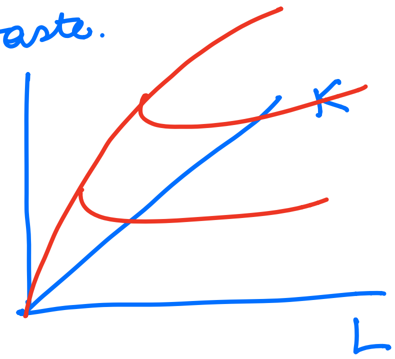
Chapter 15 Production

Production Problem: How a firm employs inputs (labor L and capital K) to
1. Maximize output Q for a given cost level C_0.
2. Minimize cost C to produce a give level of output Q_0.

- The understanding of this production problem is crucial to understanding the behavior of a firm in perfect competition and monopoly—and other markets to be discussed in EE311.

Production Function $\boxed{Q = f(L, K)}$ is a relationship between the highest output Q achievable from using labor $= L$ and capital $= K$ as efficiently as possible with the given technology

no unnecessary waste.

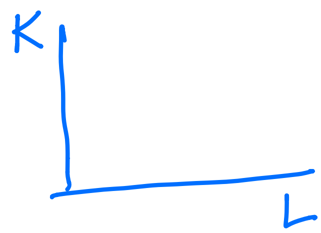


Example: $Q = f(L, K) = 10L^{0.5}K^{0.5}$,

 $L^{0.5} = 20$, $K^{0.5} = 30$

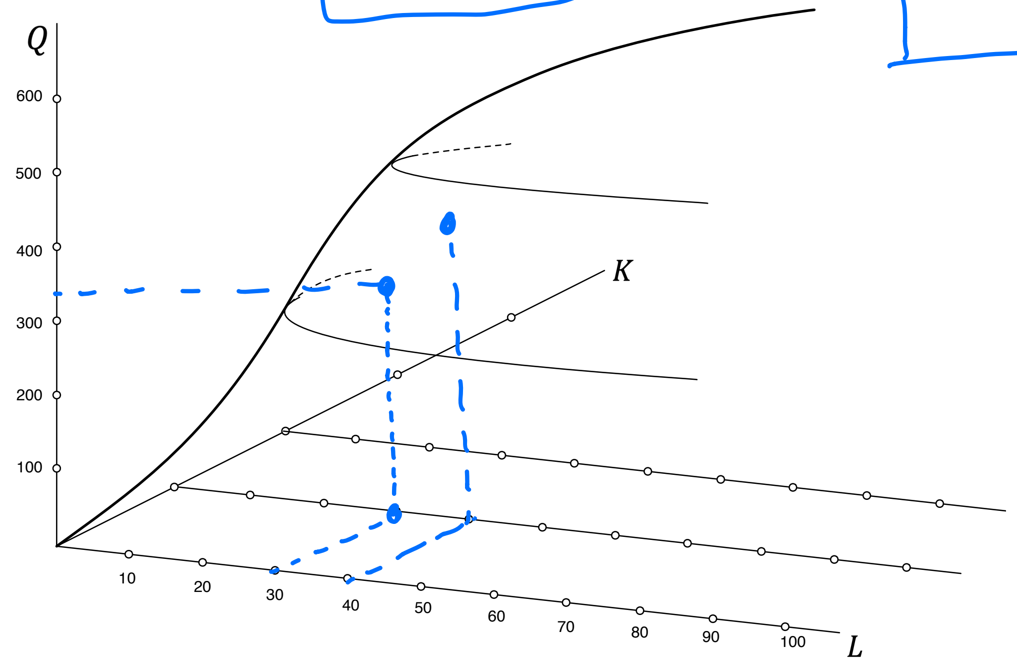
$$\left.\begin{array}{l} L = 400 \\ K = 900 \end{array}\right\} \Rightarrow Q = 6{,}000 \; units$$

$$\left.\begin{array}{l} L = 100 \\ K = 900 \end{array}\right\} \Rightarrow Q = 3{,}000 \; units$$



Graph of a Production Function in 3D

$\boxed{Q = f(L,K)}$



Eg.. Software Company.

L K.

Software Engineers. Laptop.

$Q = 100$	K_0.
$Q = 1000$	K_0.

Short-Run Production: Time frame that there is at least ***one input*** that cannot be changed (fixed factor)

- Usually in Short-Run, capital is assumed to be the fixed factor at $K = K_0$
- Fixed factor does not change with the quantity Q $\implies$ Fixed Cost.

Long-Run Production: Time frame that is long enough for the firm to vary every input
$\implies$ no fixed factor $\implies$ no fixed cost.

Total Product = TP(L)

Short-Run

<u>Short-Run Production</u> With production function

$$Q = f(L, K) = 10L^{0.5}K^{0.5}$$

Assume capital K is fixed at $\boxed{K_0 = 900}$, we have

$$Q = 10L^{0.5}\underbrace{K_0^{0.5}}_{=30}$$
$$= 300L^{0.5}$$

$Q = 300L^{0.5}$

Q	L.
3000	100
6,000	400
9,000	900
⋮	⋮

- we can change output Q by changing L in the short run
- This is called the Total Product is the output that can be produced at various labor level L, with capital fixed at K_0,

$$TP_{K_0}(L) = f(L, K_0)$$

$K_1 = 1600$..

$TP_{K_1}(L) = 10L^{0.5}\underbrace{(1600)^{0.5}}_{40}$
$= 400L^{0.5}$.

Q	L
400	100
800	400
1,200	900.

Total, Average, and Marginal

Given a Total, we can find Average and Marginal. This is the first T-A-M relation to be discussed in this class.

Given the Total Product $TP(L)$, (with subscript K_0 suppressed)

if L increases (decreases) by 1 unit of labor, output increases (decreases) by 20 unit

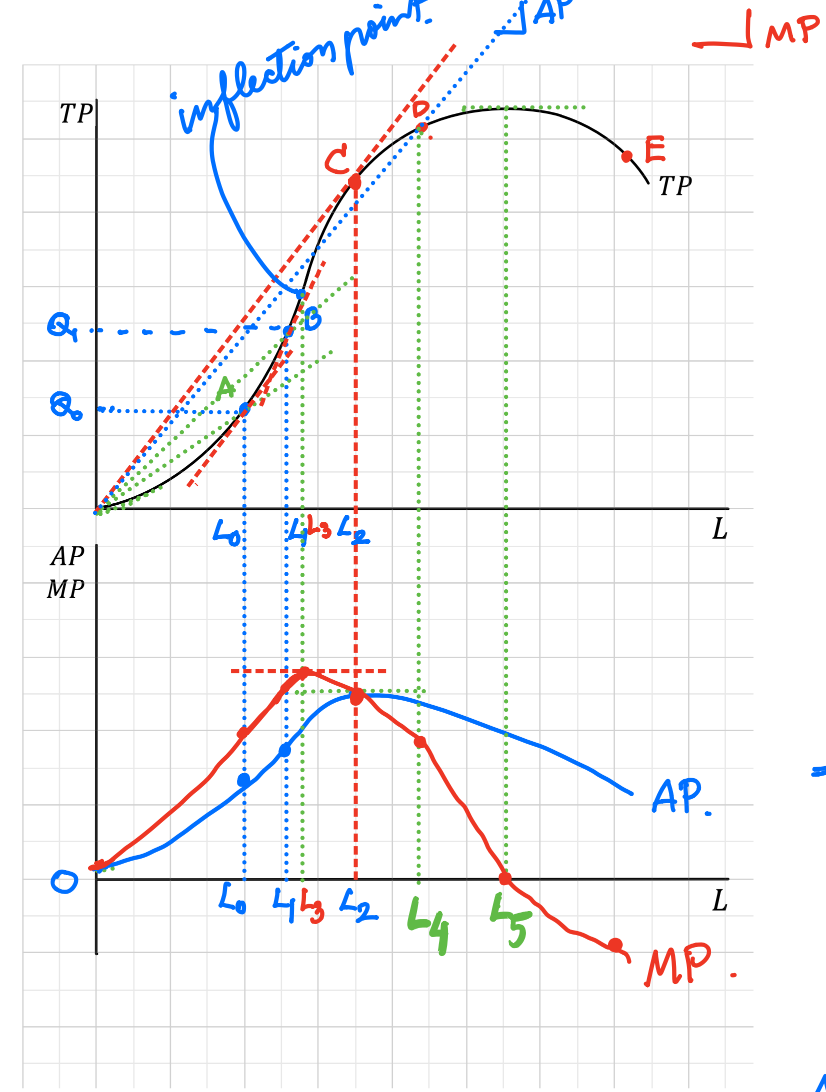
- Average Product $AP(L) = \frac{TP(L)}{L}$,
- Marginal Product $MP(L) = \boxed{\frac{dTP(L)}{dL}}$, = 20 unit of Q.

AP	TP	L.
$\frac{3000}{100} = 30$	3000	100
13.33	6,000	400
10.	9,000	900
	⋮	⋮

Marginal Product $MP(L)$ is the rate of change of the output $TP(L)$. That is, $MP(L)$ is the slope of $TP(L)$

The relationship of TP, AP and MP is demonstrated by the following graphs.

At D, AP > MP

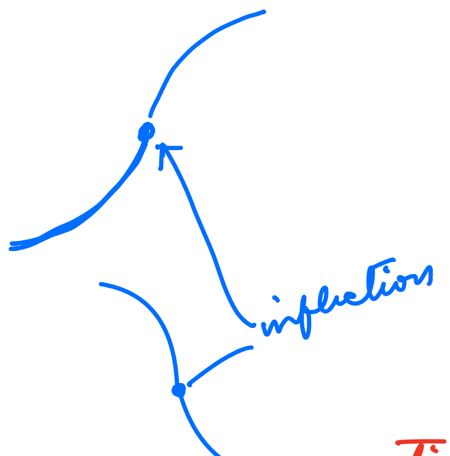


$AP(L) = \frac{TP(L)}{L}$

L	TP	AP
L_0	Q_0	Q_0/L_0
L_1	Q_1	Q_1/L_1

$AP \geq 0$ because $TP \geq 0$, $L \geq 0$.

$MP = \frac{d}{dL}(TP)$



MP can be negative.

AP - average product of all L

MP - output of the last unit of Labor.

Total	Average	Marginal
	A increasing ✓	M > A. ✓
	A decreasing ✓	M < A. ✓
	Max	M = A.
Max		M = 0
Inflection		Max.

TP = 10. from 5 workers

$AP = \frac{10}{5} = 2$

6^{th} worker contribute $\frac{1}{=}$ ~~3~~ units of output

$MP = 3 > AP = 2.$

• The relationship between Average and Marginal can also be verified by calculus. By definition, $AP = \frac{TP}{L}$

$$MP(L) = \frac{d}{dL}TP(L) = \frac{d}{dL}\left(AP(L)\cdot L\right)$$

$$= AP(L)\underbrace{\frac{dL}{dL}}_{=1} + L\frac{d\,AP(L)}{dL}$$

$$MP(L) = AP(L) + L\underbrace{\frac{d\,AP(L)}{dL}}_{\text{slope of } AP(L)} > 0$$

$= 0$