B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 1/2017

Practice Problem 3 (Differential Calculus)

1. Exercise 7.2 questions 1, 2, 4, 7, and 10 in Chiang and Wainwright (2005)

Ans. Q.1 $VC = Q^3 - 5Q^2 + 12Q;$ $\frac{dVC}{dQ} = MC = 3Q^2 - 10Q + 12$

Q.2 $C = AC \cdot Q = Q^3 - 4Q^2 + 174Q;$ $\frac{dC}{dQ} = MC = 3Q^2 - 8Q + 174$

This is a long-run cost function because the fixed cost is zero.

Q.4 (b) $= AR \cdot Q = 60Q - 3Q^2$; $MR = \frac{dR}{dQ} = 60 - 6Q$

(c) Yes.

(d) The MR curve is twice as steep as the AR curve.

Q.7 (a) $\frac{(x^2-3)}{x^2}$ (b) $-\frac{9}{x^2}$

(c) $\frac{30}{(x+5)^2}$ (d) $\frac{acx^2+2adx-bc}{(cx+d)^2}$

Q.10 (a) $MC = 6Q + 7$; $AC = 3Q + 7 + \frac{12}{Q}$

(b) $MR = 10 - 2Q$ $AR = 10 - Q$

(c) $MP = a + 2bL - cL^2$ $AP = a + bL - cL^2$

2. Find the marginal and the average functions for each of the following total functions.

a) $TC = 2Q^2 + 5Q + 11$

Ans. $MC = 4Q + 5$; $AC = 2Q + 5 + \frac{11}{Q}$.

b) $\pi = Q^2 - 10Q + 68$

Ans. $\frac{d\pi}{dQ} = 2Q - 10$; $\frac{\pi}{Q} = Q - 10 + \frac{68}{Q}$

3. Differentiate the following functions:

a) $y = \frac{x^2 - 9x + 20}{x - 5}$

Ans. $\frac{dy}{dx} = 1, x \neq 5$

b) $y = \sqrt{x + \sqrt{x}}$

Ans. $\frac{dy}{dx} = \frac{1}{2\sqrt{(x+\sqrt{x})}}\left(1 + \frac{1}{2\sqrt{x}}\right)$

c) $y = \ln(e^x + x)$

Ans. $\frac{dy}{dx} = \frac{e^x + 1}{e^x + x}$

d) $y = x^4 e^x$

Ans. $\frac{dy}{dx} = x^3 e^x (x + 4)$

4. Given the total-product function:

$$Q = 3L + 2L^2 - L^3$$

a) Find the average product (AP) function.

Ans. $AP(L) = 3 + 2L - L^2$

b) Find the marginal product (MP) function.

Ans. $MP(L) = 3 + 4L - 3L^2$

c) Determine the slopes of the AP and MP functions. What can you conclude about their relative slopes?

Ans. $\frac{d(AP)}{dL} = 2 - 2L$; $\frac{d(MP)}{dL} = 4 - 6L$

The MP curve is steeper than the AP curve.

5. A monopolist faces the demand curve given by $P = kQ^{-b}, \quad k, b > 0$. Answer the following questions.

a. Find the absolute value of elasticity of demand. Does the value decrease as the quantity of output increases? Interpret your result.

$|\varepsilon_d| = |\frac{dln(Q)}{dln(P)}| = \frac{1}{b}$. We have a constant elasticity of demand curve.

b. Find the expression for the revenue function

$R(Q) = P\,Q = kQ^{1-b}$

c. What is the average revenue function? Describe the property of average revenue as the quantity of output increases.

$AR(Q) = \frac{R(Q)}{Q} = kQ^{-b} = demand.$ ==> $Q\uparrow$ ==> $AR(Q)\downarrow$

d. Derive the marginal revenue function. How does the value of "b" determine the property of marginal revenue function?

$R'(Q) = k(1-b)Q^{-b}$. $sign(R'(Q))$ *depends on* $(1-b)$.

if $b = 1 \Rightarrow R'(Q) = 0$. That is, R(Q) doesn't change when Q changes.

if $0 < b < 1 \Rightarrow R'(Q) > 0$. That is R(Q) decreases when Q decreases.

if b > 1 $\Rightarrow$ $R'(Q) < 0$. That is R(Q) increases when Q decreases.

e. Based on the functional form of market demand equation, prove/disprove the following argument.The statement is *"Raising up the price always increases the revenue of firm, since an increase in price would always make-up with the loss in the quantity sold."* (Hint: your proof has to make uses some mathematical results regarding to the marginal revenue function.)

This is not necessarily true, so the statement is false. Here is why. First, raising up the price could be accomplished only when firm chooses to lower the production (Q drops). Second, a decrease in quantity would result in a higher revenue only if marginal revenue at that Q is negative. Third, based on "f", this could be the case only when "b" > 1, i.e. when market demand is inelastic. (Remember that the absolute value of elasticity of demand is 1/b, the inverse of "b". No, it's not "b" in our case. See the derivation above.) So, it depends on the property of market demand as summarized in "f".

6. Let the production function be $Q = -3(7-L)^5 + k$, where k is a very large value constant. Determine the level (interval) of L that the production function exhibits the law of diminishing returns.

First derivative = $15(7-L)^4$

Second derivative = $-60(7-L)^3$

The law of diminishing returns implies that second derivative is negative. That is, when $L < 7$.

7. Suppose that the consumption function can be given by, $C = 5\left(\frac{2\sqrt{Y^3}+3}{Y+10}\right)$, find the value of MPC and MPS when Y = 100

[MPC = 0.54, MPS = 1 - MPC = 0.46]

8. Suppose that total output of a firm can be given by $Q = 10\frac{L^2}{\sqrt{L^2+19}}$, where is L is the number of workers hired. Answer the following questions

a. Find the marginal product of labor (worker).

b. Suppose this firm can only charge for a constant fixed price equal to "P". Find the expression for the revenue function (TR) in term of L.

$$TR(L) = P * 10\frac{L^2}{\sqrt{L^2+19}} = 10P\frac{L^2}{\sqrt{L^2+19}}$$

c. Using the function derived above, find the marginal revenue product of labor or $\frac{dTR}{dL}$. (Hint: you may just treat "P" as a constant term in your equation.)

$$TR'(L) = 10P\frac{2L\sqrt{L^2+19} - L^2\{\frac{1}{2}(L^2+19)^{-\frac{1}{2}} * 2L\}}{L^2+19}$$

Continue with the same production function given above, but replace the assumption that the firm can only charge for a constant fixed price equal to "P" with that the firm can charge price based on the quantity of output that it sells. Assume the the pricing equation is given by $P = \frac{900}{Q+9}$. Consider the following questions.

d. Find the express of the revenue function (TR) in term of L.

$$TR(L) = P * Q = \frac{900}{Q+9} * 10\frac{L^2}{\sqrt{L^2+19}} = \frac{900}{10\frac{L^2}{\sqrt{L^2+19}}+9} * 10\frac{L^2}{\sqrt{L^2+19}}$$

e. Applying the chain rule and derive the expression of marginal revenue product of labor or $\frac{dTR}{dL}$. Then find the value of marginal revenue product of labor when L = 9.

$$TR(L) = P(Q(L)) * Q = P'(Q(L)) * Q'(L) * Q + P(Q(L)).$$

Then plug in the value of 9 into the above expression.

The answer would be 10.9.

9. Find the elasticity of consumption with respect to income when the consumption function is given by, $C = 10 + \frac{5}{8}Y - \frac{1}{2}\sqrt{Y}$ and Y = 16.

dc/dy = MPC = 9/16;

Y/C = 16/18;

Elasticity = ½

10. Suppose that market demand is $p = 300 - q^2$

a. Determine the point elasticity of demand when q = 5.

dp/dq = -10

p/q = 275/5 = 55

Elasticity = -5.5

b. For q = 5, is demand elastic, inelastic, or does it have unit elasticity?

Elastic demand (following the answer in a)

c. For what value of q does demand have unit elasticity?

q = 10

11. Suppose the average total cost function is $ATC = \frac{10}{q^2+2q} + \frac{q}{q+1} + \frac{1000}{q}$

a. What is the fixed cost?

$$TC = \frac{10}{q+2} + \frac{q^2}{q+1} + 1000$$

Fixed cost = 1005

b. What is the variable cost? What is the average variable cost?

$$VC = TC - FC = \frac{10}{q+2} + \frac{q^2}{q+1} + 1000 - 1005 \;=\; -5 + \frac{10}{q+2} + \frac{q^2}{q+1}$$

$$AVC = -5/q + \frac{10}{q^2+2q} + \frac{q^2}{q^2+q}$$

c. Find the marginal cost function.

$$MC = -\frac{10}{(q+2)^2} + \frac{q^2+2q}{(q+1)^2}$$

12. Given firm i, a perfect competition in a product market, with a short-run total cost function of $STC_i = Q_i^3 - 8Q_i^2 + 20Q_i + 40$ where Q_i is the output of firm i (in hundreds of units per day).

The market demand and supply schedules for the product are:

$Q^d = 600 - 20P$ and $Q^S = -150 + 30P$

where Q^d and Q^S are the quantities demanded and supplied in the market)in hundreds of thousands of units per day) and P is the market price (in dollars per unit). Consider the following problems.

a. What is the market equilibrium price?

Setting demand equals to supply, we yield that P*= 15.

b. Find the profit-maximizing level of output for firm i. Check the second order condition.

The profit function of competitive firm i can be given by;

$$\pi_i = 15Q_i - (Q_i^3 - 8Q_i^2 + 20Q_i + 40)$$

$$d\pi_i/dQ_i = marginal\ profit = 15 - 3Q_i^2 + 16Q_i - 20$$

$$\rightarrow Q_i^* = 1/3\ and\ 5$$

$$d^2\pi_i/dQ_i^2 = 16 - 6Q_i$$

$$Q_i^* = 5 \longrightarrow d^2\pi_i/dQ_i^2 = 16 - 30 = -14 < 0 \quad (concave\ function)$$

c. Calculate the level of profits (losses)

Total revenue is 15*5 = 75. Total cost is $(5)^3 - 8(5)^2 + 20(5) + 40 = 65$

Profit is then equal to 10

d. Given the level of economic profits, discuss the long-run adjustment in the market and for this firm. (Hint: your answer to the question could be only stated in a narrative way. You don't need solve out any numbers; provide economic intuition to support your claim.)

As the competitive firm earns positive profit in the short-run, it is likely that we have more new entrants in the long-run. Hence, the total supply might be increasing, and hence equilibrium price is gradually pushed down. In the long-run, we expect the following; (ii) an increase in the number of firms in the industry and (ii) a lower in price and hence market share that each operating firm will be acquiring.