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1. In Table 1. $X_i$ is total econometrics exam point (total points are 100) and $Y_i$ is GPA of each BE student.

**Table 1.a**

| Student | $Y_i$ | $X_i$ |
|---|---|---|
| 1 | 2.8 | 63 |
| 2 | 3.4 | 72 |
| 3 | 3 | 78 |
| 4 | 3.5 | 81 |
| 5 | 3.6 | 87 |
| 6 | 3.0 | 75 |
| 7 | 2.7 | 75 |
| 8 | 3.7 | 90 |

$(X_i - \bar{X})(Y_i - \bar{Y}) = x_i y_i$

$(63-77.625)(2.8-3.2125) = 6.0328$

$(72-77.625)(3.4-3.2125) = -1.0547$

$(78-77.625)(3-3.2125) = -0.0797$

$(81-77.625)(3.5-3.2125) = 0.9703$

$(87-77.625)(3.6-3.2125) = 3.6328$

$(75-77.625)(3-3.2125) = 0.5578$

$(75-77.625)(2.7-3.2125) = 1.3453$

$(90-77.625)(3.7-3.2125) = 6.0328$

$\Sigma x_i y_i = 17.4374$

$n = 8$

$\Sigma X_i = 621$

$\bar{X} = \frac{\Sigma X_i}{n} = \frac{621}{8} = 77.625$

$\Sigma Y_i = 25.7$

$\bar{Y} = \frac{\Sigma Y_i}{n} = \frac{25.7}{8} = 3.2125$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

From OLS, $\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2}$

$$\hat{\beta}_2 = \frac{17.4374}{511.875} = 0.0341$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 3.2125 - (0.0341)77.625$$

$$= 0.5655$$

1.2 Find $\hat{Y}_i$ $and$ $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_i = 0.5625 + 0.0341 X_i$

$\hat{Y}_1 = 0.5625 + 0.0341(63) = 2.7108$, $\hat{u}_1 = Y_1 - \hat{Y}_1 = 0.0892$

$\hat{Y}_2 = 0.5625 + 0.0341(72) = 3.0177$, $\hat{u}_2 = Y_2 - \hat{Y}_2 = 0.3823$

$\hat{Y}_3 = 0.5625 + 0.0341(78) = 3.2223$, $\hat{u}_3 = Y_3 - \hat{Y}_3 = -0.2223$

$\hat{Y}_4 = 0.5625 + 0.0341(81) = 3.3246$, $\hat{u}_4 = Y_4 - \hat{Y}_4 = 0.1754$

$\hat{Y}_5 = 0.5625 + 0.0341(87) = 3.5292$, $\hat{u}_5 = Y_5 - \hat{Y}_5 = 0.0708$

$\hat{Y}_6 = 0.5625 + 0.0341(75) = 3.12$, $\hat{u}_6 = Y_6 - \hat{Y}_6 = -0.12$

$\hat{Y}_7 = 0.5625 + 0.0341(75) = 3.12$, $\hat{u}_7 = Y_7 - \hat{Y}_7 = -0.42$

$\hat{Y}_8 = 0.5625 + 0.0341(90) = 3.6315$, $\hat{u}_8 = Y_8 - \hat{Y}_8 = 0.0685$

$$\sum_{i=1}^{8} \hat{u}_i = 0.0239 \approx 0$$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \sigma^2 = \frac{\sum u_i^2}{n-2}$$

$$= \frac{0.4348}{6}$$

$$= 0.0725$$

$$var(\hat{\beta}_1) = \frac{\sum X_i^2}{n\sum x_i^2}\sigma^2$$

$$= 48,717 \times 0.0001$$

$$= 4.8717$$

$$var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{\sigma^2}{\sum(X_i-\bar{X})^2}$$

$$= \frac{0.0725}{511.875}$$

$$= 0.0001$$

2. Data is listed in the table

| $X_i$ | $Y_i$ | $(X_i-\bar{X})(Y_i-\bar{Y}) = x_iy_i$ |
|---|---|---|
| 10 | 0 | $(10-20)(0-9.1) = 91$ |
| 12 | 2 | $(12-20)(2-9.1) = 56.8$ |
| 14 | 5 | $(14-20)(5-9.1) = 24.6$ |
| 16 | 6 | $(16-20)(6-9.1) = 12.4$ |
| 18 | 7 | $(18-20)(7-9.1) = 4.2$ |
| 22 | 10 | $(22-20)(10-9.1) = 1.8$ |
| 24 | 10 | $(24-20)(10-9.1) = 3.6$ |
| 26 | 15 | $(26-20)(15-9.1) = 35.4$ |
| 28 | 16 | $(28-20)(16-9.1) = 55.2$ |
| 30 | 20 | $(30-20)(20-9.1) = 109$ |

$$\bar{X} = \frac{\sum X_i}{n} = \frac{200}{10} = 20$$

$$\bar{Y} = \frac{\sum Y_i}{n} = \frac{91}{10} = 9.1$$

$$\sum x_iy_i = 384$$

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0,\sigma^2)$

Find estimators of $\beta_1$ and $\beta_2$ from the OLS method and interpret the meaning.

From OLS, $\hat{\beta}_2 = \frac{\sum x_iy_i}{\sum x_i^2} = \frac{384}{440} = 0.8727$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} = 9.1-(0.8727)(20) = -8.3545$$

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$\hat{Y}_i = -8.3545 + 0.8727 X_i$$

$$\hat{Y}_1 = -8.3545 + 0.8727(10) = 0.3725 \quad , \quad \hat{u}_1 = Y_1 - \hat{Y}_1 = -0.3725$$

$$\hat{Y}_2 = -8.3545 + 0.8727(12) = 2.1179 \quad , \quad \hat{u}_2 = Y_2 - \hat{Y}_2 = -0.1179$$

$$\hat{Y}_3 = -8.3545 + 0.8727(14) = 3.8633 \quad , \quad \hat{u}_3 = Y_3 - \hat{Y}_3 = 1.1367$$

$$\hat{Y}_4 = -8.3545 + 0.8727(16) = 5.6087 \quad , \quad \hat{u}_4 = Y_4 - \hat{Y}_4 = 0.3913$$

$$\hat{Y}_5 = -8.3545 + 0.8727(18) = 7.3541 \quad , \quad \hat{u}_5 = Y_5 - \hat{Y}_5 = -0.3541$$

$$\hat{Y}_6 = -8.3545 + 0.8727(22) = 10.8449 \quad , \quad \hat{u}_6 = Y_6 - \hat{Y}_6 = 0.8449$$

$$\hat{Y}_7 = -8.3545 + 0.8727(24) = 12.5903 \quad , \quad \hat{u}_7 = Y_7 - \hat{Y}_7 = -2.5903$$

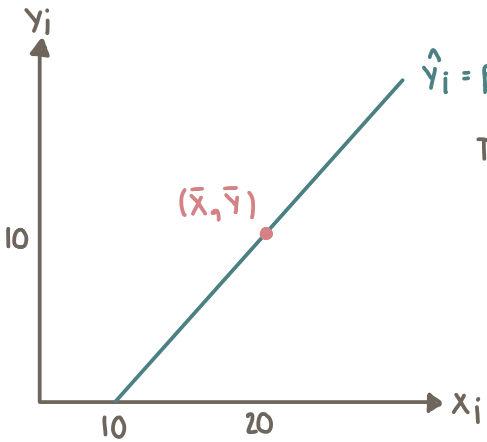
$$\hat{Y}_8 = -8.3545 + 0.8727(26) = 14.3357 \quad , \quad \hat{u}_8 = Y_8 - \hat{Y}_8 = 0.6643$$

$$\hat{Y}_9 = -8.3545 + 0.8727(28) = 16.0811 \quad , \quad \hat{u}_9 = Y_9 - \hat{Y}_9 = 0.0811$$

$$\hat{Y}_{10} = -8.3545 + 0.8727(30) = 17.8265 \quad , \quad \hat{u}_{10} = Y_{10} - \hat{Y}_{10} = 2.1735$$

$$\sum \hat{u}_i = 0$$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?




The line passes through $(\bar{x}, \bar{y})$.

2.4 If $X_i = 18$, what is the predicted Y?

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$= -8.3545 + 0.8727(18) = 7.3541$$