find the slope of the isoquant (dK/dL: marginal rate of technical substitution).

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# Elasticities

- **Two variables**

If $z = f(x, y)$, the partial elasticity of $z$ w.r.t. $x$ and $y$ are:

When all variables are positive, elasticities can be expressed as logarithmic derivatives:

- **n variables**

If $z = f(x_1, x_2, \ldots, x_n) = f(\mathbf{x})$, the elasticity of $z$ w.r.t. $x_i$ *when all other variables are held constant* is:

# Elasticities of Demand (1)

- Given the demand function $Q_1 = a - bP_1 + cP_2 + mY$

  where Y = income, $P_2$ = the price of a substitute good.

- Own price elasticity of demand:

- Income elasticity of demand:

- Cross-price elasticity of demand:

# Elasticities of Demand (2)

- **Example** : Given $Q_1 = 100 – P_1 + 0.75P_2 - 0.25P_3 + 0.0075Y$ .

At $Y = 10{,}000$, $P_1 = 10$, $P_2 = 20$, $P_3 = 40$ and $Q_1 = 100$, find the different cross-price elasticities of demand.

➢

# Output Elasticity

- Given a *linearly homogenous* Cobb-Douglas production function

$$Q = F(K, L, T) = AK^{\alpha}L^{\beta}$$

- The output elasticity of capital:

- The output elasticity of labor: