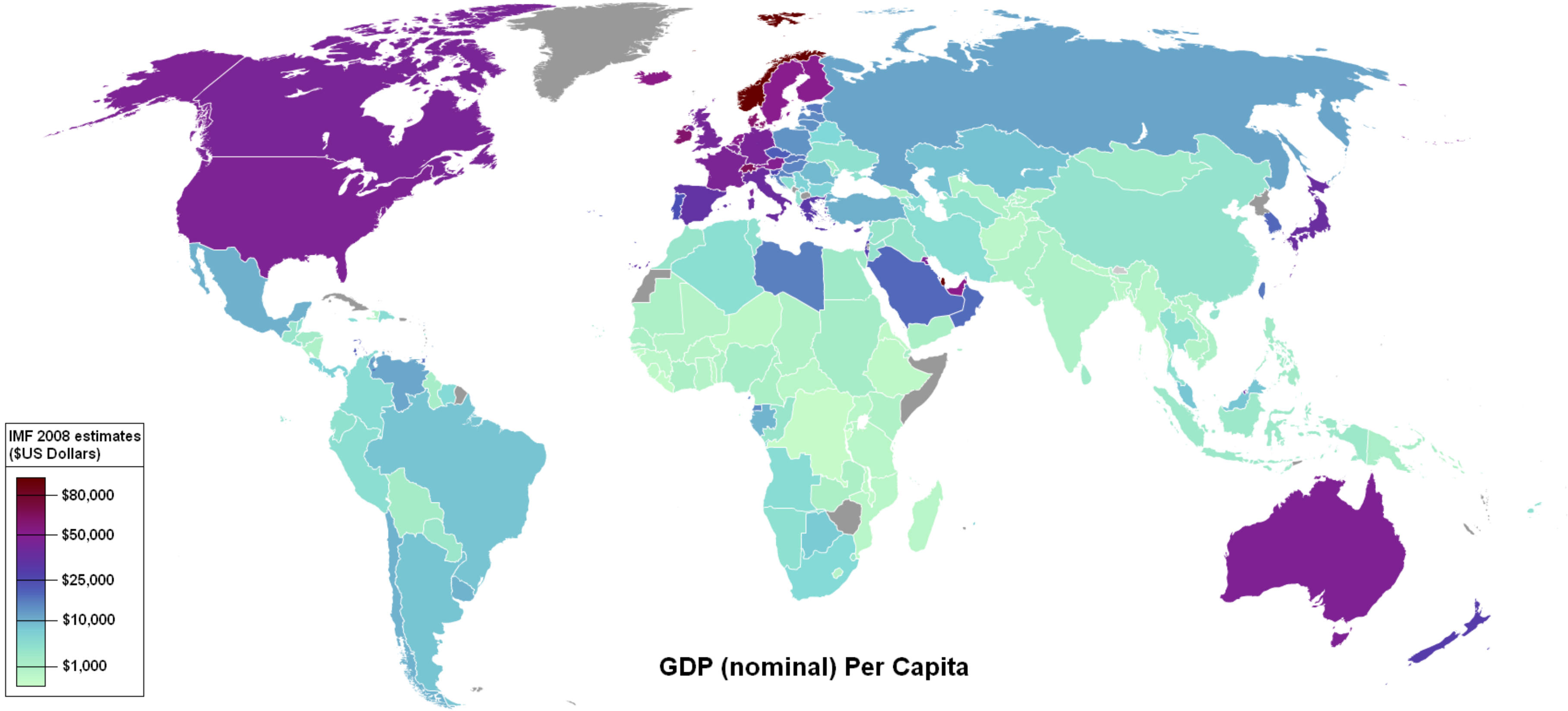
EE462 Development Macroeconomics

Lecture 2
Basic Facts
A Brief History of Modern Growth theory
The Solow Growth Model

Facts about Economic Growth



1. There is enormous variation in per capita income across countries.

Map of countries by 2008 GDP (nominal) per capita, based on IMF estimates

2. Rates of economic growth vary substantially across countries.

- Growth miracles – Japan, NICs (i.e., South Korea, Taiwan, Singapore, and Hong Kong)
- Growth disaster – Argentina, countries in sub-Saharan Africa

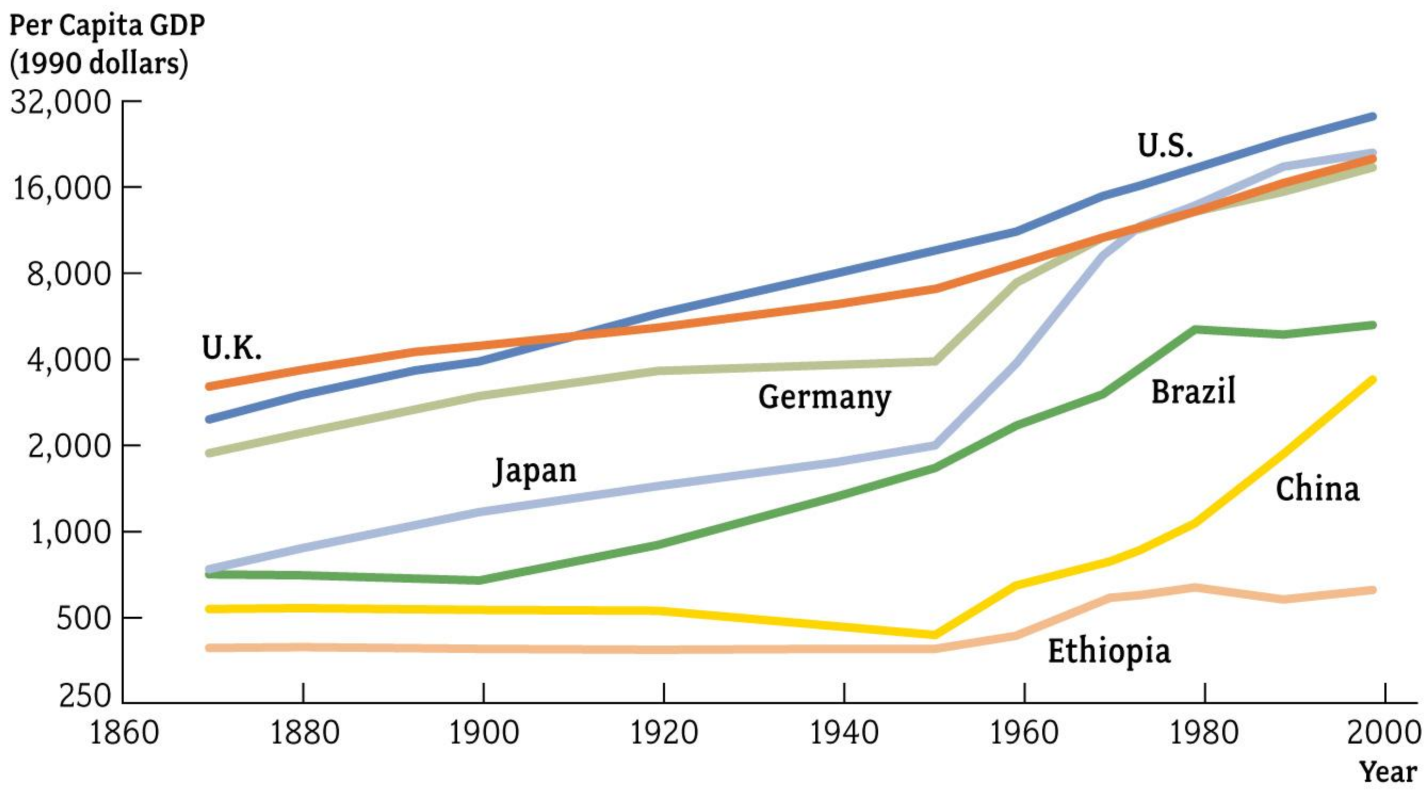
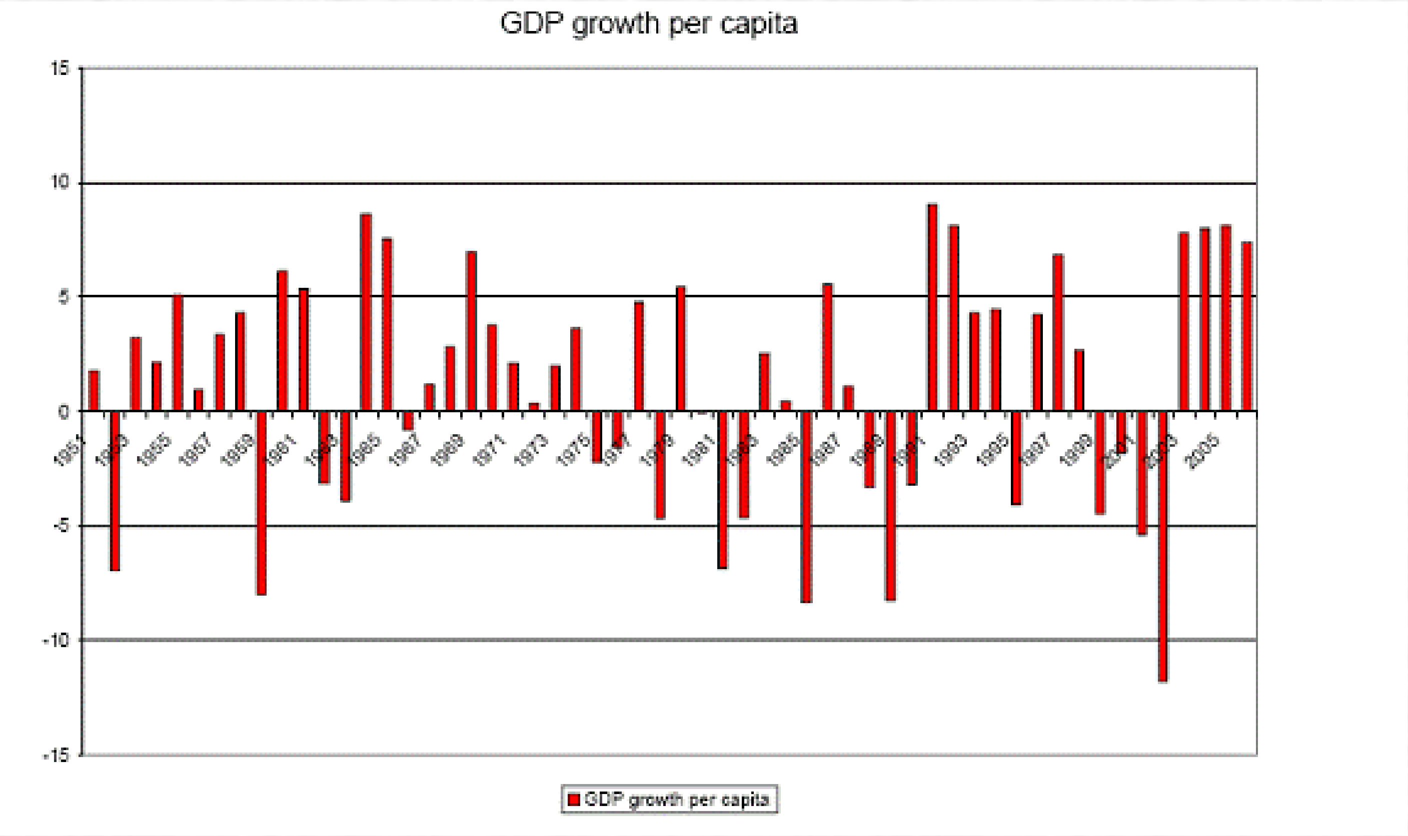


FIGURE 1.1 Per Capita GDP in Seven Countries, 1870–2000

Macroeconomics, Charles I. Jones

Copyright © 2008 W. W. Norton & Company



Argentina per capita GDP growth rate, 1951-2008

3. Growth rate are not generally constant over time.

- For the world as a whole, growth rates were close to zero over most of history but have increased sharply in the twentieth century.
- For individual countries, growth rates also change over time.

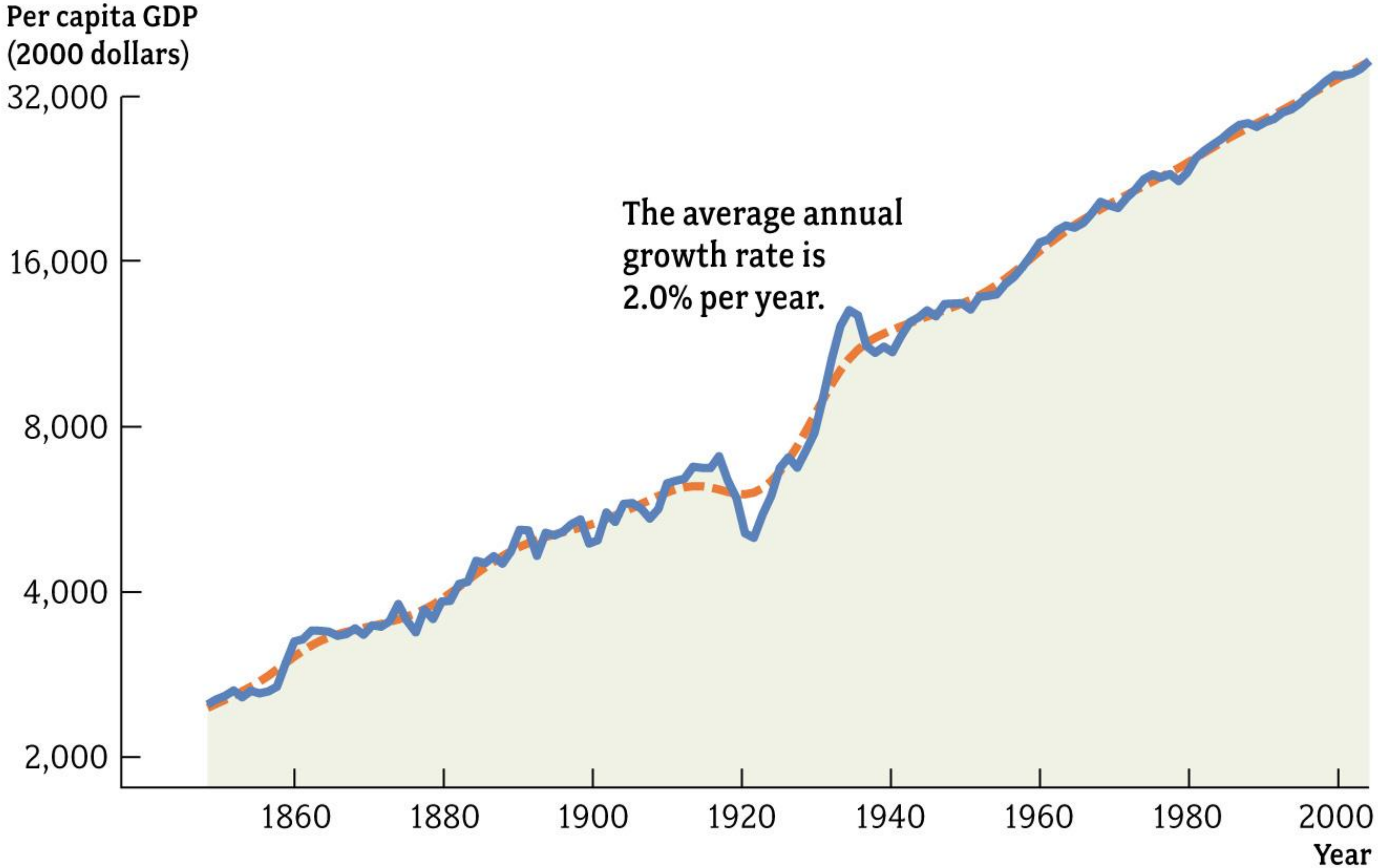
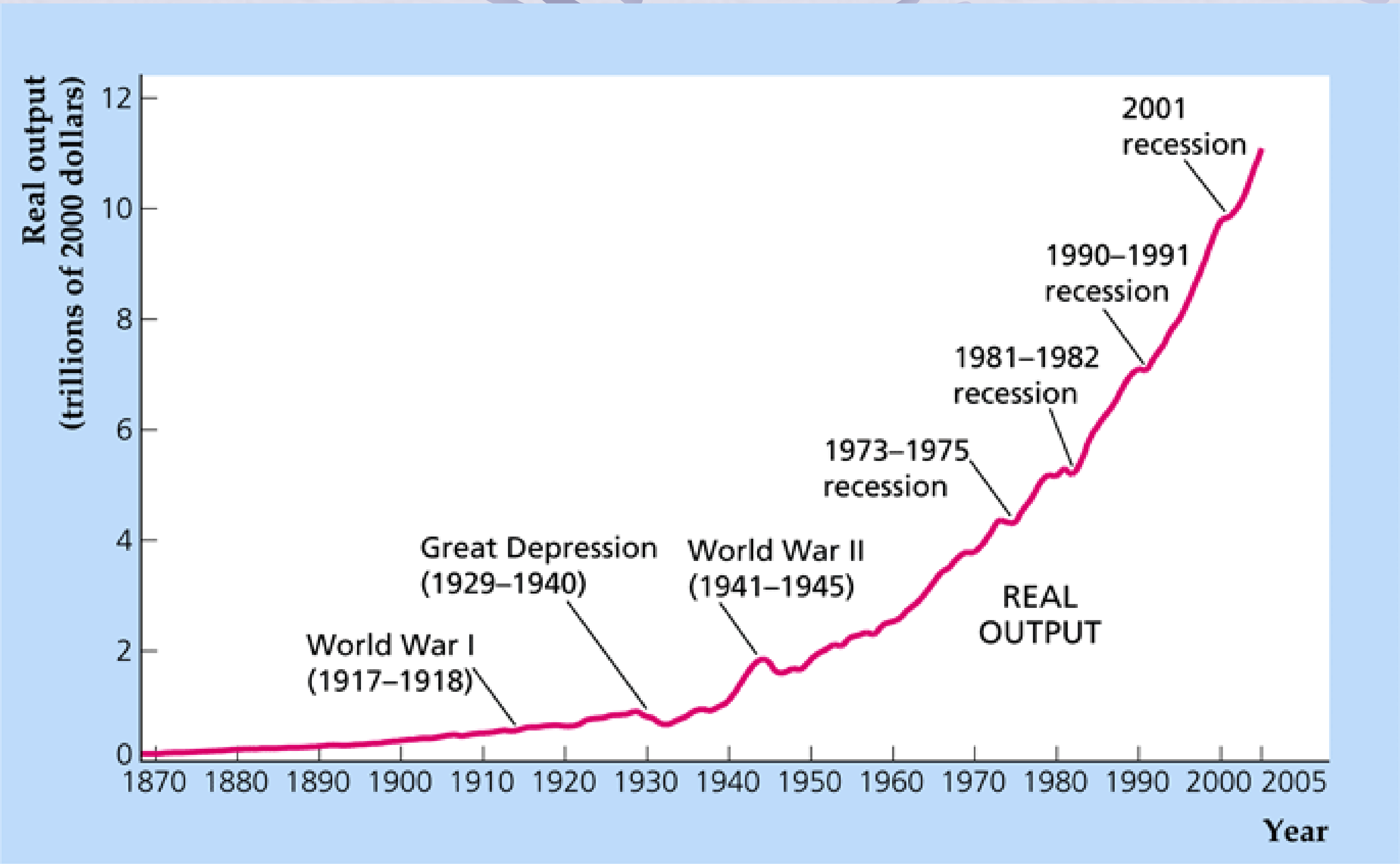


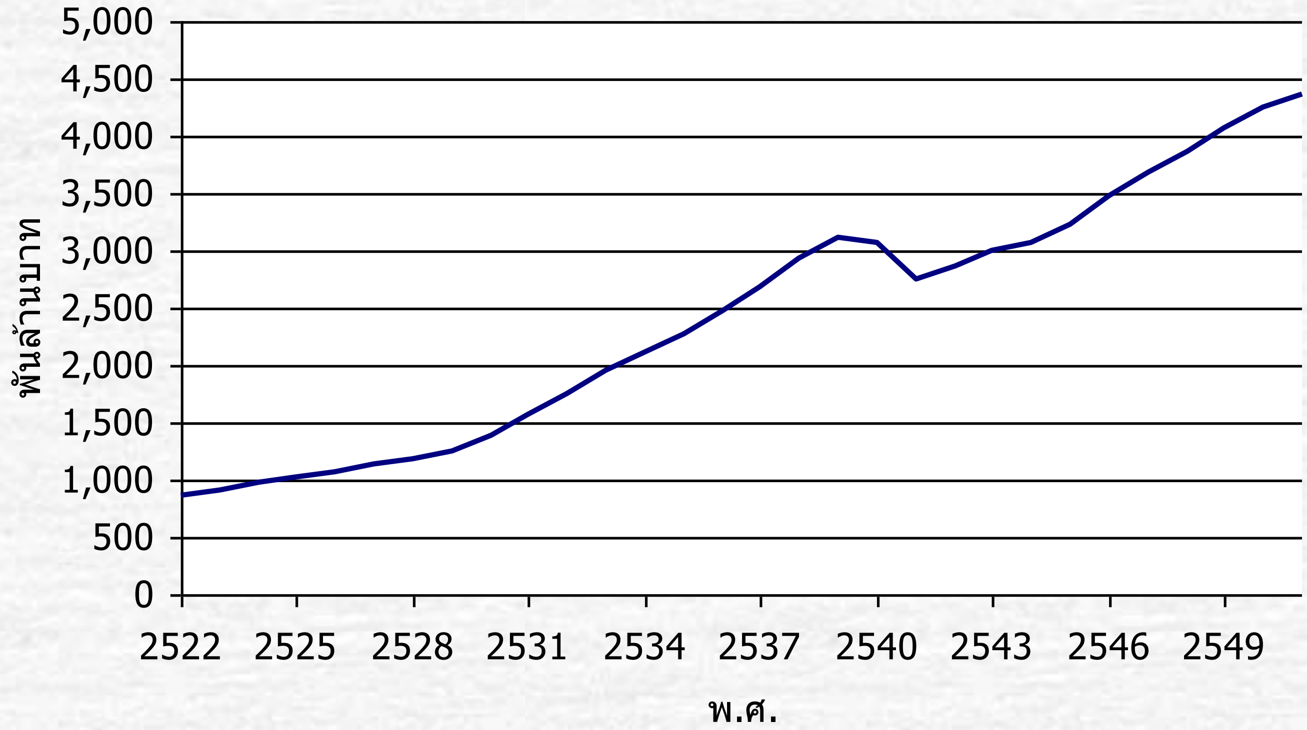
FIGURE 1.6 Per Capita GDP in the United States, 1870–2004

Macroeconomics, Charles I. Jones

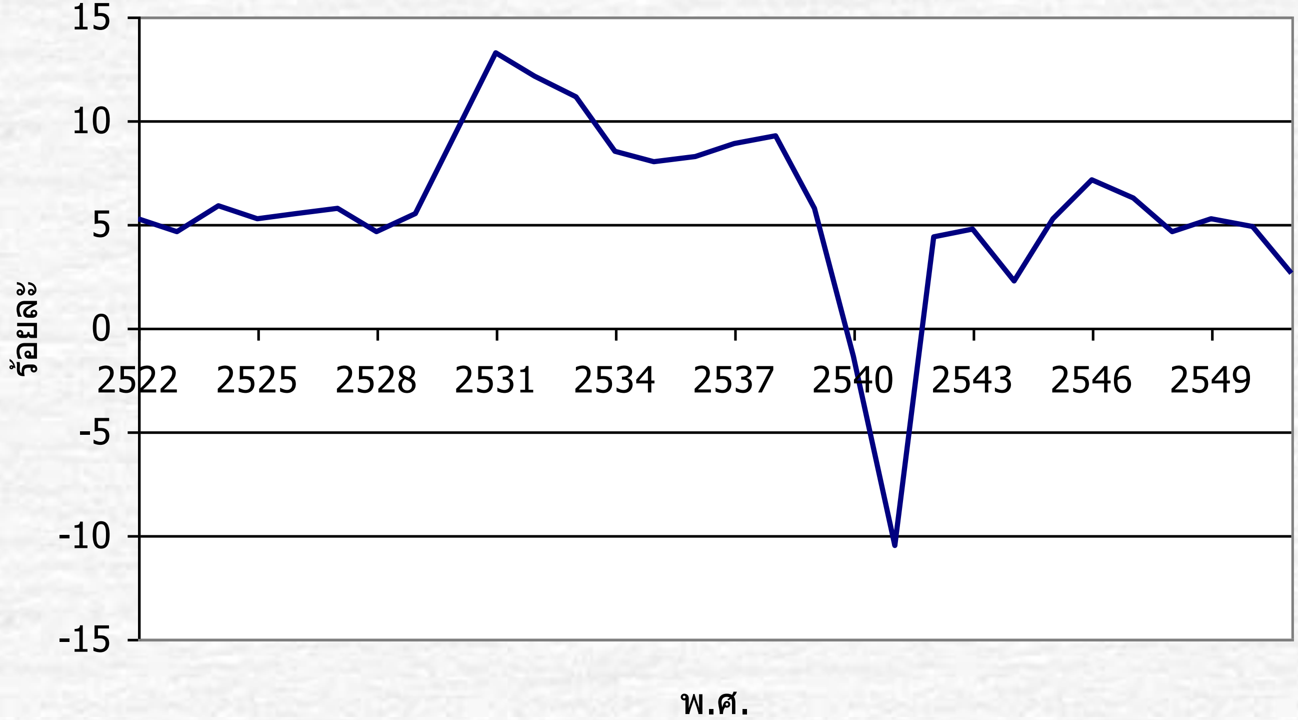
Copyright © 2008 W. W. Norton & Company



Output of the U.S. economy (2000=100), 1869–2005



Real GDP in Thailand (2531=100), 2522-2551



Real GDP Growth Rates in Thailand, 2522-2551

4. A country's relative position in the world distribution of per capita income is not immutable.

- Countries can move from being poor to being rich, and vice versa.

	Levels of real GDP per capita				
Country	1870	1913	1950	2005	Annual growth rate 1870–2005
Australia	3,645	5,715	7,493	23,868	1.4%
Canada	1,695	4,447	7,437	24,200	2.0
France	1,876	3,485	5,270	21,662	1.8
Germany	1,821	3,648	3,881	19,325	1.8
Japan	737	1,385	1,926	21,610	2.5
Sweden	1,664	3,096	6,738	22,310	1.9
United Kingdom	3,191	4,921	6,907	21,981	1.4
United States	2,445	5,301	9,561	31,242	1.9

Note: Figures are in U.S. dollars at 1990 prices, adjusted for differences in the purchasing power of the various national currencies.
Source: Data for 1870, 1913, and 1950 from Angus Maddison, *The World Economy: A Millennial Perspective*, Paris: OECD, 2001; data for 2005 from Bureau of Labor Statistics, *www.bls.gov/fls*, rescaled to 1990 prices.

Economic growth in eight major countries, 1870–2005

- What is the engine of growth?
- “Once one starts to think about [economic growth], it is hard to think about anything else.” Robert Lucas (1988)

A Brief History of Modern Growth Theory

- Classical economists, such as Adam Smith (1776), David Ricardo (1817), Thomas Malthus (1798), Frank Ramsey (1928), Allyn Young (1928), Frank Knight (1944), and Joseph Schumpeter (1934), provided many basic ingredients for modern economic growth theories.
- These classical assumptions include competitive equilibrium, constant returns to scale, and diminishing marginal product.

- Starting point of modern growth theory is Ramsey (1928) – household optimization over time
- The economic profession did not accept Ramsey's approach until the 1960s.
- Harrod (1939) and Domar (1946) integrated Keynesian analysis with elements of economic growth.
- They used production function with little substitutability among inputs to argue that capitalist system is unstable.

- Solow (1956) and Swan (1956) model is the production function that assumes constant returns to scale and diminishing returns to each inputs.
- The model also assumes constant saving rate.
- Solow and Swan model is called neoclassical growth model.

- Some predictions from Solow and Swan model:

  (1) The lower the staring level of real per capita GDP, relative to the long-run or steady-state position, the faster is the growth rate. Catch-up effect

  (2) In the absence of continuing improvements in technology, per capita growth must eventually cease.

- In the late 1950s and 1960s, technological progress was included in neoclassical growth model exogenously.
- The long-run per capita growth rate is determined outside the model.
- Cass (1965) and Koopman (1965) brought Ramsey's analysis of consumer optimization back into the neoclassical growth model and provided for an endogenous determination of the saving rate.

- The endogeneity of saving rate does not eliminate the dependence of the long-run per capita growth rate on exogenous technological progress.
- The inclusion of a theory of technological change in the neoclassical framework is difficult because the standard competitive assumption cannot be maintained.
- New ideas → Technological progress → Increasing returns to scale → violates assumption of perfect competition

- Arrow (1962) and Sheshinski (1967) constructed models in which ideas were unintended by-product of production or investment.
- Then, each person’s discoveries spill over to the entire economy.
- Romer (1986) showed that the competitive framework can be retained in the model with technological progress but Pareto optimality does not exist.

- The work of Cass (1965) and Koopman (1965) made growth theory become excessively technical and lost contact with empirical application.
- In contrast, development economists tended to use less sophisticated model but empirically useful.
- The field of economic development and economic growth drifted apart.
- By the early 1970s, growth theory died as an active research field because of the rational expectations revolution and oil shocks.

- For about 15 years, macroeconomic research focused on short-run fluctuations.
- Since the mid-1980s, research on economic growth has experienced a new born, beginning with the work of Romer (1986) and Lucas (1988).
- The recent contributions determine the long-run growth rate within the model.
- These models are endogenous growth models or new growth models.

- In these models, growth may go on indefinitely because the returns to investment do not necessary diminish as economies develop.
- Romer (1987, 1990) incorporated R&D theories and imperfect competition into the growth framework.
- The works of Aghion and Howitt (1992) and Grossman and Helpman (1991) also contributed significantly.
- In these models, technological advance results from R&D activity which is the rewards from monopoly power.

- The new research also includes models of diffusion of technology.
- Recent research makes population growth endogenous by incorporating an analysis of fertility choice in the neoclassical model.
- The distinction between the growth theory of the 1960s and that of the 1980s and 1990s is that the recent research pays attention to empirical implications and to the relation between theory and data.

- The recent theories of endogenous growth study the roles of increasing returns, R&D activity, human capital, and the diffusion of technology. (Barro และ Sala-i-Martin, 2004)

The Solow Growth Model

- The Solow model – sometimes known as the Solow-Swan model – was developed by Robert M. Solow (1956) and T.W. Swan (1956).
- The Solow model concluded that the accumulation of physical capital cannot account for either the vast growth over time in output per person (per capita output) or the vast geographic differences in output per person.

Assumptions

- Assumptions about inputs and outputs
- Assumptions concerning the production function
- Assumptions about the evolution of the inputs into production

Assumptions about Inputs and Outputs

- The model focuses on 4 variables: output (Y), capital (K), labor (L), and knowledge or effectiveness of labor (A).
- The production function takes the form:

$$Y(t) = F(K(t), A(t)L(t))$$

- Two features of the production function:

  (1) Time (t) does not enter the production function directly, but through K, L, and A. That is, output changes over time only if the inputs change.

  (2) A and L enter multiplicatively. AL is "effective labor". The way technological progress enters in this fashion is known as labor-augmenting or Harrod-neutral.
- If the production function takes the form $Y=F(AK,L)$, it is known as capital-augmenting.
- If the production function takes the form $Y=AF(K,L)$, it is known as Hicks-neutral.

Assumptions Concerning the Production Function

- The production function has constant returns to scale in its two arguments, capital and effective labor.

$$F(cK, cAL) = cF(K, AL) \qquad \text{for all } c \geq 0$$

- Set c=1/AL, the production function takes the intensive form:

$$F\left(\frac{K}{AL}, 1\right) = \frac{1}{AL} F(K, AL) = \frac{Y}{AL}$$

- Define

$$y=Y/AL$$
$$k=K/AL$$
$$f(k)=F(k,1)$$

- The production function in intensive form is

$$y=f(k)$$

- $f(0)=0$
- $f'(k)>0$
- $f'(k)$ is marginal product of capital (MPK).

- $f''(k)<0$

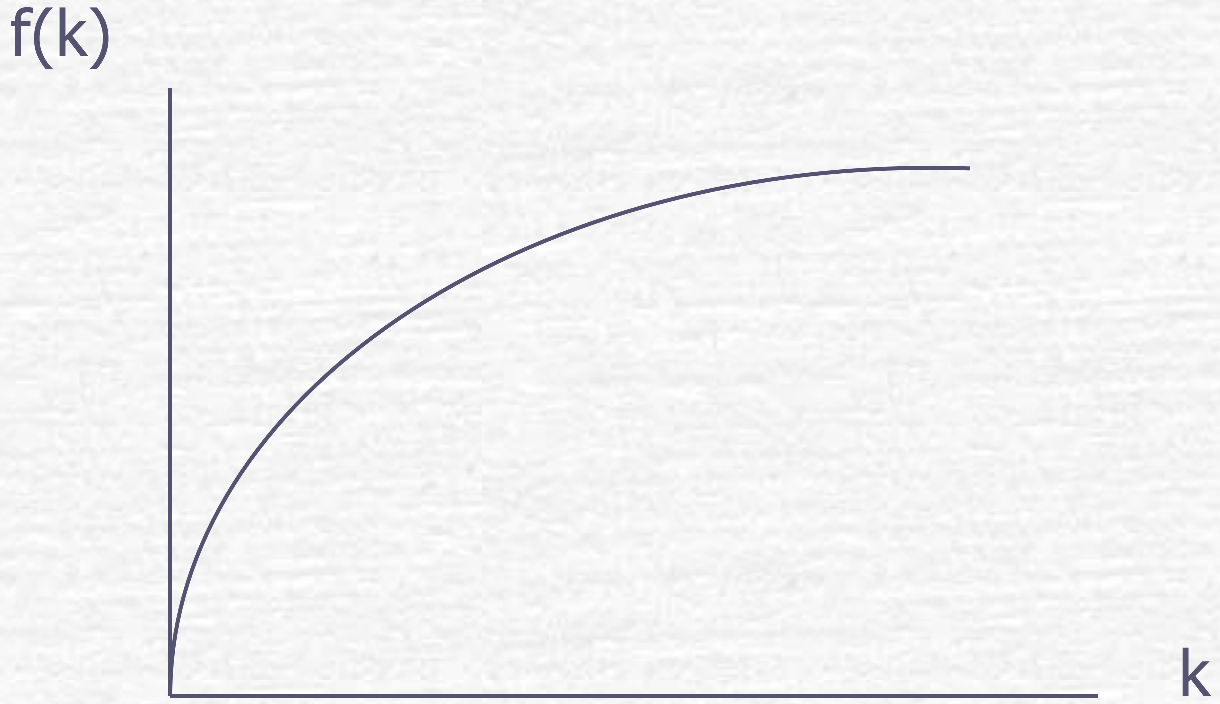
  $f''(k)$ is the slope of MPK curve. It means that MPK declines as k rises.

- $f(\bullet)$ satisfies Inada condition.

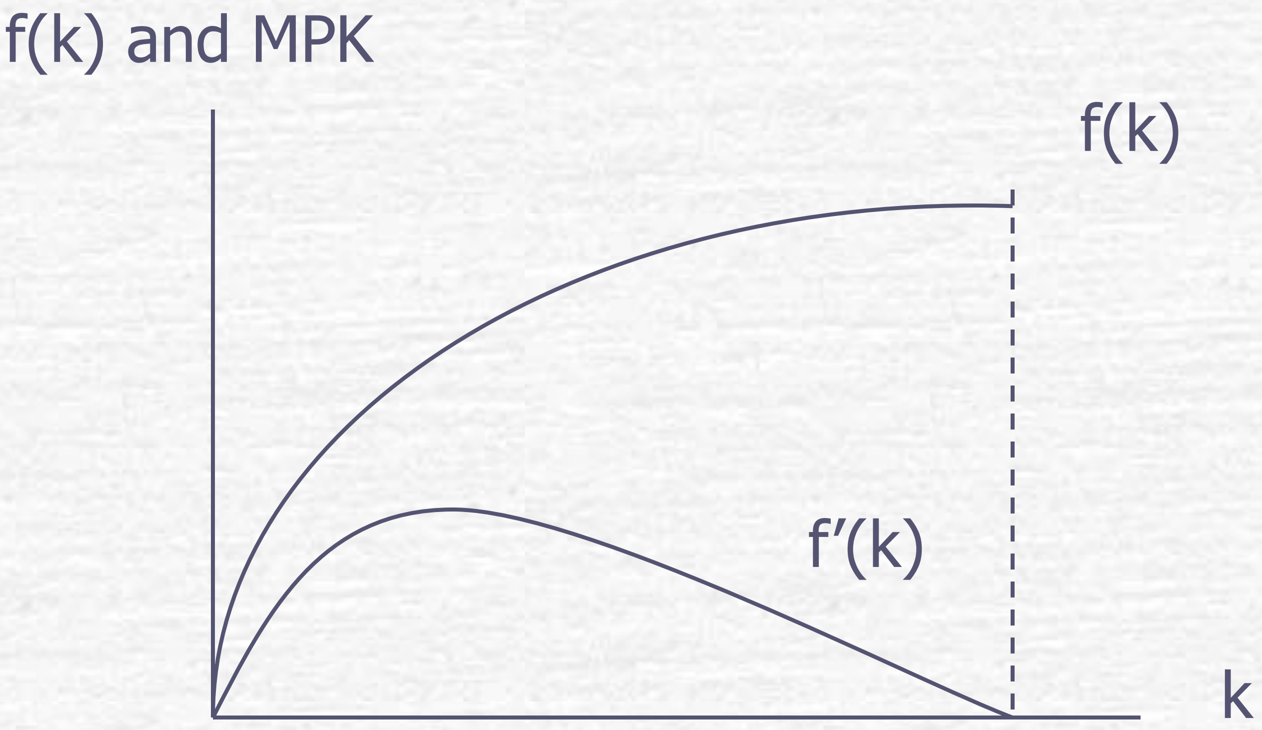
$$\lim_{k\to 0} f'(k) = \infty$$

$$\lim_{k\to \infty} f'(k) = 0$$

- MPK is very large when k is sufficiently small.
- MPK becomes very small when k becomes large.



Production Function



Production Function and MPK

- A specific example of a production is the Cobb-Douglas.

$$F(K, AL) = K^{\alpha}(AL)^{1-\alpha},\ 0 < \alpha < 1$$

- It has constant returns to scale.

$$\begin{aligned} F(cK, cAL) &= (cK)^{\alpha}(cAL)^{1-\alpha} \\ &= c^{\alpha}K^{\alpha}c^{1-\alpha}(AL)^{1-\alpha} \\ &= cK^{\alpha}(AL)^{1-\alpha} \\ &= cF(K, AL) \end{aligned}$$

- The intensive form of the Cobb-Douglas production function is

$$f(k) = k^{\alpha}$$

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

$$\frac{Y}{AL} = K^{\alpha}\frac{(AL)^{1-\alpha}}{AL}$$

$$\frac{Y}{AL} = K^{\alpha}(AL)^{1-\alpha-1}$$

$$\frac{Y}{AL} = K^{\alpha}(AL)^{-\alpha}$$

$$\frac{Y}{AL} = \frac{K^{\alpha}}{(AL)^{\alpha}} = \left(\frac{K}{AL}\right)^{\alpha}$$

$$y = k^{\alpha}$$

- When $y=f(k)=k^{\alpha}$, it implies that

$$f(k) = k^{\alpha}$$

$$f'(k) = \alpha k^{\alpha-1} = \alpha \frac{1}{k^{1-\alpha}} > 0, \quad 0 < \alpha < 1$$

$$\lim_{k\to 0} f'(k) = \infty$$

$$\lim_{k\to \infty} f'(k) = 0$$

$$f''(k) = -(1-\alpha)\alpha k^{\alpha-2} < 0$$

- That is MPK is positive and it declines when k rises.

The Evolution of Inputs into Production

- Let

$$\dot{L}(t) = nL(t)$$

$$\dot{A}(t) = gA(t)$$

$$\dot{K}(t) = sY(t) - \delta K(t)$$

- A dot over a variable denotes a derivative with respect to time.

$$\dot{X}(t) = \frac{dX(t)}{dt}$$

- n denotes the growth rate of labor.
- g denotes the growth rate of knowledge.
- n and g are exogenous parameters. Their values are determines outside the model.
- s denotes the fraction of output devoted to investment. It is exogenous and constant.
- δ denotes capital depreciation rate.
- Assume $(n+g+\delta)>0$.

Other Assumptions of the Solow Model

- There is only a single good.
- Government is absent.
- Fluctuations in employment are ignored.
- Production is described by an aggregate production function with just three inputs.
- The rates of saving, depreciation, population growth, and technological progress are constant.

The Dynamics of the Model

- Since two of the three inputs, labor and knowledge, are exogenous. The behavior of the economy is determined by the behavior of the third input, capital.
- This topic is about:

  (1) the dynamic of capital per effective labor (k) and

  (2) the balanced growth path

The Dynamic of Capital Per Effective Labor

$$k(t) = \frac{K(t)}{A(t)L(t)}$$

$$\ln k(t) = \ln K(t) - \ln A(t) - \ln L(t)$$

$$\frac{\overset{\circ}{k}(t)}{k(t)} = \frac{\overset{\circ}{K}(t)}{K(t)} - \frac{\overset{\circ}{A}(t)}{A(t)} - \frac{\overset{\circ}{L}(t)}{L(t)}$$

$$\frac{\overset{\circ}{k}(t)}{k(t)} = \frac{sY(t) - \delta K(t)}{K(t)} - g - n$$

$$\frac{\overset{\circ}{k}(t)}{k(t)} = \frac{sY(t)}{K(t)}\frac{A(t)L(t)}{A(t)L(t)} - \delta - g - n$$

$$\frac{\overset{\circ}{k}(t)}{k(t)} = s\frac{y(t)}{k(t)} - (n + g + \delta)$$

$$\overset{\circ}{k}(t) = sy(t) - (n + g + \delta)k(t)$$

$$\overset{\circ}{k}(t) = sf(k(t)) - (n + g + \delta)k(t)$$

- The key equation of the Solow model states that the change of capital stock per unit of effective labor depends on two terms:

  (1) sf(k) is savings which is equal actual investment per unit of effective labor.

  (2) $(n+g+\delta)k$ is break-even investment, the amount of investment that must be done just to keep k at existing level.

- Break-even investment is needed to prevent k from falling because:

  (1) Existing capital is depreciating. This capital must be replaced. This is δk.

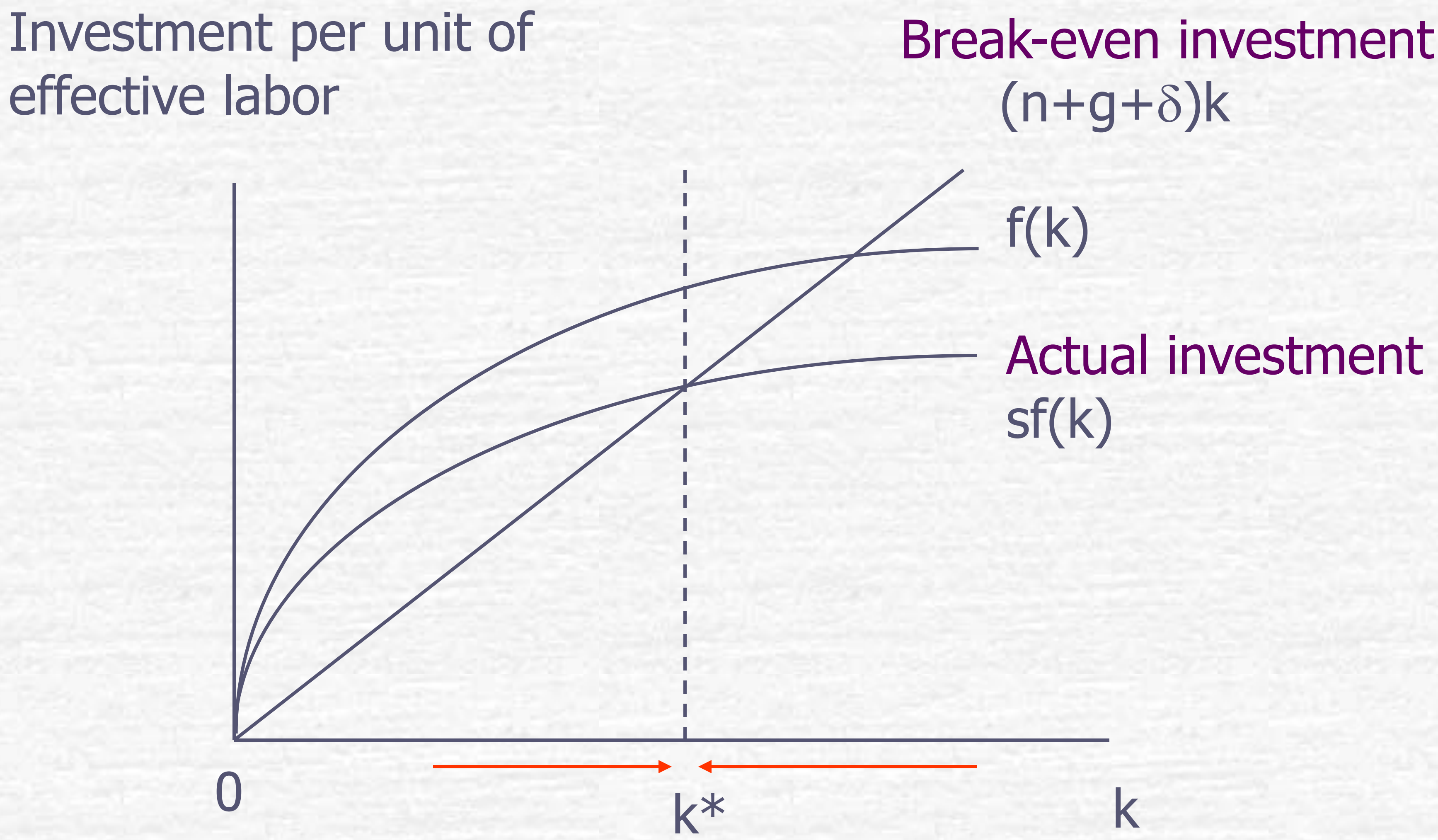
  (2) The quantity of effective labor (AL) is growing at the rate n+g. If K does not rise (no investment), k will fall because $k=K/AL$.

- According to the key equation,

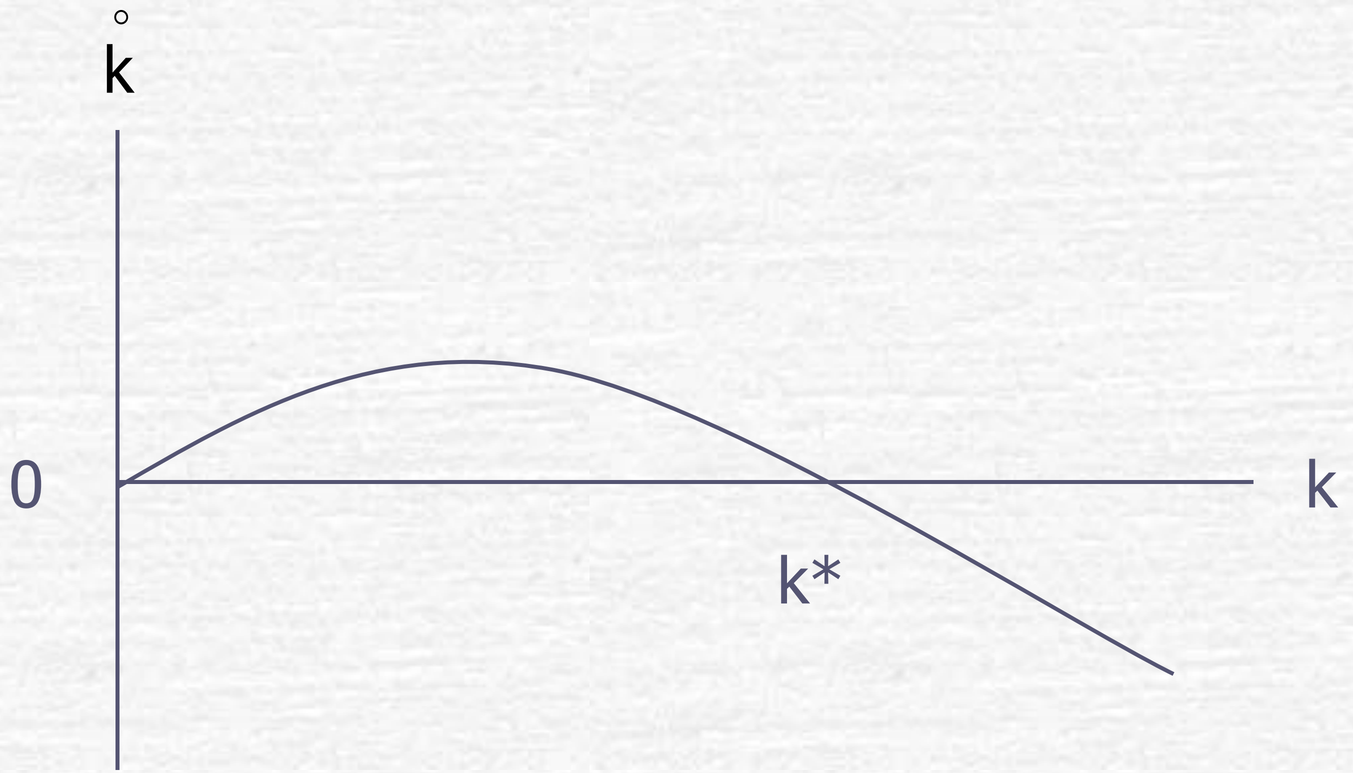
$$\overset{\circ}{k}(t) = sf(k(t)) - (n + g + \delta)k(t)$$

- It states that when the actual investment per unit of effective labor [sf(k)] is greater than the break-even investment per unit of effective labor [(n+g+δ)k], capital per unit of effective labor (k) rises.

- On the other hand, when the actual investment per unit of effective labor [sf(k)] is less than the break-even investment per unit of effective labor [(n+g+d)k], capital per unit of effective labor (k) declines.


Investment per unit of effective labor
Break-even investment
(n+g+δ)k
f(k)
Actual investment
sf(k)
0
k*
k

- Since f(0)=0, then sf(k)=(n+g+δ)k at k=0.
- Inada conditions imply that k=0, f'(k) is large. The sf(k) line is steeper than the (n+g+δ)k line. For small value of k, actual investment is larger than break-even investment.
- Inada conditions also imply that f'(k) falls toward zero as k becomes large. At some point, the slope of the actual investment line falls below the slope of the break-even investment line.



Phase diagram of k

- Phase diagram shows that if k is initially less than k* (k when actual investment exceeds greater than break-even investment), k is rising or k-dot is positive.
- If k exceeds k*, k declines or k-dot is negative.
- Thus, regardless of where k starts, it converge to k*. It is called convergence.

The Steady-State Quantity of Capital per Effective Labor and Output per Effective Labor

- For $y=f(k)=k^{\alpha}$
- In the steady state equilibrium, $\overset{\circ}{k} = 0$

$$\overset{\circ}{k} = sf(k) - (n + g + \delta)k$$

$$0 = s(k^*)^{\alpha} - (n + g + \delta)k^*$$

$$(k^*)^{1-\alpha} = \frac{s}{n + g + \delta}$$

$$k^* = \left(\frac{s}{n + g + \delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = \left(\frac{s}{n + g + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

- So, $k^*(s,n,g,\delta)$ and $y^*(s,n,g,\delta)$

The Balanced Growth Path

- When k is constant at k*, it means that K is growing at the same rate as AL because k=K/AL.
- Labor (L) and knowledge (A) grow at the rate n+g, thus capital (K) grow at the rate n+g too.
- The assumption of constant returns to scale implies that Y is also growing at the rate n+g.

- $k=K/AL$ or $k=(K/L)/A$. When k is constant at k^*, it implies that K/L is growing at the same rate as A which is g.
- Since $y=f(k)$, when k is constant, y is also constant.
- $y=Y/AL$ or $y=(Y/L)/A$. When y is constant, it implies that Y/L is growing at the same rate as A which is g.

- Thus the Solow model implies that, regardless of its starting point, the economy converge to a balanced growth path – a situation where each variable of the model is growing at a constant rate. It is sometimes called steady state equilibrium.
- On the balanced growth path, the growth rate of output per worker (Y/L) is determined by the rate of technological progress.

- Nicholas Kaldor (1961) described several facts about growth which is called Kaldor's stylized facts.
- Some facts are:

1. The growth rates of labor, capital, and output are constant.
2. The growth rates of output and capital are about equal and larger than the growth rate of labor.

- The balanced growth path of the Solow model has these properties.

The Growth Rate of Variables in the Balanced Growth Path

Variable	Growth Rate
Y	n+g
K	n+g
L	n
A	g
y	0
k	0
Y/L	g
K/L	g