Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$ RSS		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\operatorname{var}(u_i)$, $\operatorname{var}\left(\hat{\beta}_1\right)$ and $\operatorname{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \qquad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

a) $\hat{\beta_2} = \frac{\Sigma(X_i-\bar{X})(Y_i-\bar{Y})}{\Sigma(X_i-\bar{X})^2} = \frac{-174.20}{1098.8} = -0.1585$

b) $\hat{\beta_1} = \bar{Y} - \hat{\beta_2}\bar{X}$
$\hat{\beta_1} = 21.03 - (-0.1585)(12.20)$
$\hat{\beta_1} = 22.9637$

c) $\hat{Y} = 22.9637 - 0.1585 X_i$

d) interpretation of β_1, when $X=0$ then Y will 22.9637
If $X=0$, $Y = \hat{\beta_1}$

Interpretation of β_2, when $X\uparrow$ 1 unit, $Y\downarrow$ 0.1585 unit

b) Find r^2 and explain its meaning.

$R^2 = \frac{ESS}{TSS}$

$= 1 - \frac{RSS}{TSS}$

$= 1 - \frac{\Sigma \hat{u}_i^2}{\Sigma y_i^2}$

from

TSS : $\Sigma y_i^2 = \Sigma(Y_i - \bar{Y})^2$

ESS : $\Sigma \hat{y}_i^2 = \Sigma(\hat{Y}_i - \bar{Y})^2$

RSS : $(Y_i - \hat{Y})^2 = \Sigma \hat{u}_i^2$

Thus ; $r^2 = 1 - \frac{RSS}{TSS}$

$r^2 = 1 - \frac{873.14}{882.97}$

$r^2 = 0.0111$

Total variation in Y explained by the regression Model with the margin of 1.11 %.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

from the regression analysis will yeild model of $E(\hat{Y}|X=5)$

$\hat{Y} = 22.9637 - 0.1585 X_i$

thus ; when $X_i = 5$

$\hat{Y} = 22.9637 - 0.1585(5)$

$\hat{Y} = 22.1712$

when $X_i = 5$ so $\hat{Y} = 22.1712$

if $X_i = 5$, expected $\hat{Y} = 22.1712$

d) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

$\text{var}(u_i) : \sigma^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{873.16}{30.2} = 31.1836$

$\text{var}(\hat{\beta_1}) = \frac{\Sigma X_i^2}{n \cdot \Sigma(X_i - X^2)} \cdot \sigma^2 = \frac{5564 \cdot 31.1836}{30 \cdot (1098.8)} = 5.2635$

$\text{var}(\hat{\beta_2}) = \frac{\sigma^2}{\Sigma(X-\bar{X})^2} = \frac{31.1836}{1098.6} = 0.0284$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

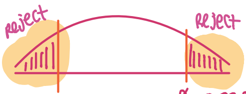
Testing β_2

1. $H_0 : \beta_2 = 0$

   $H_1 : \beta_2 \neq 0$

2. set $\alpha = 0.05$

3. Draw curve



since its equal thus α will be divided

$\frac{\alpha}{2} = 0.025$ $\frac{\alpha}{2} = 0.025$

$t_{cri} = 2.048$ $t_{cri} = 2.048$

open the table will have = $t_{28, 0.025}$ at

4. calculating t

$$t_{cal} = \frac{\hat{\beta_2} - \beta_2}{Se\ \hat{\beta_2}} = \frac{\hat{\beta_2} - \beta_2}{\sqrt{var(\hat{\beta_2})}} = \frac{-0.1585 - 0}{\sqrt{0.0284}} = -0.9405$$

5. from $|t_{cal}| < |t_{critical}|$ thus fall into reject Ho regieon

   $|0.9405| < |2.048|$

   does believe $\beta_2 = 0$; x have no effect to y at 95% confidence level

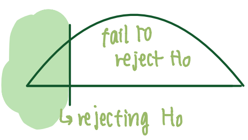
f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

1. $H_0 : \beta_2 \geq 0$

   $H_1 : \beta_2 < 0$; always assume neree

2. $\alpha = 0.01$

3. Draw curve since α thus we do not divided



a) $\alpha = 0.01$

b) find $t_{28, 0.01}$ will have -2.467

4. $$t_{cal} = \frac{\hat{\beta_2} - \beta_2}{Se\ \hat{\beta_2}} = \frac{\hat{\beta_2} - \beta_2}{\sqrt{var\ \hat{\beta_2}}} = \frac{-0.1585 - 0}{\sqrt{0.0284}} = -0.9405$$

5. In conclusion ; it makes to believe that Ho not less than 0 alienaed with the confidence level of 95% level confidence

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$
$$se(\hat{\beta}_1) \quad se(\hat{\beta}_2)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

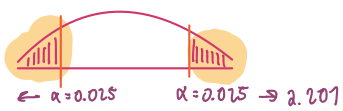
using the hypothesis

1. $H_0 : \beta_2 = 0$

   $H_1 : \beta_2 \neq 0$

2. set $\alpha = 0.05$

3. Draw curve



having $n = 11$, $t_{11, 0.025}$ thus $t_{cri} = 2.201$

4. $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{se\,\hat{\beta}_2} = \dfrac{\hat{\beta}_2 - \beta_2}{\sqrt{var\,\hat{\beta}_2}} = \dfrac{-502.4 - 0}{411.8} = -1.220$

5. thus fail to reject Ho which doesn't make economic sense if according to the regression model given.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

a) using the formula $Pr\left[\hat{Y}_0 - (t_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{Y_0}) \leq Y_0 \leq \hat{Y}_0 + (t_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{Y_0})\right]$

and

$$var(\hat{Y}_0) = \sigma^2\left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]$$

b) $X_i = 5$ thus $\hat{Y}_0 = 7836 - 502.4(5)$

$\hat{Y}_0 = 5324$

c) $var(\hat{Y}_0) = 212{,}877\left[\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73}\right]$

$var(\hat{Y}_0) = 35582.5345$

d) $\hat{\sigma}_0 = \sqrt{35582.5345}$

$\hat{\sigma}_0 = 188.6333$

e) $t_{\frac{0.05}{2}} = t_{0.025, 9} = 2.262$

f) thus range of

$= Pr[5324 - (2.262 \cdot 188.6333) \leq Y_0 \leq 5324 + (2.262 \cdot 188.6333)]$

$= Pr[4897.3115 \leq Y_0 \leq 5750.6885]$

In conclusion; the price - car with 5 years old will be within the rage of $Pr[4897.3115 \leq Y_0 \leq 5750.6885]$ with condidence level of 95%

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

standard error will change only if added will not change, contrary if multiple X will effect standard error changes thus the standard error of all X will change by 10

d) Calculate the elasticity of market price when a car is 10 years old.

a) elasticity of price $= \dfrac{\frac{dQ}{Q}}{\frac{dp}{p}} = \dfrac{dQ}{dp} \times \dfrac{p}{Q}$

b) adapting to the problem: $\dfrac{dp}{dA} \cdot \dfrac{A}{p}$

thus ; $\hat{y} = 7836 - 502.4x_i$

b1) $\dfrac{dp}{dA} = -502.4x_i$

b2) with age 10 sub age (A) = X to the regression mode will have : $\hat{y}_i = 7836 - 502.4(10)$

$\hat{y}_i = 2812$

c) elasticity will be $-502.4 \times \dfrac{10}{2812} = -1.7866$

conclusion ; elasticity of price with 10 years ≈ -1.7866, it highly elastic.