Quiz 6

1. Let

$$f(x,y,z) = \sqrt{\frac{x^2+1}{y}}\ \sin^2(xyz).$$

Find $\dfrac{\partial f}{\partial x}$.

Solution By using the **product rule**,

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{1}{\sqrt{y}}\left[(x^2+1)^{1/2}\frac{\partial}{\partial x}\sin^2(xyz) + \sin^2(xyz)\frac{\partial}{\partial x}(x^2+1)^{1/2}\right] \\
&= \frac{1}{\sqrt{y}}\left[(x^2+1)^{1/2}2\sin(xyz)\frac{\partial}{\partial x}\sin(xyz) + \sin^2(xyz)\frac{1}{2}(x^2+1)^{-1/2}\frac{\partial}{\partial x}(x^2+1)\right] \\
&= \frac{1}{\sqrt{y}}\left[(x^2+1)^{1/2}2\sin(xyz)\cos(xyz)\frac{\partial}{\partial x}(xyz) + \sin^2(xyz)\frac{1}{2}(x^2+1)^{-1/2}[2x]\right] \\
&= \frac{1}{\sqrt{y}}\left[2(x^2+1)^{1/2}\sin(xyz)\cos(xyz)[yz] + x\sin^2(xyz)(x^2+1)^{-1/2}\right] \\
&= \frac{1}{\sqrt{y}}\left[2yz(x^2+1)^{1/2}\sin(xyz)\cos(xyz) + x\sin^2(xyz)(x^2+1)^{-1/2}\right]
\end{aligned}$$

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