GAME THEORY (REVIEW)

READING: : CHURCH & WARE (PDF) → GAME THEORY & APPLICATION
: CP, CHAPTER 6

REVIEW P - A - P - I → COMPONENTS OF A GAME

- PLAYERS: $N \geq 2$
- ACTIONS
- PAYOFFS: OUTCOMES RECEIVED BY PLAYERS
- INFORMATION
  - RULE OF THE GAME, TIMING OF EVENTS
  - RATIONALITY OF PLAYERS'
  - PAYOFFS < YOURSELF / COMPETITORS'
  - HISTORY OF THE MOVES

TYPES OF GAME

(I) SIMULTANEOUS MOVE GAME Vs SEQUENTIAL MOVE GAME

(II) ONE-SHOT GAME (STATIC GAME) Vs REPEATED GAME (DYNAMIC GAME)
- FINITELY REPEATED GAME
- INFINITELY REPEATED GAME

(III) NON-COOPERATIVE GAME Vs COOPERATIVE GAME

NON-COOPERATIVE GAME: PLAYERS DO NOT COOPERATE DURING THE COURSE OF PLAY

COOPERATIVE GAME: PLAYERS DO COOPERATE
EX: WHEN POLITICAL PARTIES TRY TO FORM COALITION GOVT.

(IV) GAME OF COMPLETE INFORMATION Vs. GAME OF INCOMPLETE INFORMATION

GAME OF COMPLETE INFORMATION: ALL PLAYERS KNOW
- HIS/HER ACTION SET
- HIS/HER PAYOFFS
- OPPONENTS' ACTION SET
- OPPONENTS' PAYOFFS

GAME OF INCOMPLETE INFORMATION: PLAYERS ONLY KNOW HIS/HER OWN ACTION SET AND PAYOFFS.

EQUILIBRIUM CONCEPTS

- DOMINANT STRATEGY EQUILIBRIUM (DSE): YOU ARE DOING THE BEST YOU CAN REGARDLESS OF WHAT I'M DOING
  AND
  I'M DOING THE BEST I CAN REGARDLESS OF WHAT YOU'RE DOING

- NASH EQUILIBRIUM: YOU ARE DOING THE BEST YOU CAN GIVEN WHAT I'M DOING
  AND
  I'M DOING THE BEST I CAN GIVEN WHAT YOU'RE DOING

REPRESENTATION OF GAMES

(1) NORMAL FORM (OR PAYOFF MATRIX FORM)

CONSIDER 2 PLAYERS: 1 AND 2
EACH PLAYER HAS 2 ACTIONS: PLAYER 1 CHOOSES BET. UP AND DOWN
" " 2 " " LEFT AND RIGHT

PLAYER 1 (ROW PLAYER) \ PLAYER 2 (COLUMN PLAYER)	LEFT (L)	RIGHT (R)
UP (U)	(3, 9)	(1, 8)

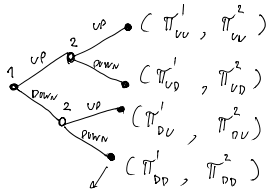
- PLAYER 1'S PAYOFF IS THE FIRST NUMBER IN EACH BOX.
- PLAYER 2'S PAYOFF IS

PLAYER 1 (OR ROW PLAYER)		UP (U)	DOWN (D)
	UP (U)	(3, 9)	(1, 8)
	DOWN (D)	(0, 0)	(0, 1)

- PLAYER 1'S PAYOFF IS THE FRONT NUMBER IN EACH BOX.
- PLAYER 2'S PAYOFF IS THE SECOND NUMBER IN EACH BOX.

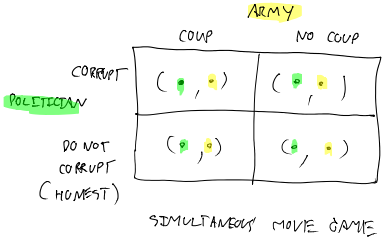
② EXTENSIVE FORM (OR GAME TREE)

CONSIDER 2 PLAYERS': ① AND ②
SUPPOSE TWO PLAYERS' ACTION SET = {UP, DOWN}
SUPPOSE FURTHER THAT PLAYER 1 MOVES FIRST AND PLAYER 2 MOVES SECOND.

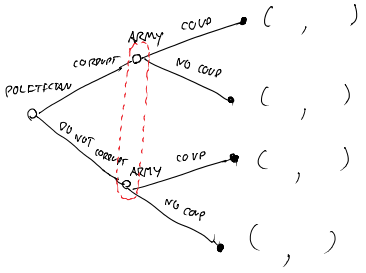


- TERMINAL NODES
- DECISION NODES

NOTES: WE CAN ALSO USE "GAME TREE" TO REPRESENT A SIMULTANEOUS MOVE GAME. LET'S SEE ... π^1 π^2



SIMULTANEOUS MOVE GAME



SOLVING GAMES

ADVERTISING GAME FORM B

FIRM A \ FIRM B	ADVERTISE	DO NOT ADVERTISE
ADVERTISE	10, 5	15, 0
DO NOT ADVERTISE	6, 8	10, 2

- SIMULTANEOUS MOVE GAME
- PROFIT: MILLION BAHT.

LET'S SOLVE THE GAME ...

- WE GOT THE SOLUTION BY USING "ELIMINATION OF (WEAKLY) DOMINATED STRATEGIES"
- FIRM A AND FIRM B, EACH HAS A DOMINANT STRATEGY: ADV.

A DOMINANT STRATEGY: THE STRATEGY THAT IS OPTIMAL, NO MATTER WHAT AN OPPONENT DOES.

- IN EQUILIBRIUM, WHEN A PLAYER HAS DOMINANT STRATEGY, HE/SHE WILL ALWAYS USE IT.

EXAMPLE 2

	ADV	NO ADV
ADV		

WILL ALWAYS USE IT.

EXAMPLE 2

1 \ 2	ADV	NO ADV
ADV	10, 5	15, 0
NO ADV	6, 8	20, 2

NASH EQUILIBRIUM

$(s^{*1}, s^{*2}) = (\text{ADV}, \text{ADV})$

$(\pi^1, \pi^2) = (10, 5)$

THE PRISONERS' DILEMMA

TWO SUSPECTS ARE CAUGHT AND PUT INTO SEPARATE ROOMS FOR INTERROGATION. THE POLICE HAD A HARD EVIDENCE TO CONVICT THEM FOR A MINOR OFFENCE (EX: STOLEN GOODS) IN WHICH THE PENALTY IS 1 YR IN JAIL.

EACH SUSPECT IS TOLD BY THE POLICE THAT

- IF HE CONFESSES, BUT ANOTHER DOES NOT, HE WILL BE "FREE" AND THE OTHER WILL BE SENTENCED FOR 20 YRS, VICE VERSA.
- IF BOTH CONFESS, THE WILL BE SENTENCED FOR 5 YRS.

A \ B	CONFESS	DO NOT CONFESS
CONFESS	(-5, -5)	(0, -20)
DO NOT CONFESS	(-20, 0)	(-1, -1)

CONSIDER A:

IF B CONFESSES, A'S BEST RESPONSE IS "CONFESS" $(-5 > -20)$

FORMALLY, $\pi^A_{\text{CONFESS, CONFESS}} > \pi^A_{\text{DO NOT, CONFESS}}$.

$(-5) \qquad (-20)$

IF B DOES NOT CONFESS, A'S BEST RESPONSE IS "CONFESS"

SINCE $\pi^A_{\text{CONFESS, DO NOT}} > \pi^A_{\text{DO NOT, DO NOT}}$

$(0) \qquad (-1)$

NEXT, CONSIDER B:

- IF A CONFESSES, B'S BEST RESPONSE IS CONFESS

SINCE $\pi^B_{\text{CONFESS, CONFESS}} > \pi^B_{\text{DO NOT, CONFESS}}$ $(-5) > (-20)$

- IF A DOES NOT CONFESS, B'S BEST RESPONSE IS CONFESS

SINCE $\pi^B_{\text{CONFESS, DO NOT}} > \pi^B_{\text{DO NOT, DO NOT}}$

$(0) > (-1)$

SO, N.E. IS $(s^{A*}, s^{B*}) = (\text{CONFESS}, \text{CONFESS})$

$(\pi^A, \pi^B) = (-5, -5)$

EX: MORAL HAZARD IN TEAMS

TWO WORKERS $i = 1, 2$, EACH CAN "WORK" $(s_i = 1)$ OR "SHIRK" $(s_i = 0)$.

TOTAL TEAM OUTPUT $Q = 4(s_1 + s_2)$ IS SHARED EQUALLY BETWEEN THE TWO WORKERS. EACH WORKER INCURS A PRIVATE COST OF 3 WHEN WORKING AND 0 WHEN SHIRKING.

WORKER 1 \ WORKER 2	WORK	SHIRK
WORK	$(1, 1)$	$(-1, 2)$
SHIRK	$(2, -1)$	$(0, 0)$

BATTLE OF SEXES (BoS)

IN PD GAME, THE MAIN ISSUE IS WHETHER OR NOT PLAYERS WILL COOPERATE. HOWEVER, IN BoS, PLAYERS AGREE THAT IT IS BETTER FOR THEM TO COOPERATE THAN NOT TO COOPERATE, BUT THEY MIGHT DISAGREE ABOUT THE BEST OUTCOME.

HUSBAND \ WIFE	BOXING	OPERA
BOXING	$(2, 1)$	$(0, 0)$
OPERA	$(0, 0)$	$(1, 2)$

HERE, BOTH (BOXING, BOXING) AND (OPERA, OPERA) ARE N.E.

MUTUALLY CORRECT EXPECTATIONS COME INTO PLAY ...

LET'S SEE THIS GAME

H \ W	B	O
B	$(2, 2)$	$(0, 0)$
O	$(0, 0)$	$(1, 1)$