Chapter 5 Multivariate Calculus: Applications

In this chapter we will examine three groups of examples:

1) Effectiveness of Monetary Policy with Balanced Budget in Closed Economy:
   a. Fixed Price without Money Market
   b. Fixed Price with Money Market (IS-LM)
   c. Flexible Price with Money Market
2) Monetary Policy Effectiveness
   a. IS-LM Model
   b. Mundell-Fleming
3) Tax Incidence in Competitive Market

5.1 Balanced-Budget Multipliers in Closed Economy.

Three cases are discussed to demonstrate the applications of Cramer's Rule and Implicit Function Theorem:
a) a fixed price, closed economy with no money market
b) a fixed price, IS-LM model with money market
c) a flexible price Aggregate Demand and Supply

- The Balanced-Budget multipliers changes as the model becomes more general
- See end-of-chapter problems analyzing effects of other parameter on endogeneous variables

5.1.1 Simple Keynesian Model: Fixed price, closed economy without money market.

$$Y = C(Y_d) + I + G$$
$$Y_d = Y - T$$
$$G = T$$

Take total differential, with $dI = 0$ and $dG = dT$

$$dY = C'(Y-T)dY - C'(Y-T)dT + dI + dG$$
$$(1 - C'(Y-T))dY = (1 - C'(Y-T))dG$$
$$\frac{dY}{dG} = 1.$$

Note: We can apply the Implicit Function Theorem to get the same result. That is, we write the implicit function

$$f\left(Y(G),G\right) = Y - C\left(Y-T\right) - I - G$$
$$= Y - C\left(Y-G\right) - I - G = 0,$$

by noting that $G = T$. Let Y^* be the equilibrium income at the level of government expenditure $\overline{G}$ and tax $\overline{T}$, i.e., $Y^* = Y\left(\overline{G}\right)$. By direct application of the Implicit Function Theorem, we have

$$\frac{\partial Y^*}{\partial G} = -\left[\frac{\partial f\left(Y^*,\overline{G}\right)}{\partial Y}\right]^{-1} \frac{\partial f\left(Y^*,\overline{G}\right)}{\partial G}$$
$$= -\left(1-C'\right)^{-1}\left(C'-1\right) = 1.$$

HW Baldani, p. 173, # 6.2.

5.1.2 IS-LM Model In the previous model of 5.1.1, there is no crowding out effect as the investment is exogeneous. The model now is,

$$Y = C\left(Y_d\right) + I\left(r\right) + G$$
$$M = L\left(Y,r\right),$$

where investment is a function of interest rate r with $I'\left(r\right) < 0$, M is the money supply, and $L\left(Y,r\right)$ is the money demand with $L_Y\left(Y,r\right) > 0$ and $L_r\left(Y,r\right) < 0$.

Take total differential of the two equations

$$dY = C'dY - C'dT + I'dr + dG$$
$$dM = L_Y dY + L_r dr.$$

With balanced budget, $dG = dT$, and the money supply being exogeneous $dM = 0$, we have

$$dY = C'dY + (1 - C')dG + I'dr$$
$$0 = L_Y dY + L_r dr,$$

and written in matrix form

$$\begin{bmatrix} 1 - C' & -I' \\ L_Y & L_r \end{bmatrix} \begin{bmatrix} dY \\ dr \end{bmatrix} = \begin{bmatrix} (1 - C')dG \\ 0 \end{bmatrix}.$$

By Cramer's Rule,

$$dY = \frac{\begin{vmatrix} (1 - C')dG & -I' \\ 0 & L_r \end{vmatrix}}{\begin{vmatrix} 1 - C' & -I' \\ L_Y & L_r \end{vmatrix}} = \frac{(1 - C')L_r}{(1 - C')L_r + L_Y I'} dG,$$

and the Balanced-Budget Multiplier is

$$\frac{\partial Y}{\partial G} = \frac{(1 - C')L_r}{(1 - C')L_r + L_Y I'} = \frac{1}{1 + \phi},$$

where $\phi = \dfrac{L_Y I'}{(1 - C')L_r} > 0$. Thus $0 < \dfrac{\partial Y}{\partial G} < 1$.

Economic Explanation:

$\Delta G = \Delta T$ $\Rightarrow$ ΔY as in the simple model
$\Rightarrow$ Shift in IS curve
$\Rightarrow$ higher equilibrium *r*
$\Rightarrow$ a reduction in *I*—crowding out effect.

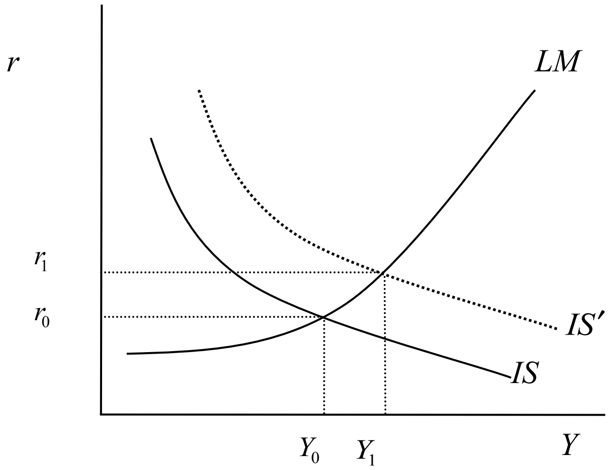

Figure 5.1 Effects of Balance-Budget Multiplier

Note: $\phi = \dfrac{L_Y I'}{(1-C')L_r}$ depends on four factors: C', I', L_Y and L_r.

HW Repeat the above analysis by applying the Implicit Function Theorem.

HW Baldani, p. 173, # 6.3, 6.4.

5.1.3 Aggregate Demand–Aggregate Supply Model:

Model with flexible aggregate price level P in a closed economy.

$$Y = C(Y_d) + I(r) + G,\ 0 < C' < 1,\ I' < 0$$

$$\frac{M}{P} = L(Y, r),\ L_Y > 0,\ L_r < 0$$

Aggregate Supply curve:

$$P = P^E + g(Y - Y^F),\ g' \geq 0,$$

where P^E is the expected price level, Y^F is the output at full employment, $g' \geq 0$ is the slope of the aggregate supply curve. There are three possibilities:

1) AS_1: $g' = 0 \Rightarrow$ IS-LM model (6.2.2)
2) AS_2: $g' = \infty \Rightarrow$ Classical model with completely flexible prices and continual full employment.

3) AS_3 : $0 < g' < \infty$ $\Rightarrow$ when g' represents the flexibility of price.

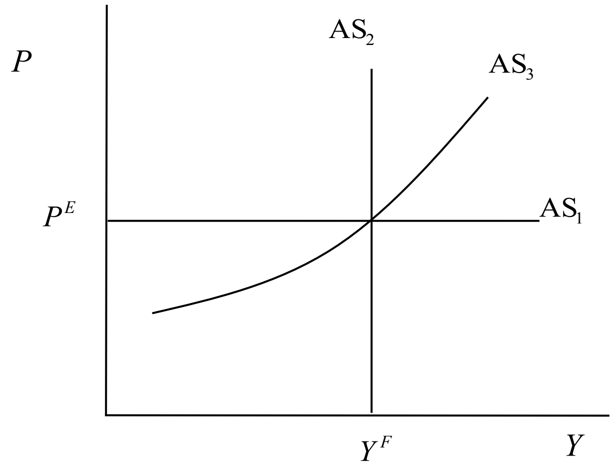

Figure 5.2 Different cases of Aggregate Supply

The model has three equations:

$$Y = C(Y_d) + I(r) + G$$
$$\frac{M}{P} = L(Y, r)$$
$$P = P^E + g(Y - Y^F).$$

The money supply, expected price and output at full employment are exogeneous and thus

$$dM = dP^E = dY^F = 0,$$

and balanced budget so $dG = dT$.

Take total differential and put it in a matrix form,

$$\begin{bmatrix} 1-C' & -I' & 0 \\ L_Y & L_r & \frac{M}{P^2} \\ -g' & 0 & 1 \end{bmatrix} \begin{bmatrix} dY \\ dr \\ dP \end{bmatrix} = \begin{bmatrix} (1-C')dG \\ 0 \\ 0 \end{bmatrix},$$

By Cramer's Rule, the Balanced-Budget Multiplier in the AD-AS model is

$$dY = \frac{\begin{vmatrix} (1-C')dG & -I' & 0 \\ 0 & L_r & \dfrac{M}{P^2} \\ 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1-C' & -I' & 0 \\ L_Y & L_r & \dfrac{M}{P^2} \\ -g' & 0 & 1 \end{vmatrix}} dG$$

$$= \frac{(1-C')L_r}{g'\dfrac{M}{P^2}I' + (1-C')L_r + L_Y I'} dG.$$

The difference between this and IS-LM model is $g'\dfrac{M}{P^2}I'$ in the denominator.

1) AS_1: $g'=0 \Rightarrow g'\dfrac{M}{P^2}I'=0$ thus

$$\left(\frac{\partial Y}{\partial G}\right)_{IS-LM} = \left(\frac{\partial Y}{\partial G}\right)_{AS-AD}.$$

2) AS_2: $g'=\infty \Rightarrow \left(\dfrac{\partial Y}{\partial G}\right)_{AS-AD} = 0$. There is no change in the income as it is already at the full employment.

3) AS_3: $0<g'<\infty \Rightarrow \left(\dfrac{\partial Y}{\partial G}\right)_{IS-LM} > \left(\dfrac{\partial Y}{\partial G}\right)_{AS-AD}$, since $g'\dfrac{M}{P^2}I'<0$.

H.W. Find the Balance-Budget Multiplier for the case of flexible price by the Implicit Function Theorem. That is, write the implicit function

$$\mathbf{f}\left(Y^*,r^*,P^*;\bar{G},\bar{M}\right)=\begin{bmatrix} f^1\left(Y^*,r^*,P^*;\bar{G},\bar{M}\right) \\ f^2\left(Y^*,r^*,P^*;\bar{G},\bar{M}\right) \\ f^3\left(Y^*,r^*,P^*;\bar{G},\bar{M}\right) \end{bmatrix}$$

$$=\begin{bmatrix} Y^*-C\left(Y_d^*\right)-I\left(r^*\right)-\bar{G} \\ \dfrac{\bar{M}}{P^*}-L\left(Y^*,r^*\right) \\ P^*-P^E-g\left(Y^*-Y^F\right) \end{bmatrix}=\mathbf{0}.$$

HW Baldani, p. 173, # 6.6, 6.8.

5.2 Monetary Policy Effectiveness

This application also illustrates the applications of the Implicit Function Theorem and Implicit Function Differentiation.

Starts with IS-LM Model in a closed economy and find comparative static effects of change in Money Supply: $\left(\dfrac{\partial Y^*}{\partial M},\dfrac{\partial r^*}{\partial M}\right)$.

Then we consider Open Economy with Balance-of-payment constraint (Mundell-Fleming Model), with Fixed and Flexible exchange rate cases under the assumption of perfect capital mobility and perfect capital immobility.

5.2.1 IS-LM Model Equilibrium in goods and money markets of a closed economy.

$$Y=C(Y)+I(Y,r)+G;\ 0<C'<1, I_Y>0, I_r<0, C'+I_Y<1$$
$$M=L(Y,r);\ L_Y>0, L_r<0$$

Writing the equilibrium income and interest rate as a function of the money supply as $Y^*(M)$ and $r^*(M)$, we have a system of two equations

$$Y^*(M)=C\left(Y^*(M)\right)+I\left(Y^*(M),r^*(M)\right)+G$$
$$M=L\left(Y^*(M),r^*(M)\right)$$

Take implicit differentiation and suppressing arguments of functions with Y and r being endogeneous and M exogeneous,

$$\frac{\partial Y^*}{\partial M} = C'\frac{\partial Y^*}{\partial M} + I_Y\frac{\partial Y^*}{\partial M} + I_r\frac{\partial r^*}{\partial M}$$
$$1 = L_Y\frac{\partial Y^*}{\partial M} + L_r\frac{\partial r^*}{\partial M}.$$

In matrix form,

$$\begin{bmatrix} 1-C'-I_Y & -I_r \\ L_Y & L_r \end{bmatrix}\begin{bmatrix} \frac{\partial Y^*}{\partial M} \\ \frac{\partial r^*}{\partial M} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

By Cramer's Rule,

$$\frac{\partial Y^*}{\partial M} = \frac{\begin{vmatrix} 0 & -I_r \\ 1 & L_r \end{vmatrix}}{\begin{vmatrix} 1-C'-I_Y & -I_r \\ L_Y & L_r \end{vmatrix}} = \frac{I_r}{L_r\left(1-C'-I_Y\right)+L_Y I_r} > 0$$

$$\frac{\partial r^*}{\partial M} = \frac{\begin{vmatrix} 1-C'-I_Y & 0 \\ L_Y & 1 \end{vmatrix}}{\begin{vmatrix} 1-C'-I_Y & -I_r \\ L_Y & L_r \end{vmatrix}} = \frac{1-C'-I_Y}{L_r\left(1-C'-I_Y\right)+L_Y I_r} < 0$$

by noting that $L_r\left(1-C'-I_Y\right)+L_Y I_r < 0$.

<u>Implicit Function Theorem Application</u>: Write the implicit functions

$$\mathbf{f}\left(Y(G,M),r(G,M),G,M\right) = \begin{bmatrix} Y - C(Y) - I(Y,r) - G \\ M - L(Y,r) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

By the Implicit Function Theorem, with equilibrium income $Y^* = Y\left(\overline{G},\overline{M}\right)$ and $r^* = r\left(\overline{G},\overline{M}\right)$ at some given levels of $\overline{G}$ and $\overline{M}$ and assuming $\nabla_{\begin{bmatrix} Y \\ r \end{bmatrix}}\mathbf{f}\left(Y^*,r^*;\overline{G},\overline{M}\right)$ is invertible, we have function $\begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} Y(G,M) \\ r(G,M) \end{bmatrix}$ such that

a) $\mathbf{f}\left(Y(G,M),r(G,M);G,M\right)=\mathbf{0}$, $\left|G-\bar{G}\right|<\varepsilon$ a n d $\left|M-\bar{M}\right|<\varepsilon$ for some $\varepsilon>0$

b) $\begin{bmatrix} Y^* \\ r^* \end{bmatrix}=\begin{bmatrix} Y(\bar{G},\bar{M}) \\ r(\bar{G},\bar{M}) \end{bmatrix}$, and

c)

$$\nabla_{\begin{bmatrix} G \\ M \end{bmatrix}}\begin{bmatrix} Y^* \\ r^* \end{bmatrix}=-\left[\nabla_{\begin{bmatrix} Y \\ r \end{bmatrix}}\mathbf{f}\left(Y^*,r^*,\bar{G},\bar{M}\right)\right]^{-1}\nabla_{\begin{bmatrix} G \\ M \end{bmatrix}}\mathbf{f}\left(Y^*,r^*\bar{G},\bar{M}\right)$$

$$=-\begin{bmatrix} 1-C'-I_Y & -I_r \\ -L_Y & -L_r \end{bmatrix}^{-1}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\frac{1}{L_r\left(1-C'-I_Y\right)+L_Y I_r}\begin{bmatrix} -L_r & I_r \\ L_Y & 1-C'-I_Y \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\frac{1}{L_r\left(1-C'-I_Y\right)+L_Y I_r}\begin{bmatrix} L_r & I_r \\ -L_Y & 1-C'-I_Y \end{bmatrix}.$$

So, we have as

$$\frac{\partial Y^*}{\partial M}=\frac{I_r}{L_r\left(1-C'-I_Y\right)+L_Y I_r}>0$$

$$\frac{\partial r^*}{\partial M}=\frac{1-C'-I_Y}{L_r\left(1-C'-I_Y\right)+L_Y I_r}<0.$$

Note that we also have

$$\frac{\partial Y^*}{\partial G}=\frac{-L_r}{L_r\left(1-C'-I_Y\right)+L_Y I_r}>0$$

$$\frac{\partial r^*}{\partial G}=\frac{L_Y}{L_r\left(1-C'-I_Y\right)+L_Y I_r}<0.$$

Comparative Static Analysis:

- if I_Y increases

$$I_Y\uparrow\Rightarrow\left(1-C'-I_Y\right)\downarrow\Rightarrow\left(\frac{\partial Y^*}{\partial M}=\frac{I_r}{L_r\left(1-C'-I_Y\right)+L_Y I_r}>0\right)\uparrow$$

- if I_r decreases, (a bigger negative number)

$$I_r \downarrow \Rightarrow \left(\begin{aligned} \frac{\partial Y^*}{\partial M} &= \frac{I_r}{L_r\left(1 - C' - I_Y\right) + L_Y I_r} \\ &= \frac{1}{\frac{L_r\left(1 - C' - I_Y\right)}{I_r} + L_Y} \end{aligned} \right) \uparrow$$

HW Baldani, p. 174, #6.14, 6.15.

5.2.2 Mundell-Fleming Model with Flexible Exchange Rate We consider Open Economy with Balance-of-payment constraint (Mundell-Fleming Model), with Fixed and Flexible exchange rate cases under the assumption of perfect capital mobility and perfect capital immobility.

Balance of Payment (BoP)

- keeps track of international exchange of currencies when there is international trade or when capital flows from one country to another.
- combines trade balance with capital account to see people want to exchange ฿ more or less than foreign currency $.
- When exchange rate is flexible, trade balance must offset capital account balance so there is no excess demand or supply of ฿ in the foreign exchange market.

Let f = net inflow of capital into Thailand (which requires foreign currencies to be cxchanged to ฿)

X = net export (when positive it requires foreign currencies to be exchanged to ฿)

BoP equation when the exchange rate is flexible:

$$X + f = 0,$$

when the exchange rate is fixed, $X + f$ is not necessarily zero; the central bank must buy or sell ฿ to offset the imbalance. Thus, when the exchange rate is fixed,

$$X + f = \Delta F$$

which is the central bank holdings of foreign currencies.

- $\Delta F > 0 \Rightarrow$ excess demand for ฿, so central bank sells ฿ and buys foreign currencies.

- Net export X is a function of income and exchange rate e,

$$X = X(Y, e),\ \frac{\partial X}{\partial Y} < 0,\ \frac{\partial X}{\partial e} > 0,$$

  where $e =$ exchange rate of domestic (฿) for foreign currency ($), i.e., $e = \frac{฿}{\$}$. If e increases, it means ฿ depreciates.

- $f = f(r - r^F) =$ net capital inflows as a function of interest rate diffentials, where $f' > 0$ and
  r = interest rate,
  r^F = rate of return earned on foreign assets.

  The BoP equilibrium condition

$$X(Y, e) + f(r - r^F) = \Delta F,$$

- Flexible exchange rate $\Rightarrow \Delta F = 0$ and e is endogeneously determined by demand and supply of ฿ in foreign exchange market,

- Fixed exchange rate $\Delta F \lesseqgtr 0$, endogeneously determined by demand and supply of ฿ and e is exogeneous.

For flexible exchange rate model, the endogeneous variables are e^*, Y^*, r^* at equilibrium with implicit functions at r^F, G, M is given by

$$\mathbf{f}\left(e^*,Y^*,r^*;r^F,G,M\right)=\begin{bmatrix} X\left(Y^*,e^*\right)+f\left(r^*-r^F\right)-\Delta F \\ Y^*-C\left(Y^*\right)-I\left(Y^*,r^*\right)-G-X\left(Y^*,e^*\right) \\ M-L\left(Y^*,r^*\right)\end{bmatrix}=\mathbf{0}.$$

By Implicit Function Theorem,

$$\nabla_{\begin{bmatrix} r^F \\ G \\ M\end{bmatrix}}\begin{bmatrix} e^* \\ Y^* \\ r^*\end{bmatrix}=-\begin{bmatrix}\nabla_{\begin{bmatrix} e \\ Y \\ r\end{bmatrix}}\mathbf{f}\end{bmatrix}^{-1}\nabla_{\begin{bmatrix} r^F \\ G \\ M\end{bmatrix}}\mathbf{f}$$

$$=-\begin{bmatrix} X_e & X_Y & f' \\ -X_e & 1-C'-I_Y-X_Y & -I_r \\ 0 & -L_Y & -L_r\end{bmatrix}^{-1}\begin{bmatrix} -f' & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1\end{bmatrix}$$

We have

$$\begin{bmatrix} e_M^* \\ Y_M^* \\ r_M^*\end{bmatrix}=-\begin{bmatrix} X_e & X_Y & f' \\ -X_e & 1-C'-I_Y-X_Y & -I_r \\ 0 & -L_Y & -L_r\end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ 1\end{bmatrix},$$

and thus

$$e_M^*=-\frac{\begin{vmatrix} 0 & X_Y & f' \\ 0 & 1-C'-I_Y-X_Y & -I_r \\ -1 & -L_Y & -L_r\end{vmatrix}}{\begin{vmatrix} X_e & X_Y & f' \\ -X_e & 1-C'-I_Y-X_Y & -I_r \\ 0 & -L_Y & -L_r\end{vmatrix}}=\frac{-X_YI_r-f'\left(1-C'-I_Y-X_Y\right)}{X_e\left(L_r\left(1-C'-I_Y\right)+L_Y\left(I_r-f'\right)\right)}>0.$$

Similarly,

$$\begin{bmatrix} e_G^* \\ Y_G^* \\ r_G^*\end{bmatrix}=-\begin{bmatrix} X_e & X_Y & f' \\ -X_e & 1-C'-I_Y-X_Y & -I_r \\ 0 & -L_Y & -L_r\end{bmatrix}^{-1}\begin{bmatrix} 0 \\ -1 \\ 0\end{bmatrix},$$

and

$$\begin{bmatrix} e_G^* \\ Y_G^* \\ r_G^* \end{bmatrix} = -\begin{bmatrix} X_e & X_Y & f' \\ -X_e & 1-C'-I_Y-X_Y & -I_r \\ 0 & -L_Y & -L_r \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$e_G^* = -\frac{\begin{vmatrix} 0 & X_Y & f' \\ -1 & 1-C'-I_Y-X_Y & -I_r \\ 0 & -L_Y & -L_r \end{vmatrix}}{\begin{vmatrix} X_e & X_Y & f' \\ -X_e & 1-C'-I_Y-X_Y & -I_r \\ 0 & -L_Y & -L_r \end{vmatrix}} = \frac{X_Y L_r - f'L_Y}{X_e\left(L_r\left(1-C'-I_Y\right)+L_Y\left(I_r-f'\right)\right)}.$$

- we can similarly compute Y_G^* and r_G^*.

Comparison of Mundell-Fleming and IS-LM Models

$$Y_M^*\Big|_{MF} = \frac{I_r - f'}{L_r\left(1-C'-I_Y\right)+L_Y\left(I_r-f'\right)} = \frac{1}{L_r\dfrac{1-C'-I_Y}{I_r-f'}+L_Y} > 0$$

$$Y_M^*\Big|_{IS-LM} = \frac{I_r}{L_r\left(1-C'-I_Y\right)+L_Y I_r} = \frac{1}{L_r\dfrac{1-C'-I_Y}{I_r}+L_Y} > 0.$$

Since $f' > 0$, we have $Y_M^*\Big|_{MF} > Y_M^*\Big|_{IS-LM}$.

$$r_M^*\Big|_{MF} = \frac{1-C'-I_Y}{L_r\left(1-C'-I_Y\right)+L_Y\left(I_r-f'\right)} = \frac{1}{L_r+L_Y\dfrac{I_r-f'}{1-C'-I_Y}}$$

$$r_M^*\Big|_{IS-LM} = \frac{1-C'-I_Y}{L_r\left(1-C'-I_Y\right)+L_Y I_r} = \frac{1}{L_r+L_Y\dfrac{I_r}{1-C'-I_Y}}.$$

Thus $r_M^*\Big|_{MF} < r_M^*\Big|_{IS-LM}$.

2 Special Cases

a) $f' = 0 \Rightarrow$ perfect capital immobility so there is no capital flow across borders.

$$Y_M^*\Big|_{MF} = Y_M^*\Big|_{IS-LM}$$
$$r_M^*\Big|_{MF} = r_M^*\Big|_{IS-LM}$$

- This is true even the Mundell-Fleming model includes net exports because with flexible exchange rates and no capital flow, the BoP = 0—achieved by exchange rate adjusting as much as needed so BoP = 0, i.e., $X=0$. Effectively, we are in the IS-LM model.

b) $f' \to \infty \Rightarrow r_M^*\Big|_{MF} = \dfrac{1}{L_r + L_Y \dfrac{I_r - f'}{1-C'-I_Y}} = 0$

- when capital is perfectly mobile, interest rates must be equal everywhere. This is because, if $r > r^F$, capital inflows cause interest to comedown.
- Monetary policy has no effect on domestic interest *r*. Thus, higher $f' \Rightarrow$ lower change in *r* due to change in the money supply *M*.

$$f' \to \infty \Rightarrow Y_M^*\Big|_{MF} = \frac{1}{L_r \dfrac{1-C'-I_Y}{I_r - f'} + L_Y} = \frac{1}{L_Y}$$

- i.e., since *r* does not change, the money market can be in equilibrium again if equilibrium output Y^* rises just enough to create enough demand for money so $L = M$.

$$
\begin{aligned}
& M \uparrow \Rightarrow r \downarrow \\
& \quad \Rightarrow \text{Capital Outflows} \\
& \quad \Rightarrow e = \frac{\text{B}}{\$} \uparrow \\
& \quad \Rightarrow X \uparrow \\
& \quad \Rightarrow Y \uparrow \Rightarrow \begin{cases} \text{Import } \uparrow \text{ offsetting increase in } X \text{ until } X = 0 \\ \\ L \uparrow \Rightarrow \begin{cases} r \uparrow \text{ offsetting decrease in } r \\ L = M \end{cases} \end{cases}
\end{aligned}
$$

HW Baldani, p. 175, #6.19 Find $Y_M^*\Big|_{MF}$ and $r_M^*\Big|_{MF}$ of Mendell-Fleming model when the exchange rate is fixed, and a) $f' = 0$ and b) $f' \to \infty$. That is, the endogeneous variables are $(Y, r, \Delta F)$ and exogenous variables are (r^F, G, M, e).

5.3 Tax Incidence in Supply-Demand Model

The standard competitive Supply-Demand model gives 3 important results of tax incidence analysis

1. after tax equilibrium is the same for tax paid by buyers or sellers.
2. consumers and producers shares the burden
3. consumers' burden increases as the elasticity of demand decreases and/or elasticity of supply increases.

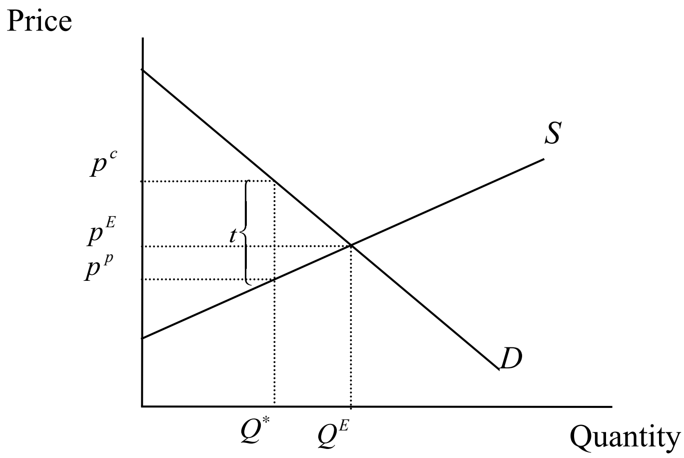

Figure 5.3 Supply-Demand model with specific tax.

p^c = price paid by consumers after tax

p^p = price received by producers after tax

t = specific tax rate

There are three equations:

$$p^c = D(Q)$$
$$p^p = S(Q)$$
$$p^c = p^p + t,$$

with Q, p^c, p^p being endogeneous and t exogeneous and we can write the implicit function

$$\mathbf{f}(Q_t, p_t^c, p_t^p; t) = \begin{bmatrix} p^c - D(Q) \\ p^p - S(Q) \\ p^c - p^p - t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

By Implicit Function Theorem,

$$\begin{bmatrix} Q_t' \\ p_t'^c \\ p_t'^p \end{bmatrix} = -\begin{bmatrix} -D' & 1 & 0 \\ -S' & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

We have, if $D' < 0$ and $S' > 0$

$$Q_t' = -\frac{\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} -D' & 1 & 0 \\ -S' & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}} = \frac{1}{D' - S'} < 0,$$

$$p_t'^c = -\frac{\begin{vmatrix} -D' & 0 & 0 \\ -S' & 0 & 1 \\ 0 & -1 & -1 \end{vmatrix}}{\begin{vmatrix} -D' & 1 & 0 \\ -S' & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}} = \frac{D'}{D' - S'} > 0,$$

$$p_t'^p = -\frac{\begin{vmatrix} -D' & 1 & 0 \\ -S' & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} -D' & 1 & 0 \\ -S' & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}} = \frac{S'}{D' - S'} < 0.$$

- if $D' < 0$ and $S' > 0$, then

$$\left. \begin{aligned} 0 < p_t'^c = \frac{1}{1 - S'/D'} < 1 \\ -1 < p_t'^p = \frac{1}{D'/S' - 1} < 0 \end{aligned} \right\} \Rightarrow \text{Result 2: Consumers \& Producers share burden}$$

- We always have $D' < 0$ for market demand and thus if

1. $S' = 0 \Rightarrow p_t'^c = 1,\ p_t'^p = 0$--the burden is only on the consumers.

2. $S' < 0$ and if

a) $D' < S' < 0$ then $p_t'^c > 1$ so consumers' burden > t, and $p_t'^p > 0$ producers get higher price,

b) $S' < D' < 0$ then $p_t'^p < -1$ so producers receive a reduction in price bigger than t, and $p_t'^c < 0$ so consumers pay less after tax.

Result 3: Burden are shared and depends on elasticities.

$$\eta_D = -\frac{1}{D'}\frac{p^E}{Q^E} \Rightarrow D' = -\frac{p^E}{\eta_D Q^E}$$
$$\eta_S = \frac{1}{S'}\frac{p^E}{Q^E} \Rightarrow S' = \frac{p^E}{\eta_S Q^E}$$

Thus

$$p_t'^c = \frac{D'}{D' - S'} = \frac{\eta_S}{\eta_D + \eta_S} > 0,$$
$$p_t'^p = \frac{S'}{D' - S'} = -\frac{\eta_D}{\eta_D + \eta_S} < 0,$$

and $p_t'^c - p_t'^p = 1$--total burden is equal to the tax.

HW Supply-Demand Model with ad-valorem tax. If the tax is levied according to the price of the good, the model becomes

$$p^c = D(Q)$$
$$p^p = S(Q)$$
$$p^c = (1+t)\,p^p.$$

Recompute Q_t', $p_t'^c$, and $p_t'^p$.