FN 312 Investment
Practice Midterm Exam Solutions (Fall 2016)

Question 1

a) Column 1 represents the date the bill will be redeemed and the investor paid the face value amount.
Column 2 represents the days to maturity calculated from the settlement date.
Column 3 represents the interest rate that the buyer wants to receive. (The bid price is the price at which you can sell.)
Column 4 represents the interest rate that the seller is willing to pay. (The ask price is the price at which you can buy)
Column 5 represents the difference from the prior bid
Column 6 represents the annualized rate of return if the T-bill is held until maturity based on the ask price

b) The T-bill bid price can be calculated from the bank discount yield formula:

$r\_BDY = (FV – P)/FV * 360/n$

Rewriting to calculate the bid price gives:

$P = FV * (1-n/360 * r\_BDY) = 1000000 * (1-155/360*0.0352) = 984,844.44$

Verify that you can calculate the ask price to be 984,877.50

c) The bank discount yield is a measure of a bond's percentage return. The bank discount yield based on the ask price is 3.51 whereas the bond equivalent yield based on the ask price is 3.61. The bank discount yield is not a meaningful measure of the return on investment because: (i) it is based on the face value of the T-bill, not on the purchase price, and so the return on investment is not measured based on the cost of investment; (ii) it is annualized using a 360 day year, not 365; (iii) it annualizes with simple interest and ignores the effect of compounding interest.

d) Earlier, using the bid discount rate, we calculated a bid price for 155-day T-bill to be 984,844.44.

- At maturity, this T-bill will be worth $1,000,000
- You will earn $15,155.56 of interest on an investment of $984,844.44 over 155 days, a percentage return of 1.5389%.
- In a 365 day year, there are 365/155 = 2.3548 periods of 155 days in length.
- 1.5389 times 2.3548 is 3.6238% (this is the bond equivalent (bid) yield).

The effective annual rate is corresponding to the bid yield is:

$$1 + EAR = \left(1 + \frac{APR}{m}\right)^{m}$$

$$1 + EAR = \left(1 + \frac{0.036238}{2.3548}\right)^{2.3548}$$

$$= 1.015389^{2.3548}$$

$$= 1.036616$$

so, the $EAR = 3.6616\%$.

Note that when interest rates are low, the APR will be close to the EAR.

Make sure you know how to do this for the ask yield.

e) Factors that are known to move T-bill rates are: 1) Demand for risk-free fixed income securities (higher demand increases the prices of bonds and lowers interest rates) 2) Supply of bonds by the US government (when the Fed purchases bonds, it reduces its supply and injects liquidity into the economy causing interest rates to be lower) 3) Economic conditions (rates tend to rise in expansions, fall in recessions) 4) Inflation (high inflation yields high interest rates and vice versa)

During the recent global financial crisis a number of investors were concerned about default and liquidity risks of other assets causing them to demand more risk-free securities. This phenomenon is known as a "flight to safety" which pushed down the short-term T-bill rates.

At the same time, due to the massive quantitative easing program and the Fed's policy of low interest rates, the interest rates in the economy are lower due to the reduced supply of bonds.

Due to prevailing low inflation rates, investors also expect inflation rates to be low in the future, and may not expect high yields on bonds to cover for the lost of purchasing power.

Question 2

a) How many shares of each company would you buy if you want to construct an equal-weighted portfolio? How many shares of each company would you buy if you want to construct a value-weighted portfolio?

ans: To construct an equal-weighted portfolio, we need to an equal number of dollars in each company. Since we have $10,000 to invest and there are two companies, we need to invest $5,000 in each company's stock. We will purchase 5,000/100 = 50 shares of the first company and 5,000/50 = 100 shares of the second company.

ans: To construct a value-weighted portfolio, we need to purchase an equal percentage of each company's market capitalization. The market capitalization of the first is 10,000*100 = 1,000,000, and the market capitalization of the second is 60,000*50=30,000*100=3,000,000. If x is the equal percentage of each company's market capitalization that we'll invest in, then 1,000,000*x+3,000,000*x=10,000. x=1/400, so we much purchase 1,000,000/400 = $2,500 of the first company's stock and 3,000,000/400 = $7,500 of the second company's stock. This translates into buying 2,500/100 = 25 shares of the first company and 7,500/50 = 15,000/100= 150 shares of the second company.

b) A year later, company 1's share price has risen to $150 while company 2's share price is unchanged at $50. Neither company has paid a dividend. What are the returns on the equal-weighted portfolio and the value-weighted portfolio you constructed in part a)? Explain the difference.

ans: The (net simple) return of company 1's shares is (150-100)/100 = .5, and the return of company 2's shares is 0. Now we apply the idea that portfolio return is the weighted average of its individual stock returns where the weights are the shares of wealth invested in each stock. The equal-weighted portfolio return is .5*.5+.5*0=25%. The value-weighted portfolio return is .25*.5+.75*0=12.5%. The equal-weighted portfolio has a higher return because it held a greater share of wealth in the stock with the higher return.

c) How must you trade a year later to keep the equal-weighted portfolio equal-weighted? How must you trade to keep the value-weighted portfolio value-weighted? Explain. (Note: It is sufficient to explain whether you need to buy or sell the shares of either company. You do not need to calculate the exact amounts.)

ans: To keep the equal-weighted portfolio equal-weighted you must sell some of the first company's shares and buy some more of the second company's shares. Since the first company's stock price appreciated and the second company's did not, the wealth share held in the first company's stock before rebalancing is greater than 50%.

ans: To keep the value-weighted portfolio value-weighted nothing must be done. Since the number of shares outstanding hasn't changed for either company, the relative market capitalization has changed in proportion to the change in share prices. The relative wealth share held in each company has also changed in proportion to the change in share price.

d) In this question, there is a typo. I meant to ask for examples of price-weighted stock indexes (eg. Dow Jones, Nikkei). Examples of value-weighted indexes are the S&P 500 and the NASDAQ. The value-weighted index is preferred because stocks with larger market-cap should get larger weight in the index. The value-weighted index also does not have to use an adjustment factor to account for stock splits and dividends.

Question 3

(a) Calculate the expected payoff on a unit of the asset.

Answer:

Denote the payoff by V. Then, the expected payoff is:

$$\mathrm{E}[V] = 1 \times 0.3 + 4 \times 0.5 + 5 \times 0.2 = 3.3.$$

(b) Calculate the variance and standard deviation of the payoff on the asset.

Answer:

The variance of V is defined by:

$$\mathrm{var}(V) \equiv \mathrm{E}[(V - \mathrm{E}[V])^2] = \mathrm{E}[V^2] - (\mathrm{E}[V])^2.$$

Hence

$$\begin{aligned} \mathrm{var}(V) &= (1 \times 0.3 + 16 \times 0.5 + 25 \times 0.2) - (3.3)^2 \\ &= 13.3 - 10.89 = 2.41 \end{aligned}$$

Standard deviation $\equiv \sqrt{\mathrm{var}(V)} = \sqrt{2.41} \approx 1.552.$

(c) Suppose that the current price of the asset is £2. Calculate the expected rate of retur the variance of the rate of return on the asset.

Answer:

The rate of return on the asset is defined by $V/P - 1$. Hence:

$$\mathrm{E}\left[\frac{V}{P} - 1\right] = \mathrm{E}\left[\frac{V}{P}\right] - \mathrm{E}[1] = \frac{\mathrm{E}[V]}{P} - 1 = \frac{3.3}{2} - 1 = 0.65 = 65\%.$$

Also:

$$\mathrm{var}\left(\frac{V}{P} - 1\right) = \mathrm{var}\left(\frac{V}{P}\right) - \mathrm{var}(1) = \frac{\mathrm{var}(V)}{P^2} - 0 = \frac{2.41}{4} = 0.6025.$$

Hence the standard deviation of the rate of return is: $\sqrt{0.6025} \approx 0.776.$

d) The problem with using the standard deviation as a risk measure is that it does not differentiate between good and bad surprises. Also, the standard deviation may not be a measure of risk that reflects what really troubles investors. Investors care about downside risk of a crash or a poor market, while returns above the average are viewed desirable.

The semi-variance measure is a risk measure that accounts for downside risk. It takes the average of the squared deviations below the mean. Another one is the

Value at risk (VaR) measure which estimates how much an investor might lose given normal market conditions in a set time period such as in a day. For example, if a portfolio of stocks has a one-day 5% VaR of $1 million, that means that there is a 0.05 probability that the portfolio will fall in value by more than $1 million over a one-day period if there is no trading.

Question 4

(a) $u(W) = 2W + 5$

ans: $u'(W) = 2 > 0$, and $u''(W) = 0$. Thus, the utility function is increasing at a constant rate. Such a utility function exhibits risk neutral behavior. Note that the stated gamble is a fair gamble since $E[\Delta\tilde{W}] = 0.5 * (100) + 0.5 * (-100) = 0$. Since

$$u(1000) = 2005 = E[u(gamble)] = 0.5(2205) + 0.5(1805) = 2005$$

someone with the given utility function would be indifferent to accepting the gamble.

(b) $u(W) = W^3$

ans: $u'(W) = 3W^2 > 0$, and $u''(W) = 6W > 0$. Thus, the utility function is increasing at an increasing rate. Such a utility function exhibits risk-loving behavior. Since

$$u(1000) = 1,000,000,000 < E[u(gamble) = 0.5(1,100)^3 + 0.5(900)^3$$

someone with the given utility function would choose to take the gamble.

(c) $u(W) = -(1/2) * e^{-2W}$

ans: $u'(W) = e^{-2W} > 0$, and $u''(W) = -2 * e{-}2W < 0$. Thus, the utility function increases at a decreasing rate. Such a utility function exhibits risk averse behavior. Since

$$u(1000) = -0.5 * e^{-2000} > E[u(gamble)] = -0.25 * e^{-2200} - 0.25 * e^{-1800}$$

someone with the given utility function would not be willing to accept the gamble.

d) An underlying assumption for Markowitz portfolio optimization is that investors are risk-averse and therefore only the utility function in c) would work.

Question 5

a)

Mean = (0.30 x 7%) + (0.7 x 17%) = 14% per year.
Standard deviation = 0.70 x 27% = 18.9% per year.

b)

Mean return on portfolio = $R_f + (R_p - R_f)y$
= 7% + (17% - 7%)y = 7% + 10%y

If the mean of the portfolio is equal to 15%, then solving for y we will get:

$15\% = 7\% + 10\% y$ => $y = (15\% - 7\%)/10\%$ => $y = 0.8$

Thus, in order to obtain a mean return of 15%, the client must invest 80% of total funds in the risky portfolio and 20% in Treasure bills.

Investment proportions of the client's funds:

- 20% in T-bills
- 0.8 x 27% = 21.6% in Stock A
- 0.8 x 33% = 26.4% in Stock B
- 0.8 x 40% = 32.0% in Stock C

c)

Portfolio standard deviation = y x 27%. If your client wants a standard deviation of 20%, then

$y = (20\%/27\%) = 0.7407 = 74.07\%$ in the risky portfolio.

Mean return = 7% + (17% - 7%)y = 7% + 10% (0.7407) = 7% + 7.407% = 14.407%.

d)

$y^* = (R_p - R_f)/0.01A\sigma^2$ => $y^* = (17 - 7)/(0.01 \times 3.5 \times 27^2) = 10/25.515 = 0.3919$

e)

The slope of the CML = (13% - 7%)/25% = 0.24. You can draw a graph as we did in class. My fund allows an investor to achieve a higher mean for any given standard deviation than would a passive strategy, that is, a higher expected return for any given level of risk. Indeed, my fund's reward-to-variability ratio is (17% - 7%)/27% = 0.37, which is much better than 0.24 (the RTRR of the S&P 500).

f)

The fee would reduce the reward-to-variability ratio, that is, the slope of the CAL. Clients will be indifferent between your fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the maximum fee. In this case:

Slope of CAL with fee = (17% - 7% - f)/27 = (10 - f)/27%.

Slope of CML (which requires no fee) = (13% - 7%)/25% = 0.24

Setting these slopes equal, then we get:

(10% - f)/27% = 0.24 => 10% - f = 27% x 0.24 => f = 10% - 6.48% = 3.52% per year.

Question 6

(a) Explain how to construct the efficient portfolio frontier for the cases in which the correlation coefficient between the returns, ρ_{12}, is equal to $+1$ and also when it is equal to -1.

Answer:

The expected rate of return on the portfolio is given by:

$$\mu_P = \frac{1}{5}a + \frac{3}{5}(1-a),$$

where a is the proportion of the portfolio invested in asset 1.

The variance of the rate of return on the portfolio is:

$$\sigma_P^2 = 4a^2 + 36(1-a)^2 + 2a(1-a) \times 2 \times 6 \times \rho_{12},$$

where ρ_{12} is the correlation coefficient between the rates of return on the two assets.

Case: $\rho_{12} = +1$:

$$\begin{aligned} \sigma_P^2 &= 4a^2 + 36(1-a)^2 + 24a(1-a) && (1) \\ &= (2a + 6(1-a))^2 && (2) \\ \sigma_P &= 2a + 6(1-a) && (3) \end{aligned}$$

Case: $\rho_{12} = -1$:

$$\begin{aligned} \sigma_P^2 &= 4a^2 + 36(1-a)^2 - 24a(1-a) && (4) \\ &= (2a - 6(1-a))^2 && (5) \\ \sigma_P &= \pm(2a - 6(1-a)) && (6) \\ \sigma_P &= +(2a - 6(1-a)) \quad \text{for} \quad a \geq \frac{3}{4} && (7) \\ \sigma_P &= -(2a - 6(1-a)) \quad \text{for} \quad a < \frac{3}{4} && (8) \end{aligned}$$

Case: $\rho_{12} = +1$:

$$\sigma_P = 6 - 4a$$

Hence:

$$a = \frac{6 - \sigma_P}{4} \qquad a = \frac{-2 + \sigma_P}{4}$$

$$\begin{aligned} \mu_P &= \frac{1}{5}\left(\frac{6 - \sigma_P}{4}\right) + \frac{3}{5}\left(\frac{-2 + \sigma_P}{4}\right) && (9) \\ &= \frac{6 - \sigma_P - 6 + 3\sigma_P}{20} = \frac{2\sigma_P}{20} = \frac{\sigma_P}{10}. && (10) \end{aligned}$$

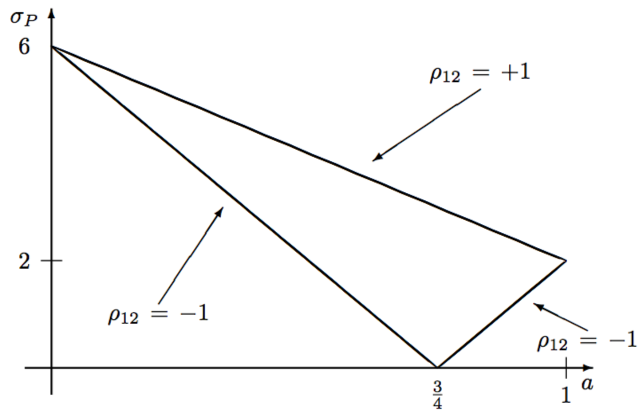

Figure 1: Standard deviation and a with $\rho_{12} = \pm 1$.

Case: $\rho_{12} = -1$ and $a \geq 3/4$:

$$\sigma_P = 2a - 6(1-a) = -6 + 8a$$

Hence:

$$a = \frac{6+\sigma_P}{8} \qquad a = \frac{2-\sigma_P}{8}$$

$$\begin{aligned} \mu_P &= \frac{1}{5}\left(\frac{6+\sigma_P}{8}\right) + \frac{3}{5}\left(\frac{2-\sigma_P}{8}\right) && (11) \\ &= \frac{6+\sigma_P+6-3\sigma_P}{40} = \frac{12-2\sigma_P}{40} = \frac{3}{10} - \frac{\sigma_P}{20}. && (12) \end{aligned}$$

Case: $\rho_{12} = -1$ and $a < 3/4$:

$$\sigma_P = -2a + 6(1-a) = 6 - 8a$$

Hence:

$$a = \frac{6-\sigma_P}{8} \qquad a = \frac{2+\sigma_P}{8}$$

$$\begin{aligned} \mu_P &= \frac{1}{5}\left(\frac{6-\sigma_P}{8}\right) + \frac{3}{5}\left(\frac{2+\sigma_P}{8}\right) && (13) \\ &= \frac{6-\sigma_P+6+3\sigma_P}{40} = \frac{12+2\sigma_P}{40} = \frac{3}{10} + \frac{\sigma_P}{20}. && (14) \end{aligned}$$

(b) Describe, in general terms, how to construct the portfolio frontier when $-1 < \rho < +1$.

Answer:

When $-1 < \rho < +1$, the portfolio proportions a and $1-a$ are chosen to minimize σ_P (or σ_P^2) for each level of μ_P. For each level of μ_P, the solution provides one point on the portfolio frontier. As μ_P is chosen at different levels, so the frontier is traced out. The multiple asset case is similar, except that now there are n portfolio proportions to choose: $a_1, a_2, \ldots, a_n$ (such that the proportions sum to 1).

For $-1 < \rho < +1$, the frontier is a hyperbola in the space of (μ_P, σ_P):

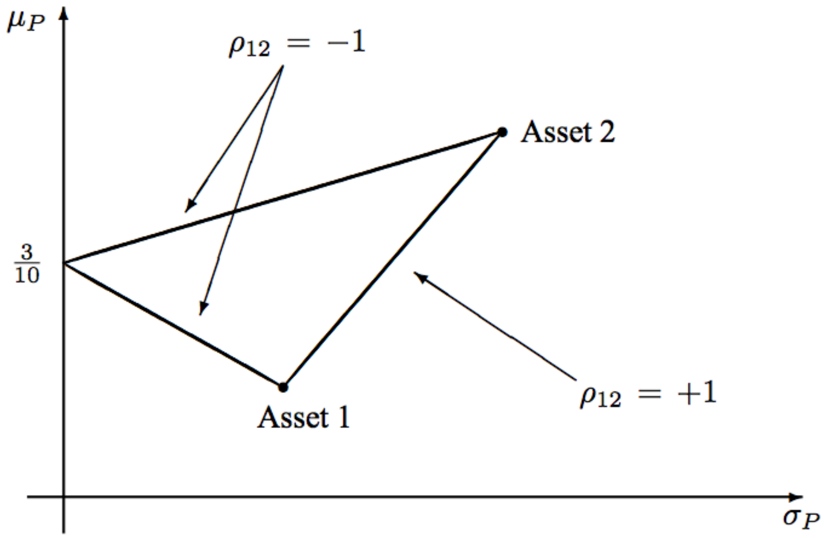

Figure 2: Efficiency Frontier with Two Assets and $\rho_{12} = \pm 1$.

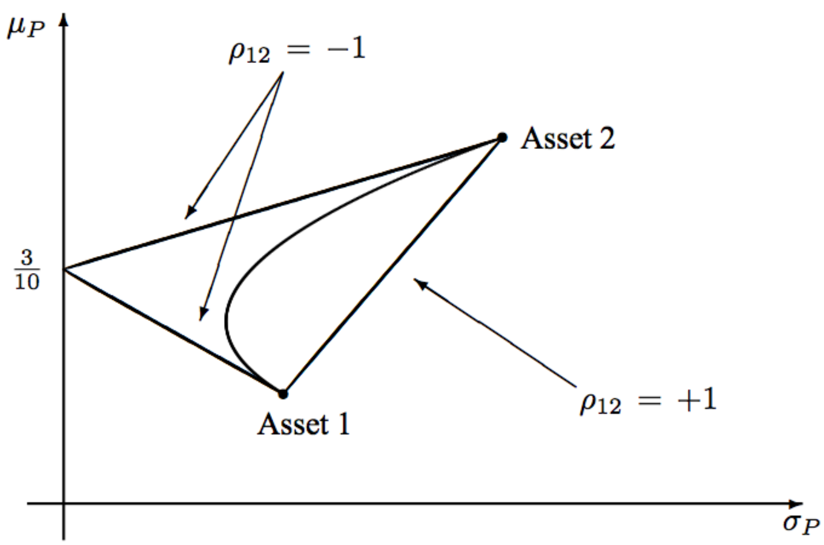

Figure 3: Efficiency Frontier with Two Assets and $\rho_{12} = \pm 1$.

Question 7

(a) Construct the efficient portfolio frontier.

Answer:

First, construct the frontier for a portfolios consisting of risky assets only. This is a hyperbola, depicted as FF in the following figures.

Next construct the set of efficient portfolios when lending occurs (i.e. when there is a non-negative proportion of the portfolio invested in the risk-free asset). To do this, with a lending rate of r_0^L, start from the point r_0^L on the vertical axis: this corresponds to a portfolio invested entirely in the risk-free asset. Now draw a straight line (ray) from this point to any point on the FF frontier. Any point on the line is a feasible portfolio. Try drawing a sequence of rays, each with higher μ_P for a given σ_P (i.e. steeper straight lines). The steepest ray that connects with FF will be a ray that is just tangential to the frontier FF. This ray depicts the efficient portfolios for the given lending rate. In the figure it is depicted by the line $r_0^L Y$.

Note that, because the investor can lend *but not borrow* at r_0^L, only the segment of the line between r_0^L and Y is relevant. Points to the right of Y are not feasible, because the investor would be *borrowing*.

The efficient portfolios with borrowing at r_0^B are constructed in a similar way to provide the set depicted by the straight line $r_0^B Z$. For this set only points to the right of Z are feasible: the investor can *borrow* but not lend at the rate r_0^B.

Hence the overall set of efficient portfolios is depicted by the line $r_0^L YZE$. In the segment $r_0^L Y$ the portfolio has a positive amount of the risk-free asset (lending). In the segment YZ the portfolio is invested entirely in risky assets (neither borrowing nor lending). In the segment ZE the portfolio has a negative amount of the risk-free asset (borrowing).

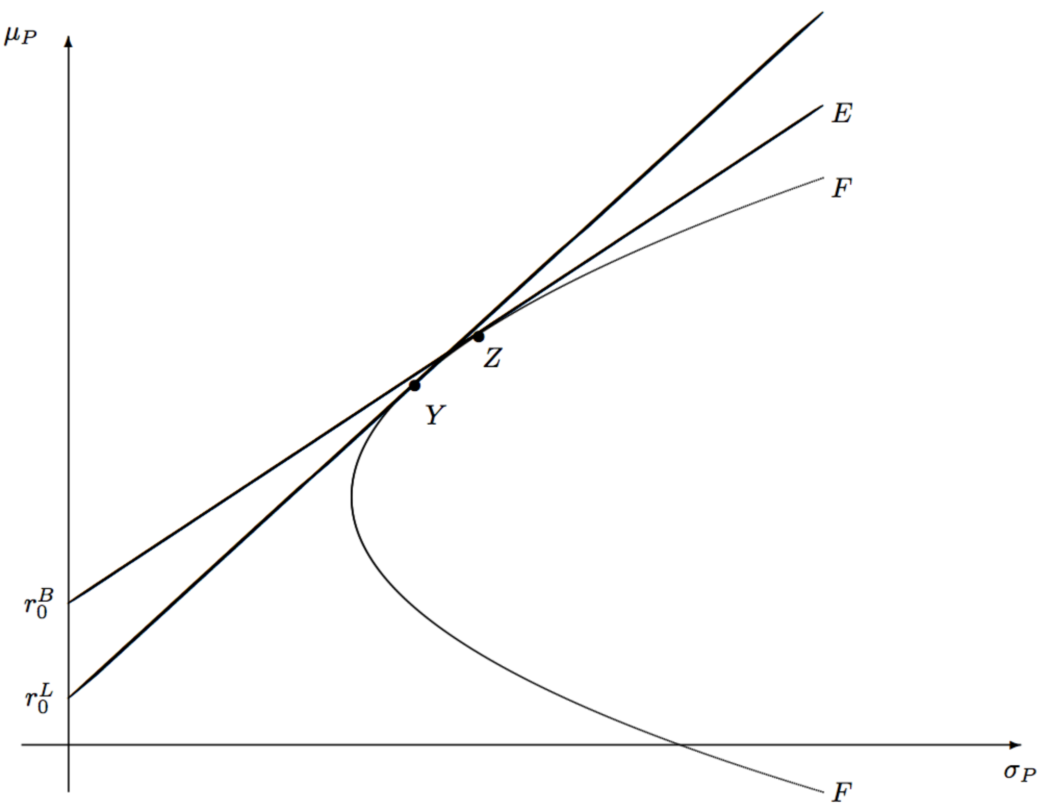

Figure 4: The Efficient Portfolios with different lending and borrowing rates.

(b) Depict an equilibrium for an investor who chooses to borrow.

Answer:

Begin with the previous diagram and imagine that there is a set of indifference curves in (μ_P, σ_P) space, representing preferences. Suppose that preferences are such that there is an indifference curve II' tangential to the line ZE (see figure 5 on page 6).

The point of tangency represents an optimal portfolio for an investor that borrows. (No higher indifference curve is attainable.)

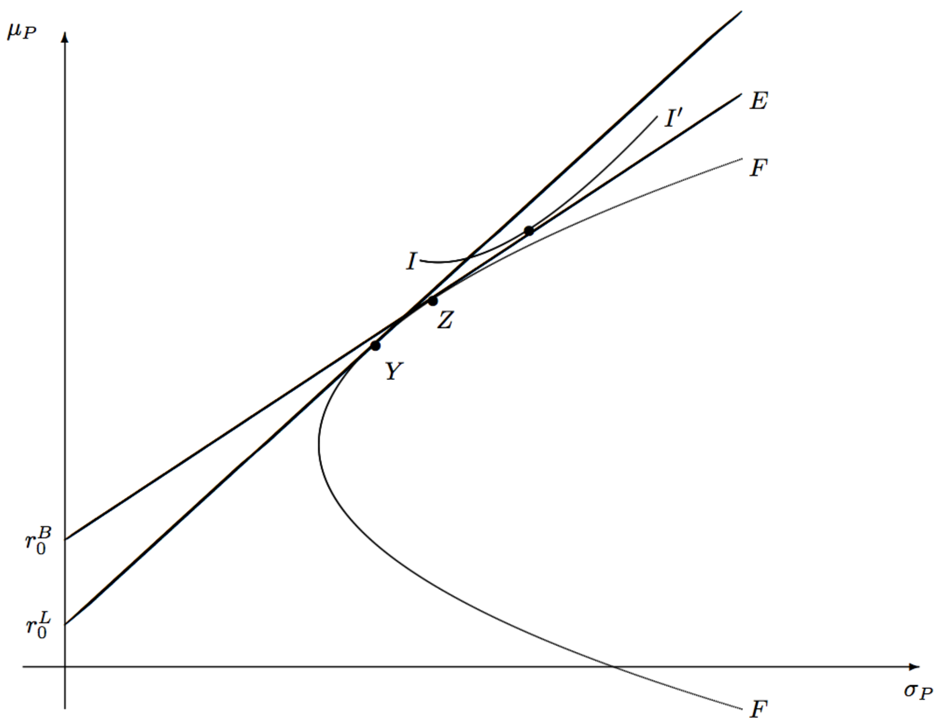

Figure 5: An optimal portfolio depicted by tangency of an indifference curve with the efficient portfolio constraint.

(c) Suppose that the interest rate at which the investor can borrow increases. Examine the implications for the investor's optimal investment decisions.

Answer:

Suppose that the borrowing rate increases from r_0^B to $r_0^{B'}$ (see figure 6 on page 7). Now the set of efficient portfolios is depicted by the line: $r_0^L YZ'E'$. Every investor is certainly no better off than before the rate increase. An investor who is a borrower at the initial rate is worse off (will be on a lower indifference curve).

If the investor continues as a borrower at the higher interest rate, there will be a point of tangency between a different (lower) indifference curve and the segment of the line $Z'E'$.

The investor will be worse off as a consequence of the increase in the borrowing rate but may end up borrowing more (taking more risk) or less depending on "substitution" and "scale" effects. The substitution effect (a movement along a given indifference curve) of an increase in the borrowing rate will be to *reduce* borrowing. The scale ("income") effect could result in either an increase or decrease in borrowing.

Note that it is possible that borrowing is reduced to zero as a consequence of the increase in the borrowing rate.

Question 8

(a) In the context of the Capital Asset Pricing Model (CAPM), define the 'beta-coefficient', β_j, corresponding to asset j. Discuss how assets' beta-coefficients should be interpreted and explain how their values can be obtained in practice.

Answer:

The beta-coefficient can be defined in any of the following equivalent ways:

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2} = \frac{\rho_{jM}\sigma_j\sigma_M}{\sigma_M^2} = \rho_{jM}\frac{\sigma_j}{\sigma_M},$$

where σ_{jM} is the covariance between the rate of return on asset j and the market rate of return, σ_M is the standard deviation of the market rate of return, σ_j is the standard deviation of the rate of return on asset j, and ρ_{jM} is the correlation coefficient between the rate of return on asset j and the market rate of return.

An asset's beta-coefficient is a measure of the relationship between its rate of return and the market rate of return. It can be interpreted as a measure of the asset's risk, relative to the market as a whole. An asset's beta-coefficient is formally the slope co-efficient on the excess rate of return on the market in a regression of the excess rate of return on asset j on the excess rate of return on the market:

$$r_j = r_0 + (r_M - r_0)\beta_j + \varepsilon_j, \qquad j = 1, 2, \ldots, n,$$

where ε_j is an unobserved random error. It is assumed that $\mathrm{E}[\varepsilon_j \,|\, r_M] = 0$, that is, the expected value of the error, conditional upon the rate of return on the market portfolio, is zero.

Typically (almost always) beta-coefficients are estimated from data on past rates of return (in the regression described above).

(b) Assuming that a risk-free asset is available, explain and interpret the Security Market Line (SML) in the context of the CAPM. Construct the SML from the given information and interpret the values of its coefficients.

Answer:

The CAPM predicts that:

$$\mu_j = r_0 + (\mu_M - r_0)\beta_j,$$

where μ_j is the expected rate of return on asset j, μ_M is the expected rate of return on the market portfolio, and r_0 is the risk-free rate of return The SML treats μ_j as a function of β_j and shows how the expected rate of return on each asset differs according to its beta-coefficient. The slope of the SML is then a measure of the market 'price' of risk.

See figure 1.

The data in the question must satisfy:

$$0.066 = r_0 + 0.4(\mu_M - r_0), \quad \text{and} \quad 0.098 = r_0 + 1.2(\mu_M - r_0).$$

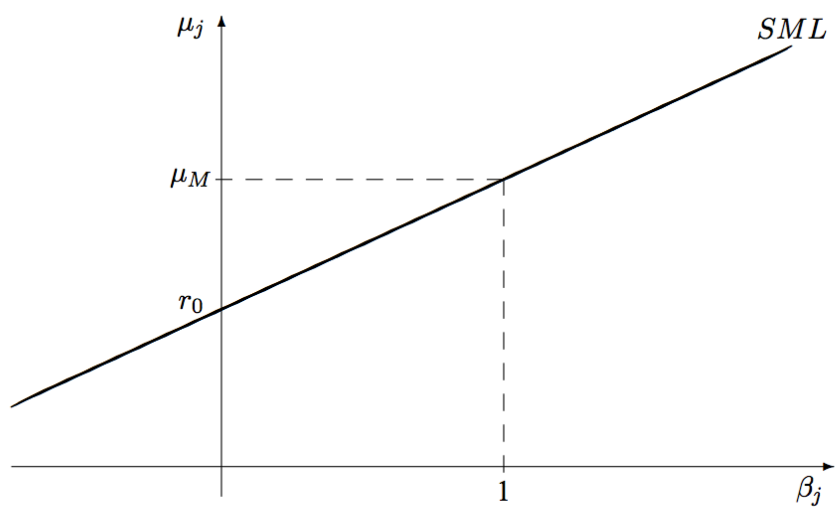

Figure 1: The Security Market Line, SML

Hence it must follow that: $\mu_M = 0.09$ and $r_0 = 0.05$. Thus, in this example the market price of risk is 4%. Hence the SML is:

$$\mu_j = 0.05 + 0.04\beta_j.$$

(Check that the data for asset 3 also satisfy the SML.)

c)

You are informed that a fourth asset, with $\beta_4 = 0.8$, is available. Recent observations reveal that its average rate of return is 7.0%. What inferences, if any, would you draw from this information? [Your answer may be in the context of either (b) or (c), above.]

Answer:

The CAPM predicts that the expected rate of return on the fourth asset is:

$$0.082 = 0.05 + 0.04 \times 0.8.$$

But the observed average rate is $7.0\% < 8.2\%$. Hence, the fourth asset is *overpriced*. This evidence could be indicative either that the market is in disequilibrium or that the CAPM is not a good representation of the market.

Question 9

This is a typical final examination question for which there are many "correct" answers of varying standards (as well as even more bad answers). What follows are some pointers about how you should set about answering a question like this:

(a) *Read* the question carefully and try to answer it, not just write about the CAPM. This question focuses on the *predictions* of the CAPM and the underlying *assumptions* that generate the predictions.

(b) Begin by defining the most important terms in the question. Then define the concepts you need. In answering this question, obviously you will concentrate on the CAPM.

Describe, briefly, the CAPM in terms of its origins in mean-variance analysis. That is, the CAPM is a model of market equilibrium in which investors choose their portfolios according to a mean-variance criterion and in which they all agree about the means and variances (i.e. homogeneous beliefs).

(c) Now you are ready to state the main predictions. These can be summarised according to the three "lines": the Capital Market Line, the Characteristic Line and the Security Market Line. Your answer should contain a brief statement of each of these. (Refer to chapter 6 of EFM. Then put EFM aside and then try to write a short paragraph on each.)

It would make your answer coherent to tie the predictions together in terms of the equation:

$$\mu_j - r_0 = (\mu_M - r_0)\beta_j, \quad \text{where} \quad \beta_j = \rho_{jM}\frac{\sigma_j}{\sigma_M}.$$

In your answer be sure to define what the symbols mean! The Capital Market Line (CML) is such that "j" denotes an efficient portfolio. The rate of return on any efficient porfolio, say E, is perfectly correlated with the market return. Hence, $\beta_E = \sigma_E/\sigma_M$ and the prediction becomes:

$$\frac{\mu_E - r_0}{\sigma_E} = \frac{\mu_M - r_0}{\sigma_M},$$

which is the equation of the CML.

The Characteristic Line, treats $\mu_j - r_0$ as a function of $\mu_M - r_0$, with slope β_j. This is useful for estimating β_j.

The Security Market Line treats $\mu_j - r_0$ as a function of β_j, with slope $\mu_M - r_0$. This is useful for testing the cross-section patterns of asset returns.

(d) Next move on to describing the assumptions. While it is not wrong to just give a long list of assumptions, the examiners will be more impressed if you can group the assumptions into catagories and offer some appraisal of their role. (Check chapter 6 of EFM. Then put EFM aside and write a few paragraphs describing the assumptions.)

(e) The crucial assumptions are (i) that there is market equilibrium in the sense of a balance between the demand and supply to hold assets, (ii) that all investors choose portfolios according to a mean-variance criterion, and (iii) that they have the same beliefs ('homogeneous' or 'unanimous' beliefs) about asset returns.

(f) What is the role of these assumptions? The mean variance assumption implies that for each investor:

$$\frac{\mu_j - r_0}{\beta_j \sigma_Z} = \frac{\mu_Z - r_0}{\sigma_Z} \quad \text{or} \quad \mu_j - r_0 = (\mu_Z - r_0)\beta_j, \quad j = 1, 2, \ldots, n,$$

where Z is the efficient portfolio comprising risky assets only. Note that, without further assumptions, μ_j, β_j and Z could differ from one investor to another.

In your answer you should now describe briefly (check EFM, chapter 6, if necessary) *why* in market equilibrium the portfolio Z can be understood as the market portfolio. Also, the assumption of homogeneous beliefs implies that μ_j and β_j are the same for each investor.

(g) Finally, you could conclude your answer by briefly mentioning the extensions of the CAPM, for example to allow for cases when it is unreasonable to assume that all investors can borrow or lend at a risk-free rate, or to encompass intertemporal planning

Question 10

a) Based on the regression results the alpha coefficient is estimated to be -0.058 with t-statistic -0.14 meaning that the null that the coefficient is zero cannot be rejected at standard levels of significance. Since the CAPM implies that alpha is equal to zero (the returns lie on the SML), the CAPM holds for these regression results.
b) 10 number of observations are too little to get reliable results. Also, using annual frequencies aggregate the returns on too high of a level. Typically, for CAPM regressions at least 5 years of monthly data should be used for the regression.
c) CAPM may not hold in the data because stock returns are extremely volatile, making it difficult to estimate alpha and beta accurately. In other words, estimates of alpha and beta may be associated with large standard errors. Furthermore, the market index such as the S&P 500 may not be the true "market portfolio" since it does not capture all investments of the average investor such as investment on human capital (education). The Roll critique for example postulates that the market portfolio is not identifiable. Another

assumption of the CAPM that does hold in practice is that investors cannot freely borrow and lend at the risk free rate.

Some anomalies for CAPM includes the size effect, the valuation effect and the momentum effect. Please see the last few slides of Lecture 6 for an explanation of these anomalies.