

Assignment 1

Due 25/8/2020

From the given data set, estimate the following models:

Capital Asset Pricing Model (CAPM)

$$\text{CAPM: } r_{jt} = \alpha_j + \beta_{j1}r_{mt} + \varepsilon_{jt} \quad (1)$$

Fama & French three-factor Model (FF)

$$\text{Fama & French: } r_{jt} = \alpha_j + \beta_{j1}r_{mt} + \beta_{j2}r_{smbt} + \beta_{j3}r_{hmlt} + \varepsilon_{jt} \quad (2)$$

Where: r_{jt} = excess return on portfolio j at time t and
 r_{mt} = excess return on market portfolio at time t – representing market risk premium.
 r_{smbt} = return on a small-stock portfolio minus the return on a large-stock portfolio (Small Minus Big) at time t – representing size premium.
 r_{hmlt} = return on a value-stock portfolio minus the return on a growth-stock portfolio (High Minus Low) at time t – representing value premium.

- (1) Determine whether there exists significant Jensen Alpha.
- (2) Determine whether portfolio j has the same risk as the market.
- (3) Determine whether there exists significant size premium.
- (4) Determine whether there exists significant growth (value) premium.
- (5) Compare CAPM and FF models and determine which model is the most appropriated model. why?

To study calendar effect (January effects) from the data set, estimate the following models:

$$r_{jt} = \alpha_j + \gamma_j D_{1t} + \beta_{j1}r_{mt} + \beta_{j2}r_{smbt} + \beta_{j3}r_{hmlt} + \varepsilon_{jt} \quad (3)$$

where: $D_{1t} = 1$ on January and $= 0$ otherwise.

- (6) Determine whether there exist significant January effects.
- (7) Make interpretation of estimated result of model (3) (including (1) sign, (2) overall test, (3) R-square, and (4) individual test).

$$r_{jt} = \alpha_j + \beta_{j1}r_{mt} + \beta_{j2}r_{smbt} + \beta_{j3}r_{hmlt} + \gamma_j D_{1t} + \beta_{j1} D_{1t} r_{mt} + \beta_{j2} D_{1t} r_{smbt} + \beta_{j3} D_{1t} r_{hmlt} + \varepsilon_{jt} \quad (4)$$

- (8) Perform Chow-test (using Intercept and Slope Dummy) whether January and other month share the same structure of the Fama-French model (Model (2) vs Model (4)).

Assume 95% confidence level for all questions.

CAPM Model

```
. reg rj rm
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Source	SS	df	MS	Number of obs	=	11,959
Model	11449.5344	1	11449.5344	F(1, 11957)	=	5988.94
Residual	22859.1346	11,957	1.91177842	Prob > F	=	0.0000
				R-squared	=	0.3337
				Adj R-squared	=	0.3337
Total	34308.669	11,958	2.86909759	Root MSE	=	1.3827

rj	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
rm	.9947206	.0128536	77.39	0.000	.9695254 1.019916
_cons	.0084273	.0126552	0.67	0.505	-.0163789 .0332335

$$\hat{r}_t = 0.0084 + 0.9947 r_{mt} + \hat{u}_t$$

FF Model

```
. reg rj rm smb hml
```

Source	SS	df	MS	Number of obs	=	11,959
Model	11681.1999	3	3893.73328	F(3, 11955)	=	2057.22
Residual	22627.4691	11,955	1.89272013	Prob > F	=	0.0000
				R-squared	=	0.3405
				Adj R-squared	=	0.3403
Total	34308.669	11,958	2.86909759	Root MSE	=	1.3758

rj	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
rm	1.005554	.0128271	78.39	0.000	.9804104 1.030697
smb	.0371377	.0061189	6.07	0.000	.0251437 .0491318
hml	.0562866	.00609	9.24	0.000	.0443492 .068224
_cons	.0073088	.0125928	0.58	0.562	-.0173752 .0319928

$$\hat{r}_{jt} = .0073 + 1.0056 r_{mt} + 0.0371 r_{smbt} + 0.0563 r_{hmlt} + \hat{u}_t$$

- (1) $H_0: \alpha = 0$
 $H_a: \alpha \neq 0$

By using estimated CAPM model,

Since t-test falls into rejection region i.e. $P(t > 0.67) > 0.05$

Then, the null hypothesis is not rejected. \therefore Jensen Alpha is not significant.

- (2) $H_0: \beta_1 = 1$ Under CAPM model
 $H_a: \beta_1 \neq 1$

$$\text{t-test: } t = \frac{0.9947 - 1}{0.0129} = -0.4109$$

$$P\text{-value} = 0.6811 > \frac{0.05}{2}$$

Since -0.4109 does not fall into rejection region, $\beta_1 = 1$ is not rejected.

\therefore Portfolio have the same risk as the market.

(3) Under FF model

$$H_0: \beta_2 = 0$$

$$H_a: \beta_2 \neq 0$$

$$t = \frac{0.0371377 - 0}{0.0061189} = 6.07$$

$$P(t > 6.07) < \frac{0.05}{2}$$

According to the STATA table, t-value falls into rejection region, therefore β_2 is significant.

(4) Under FF model

$$H_0: \beta_3 = 0$$

$$H_a: \beta_3 \neq 0$$

$$t = \frac{0.0562866 - 0}{0.00609} = 9.24$$

$$P(t > 9.24) < \frac{0.05}{2}$$

According to the STATA table, t-value falls into rejection region, therefore β_3 is significant.

(5)

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. reg rj rm smb hml
      Source |         SS          df           MS       Number of obs =      11,959
-----+-----+-----+-----+-----+-----
      Model |    11681.1999         3     3893.73328       F(3, 11955) =    2057.22
      Residual |    22627.4691    11,955     1.89272013       Prob > F =      0.0000
-----+-----+-----+-----+-----
      Total |    34308.669     11,958     2.86909759       R-squared =      0.3405
                                          Adj R-squared =    0.3403
                                          Root MSE =      1.3758

      rj |
-----+-----+-----+-----+-----+-----
      rm |    1.005554     .0128271     78.39     0.000     .9804104     1.030697
      smb |    .0371377     .0061189      6.07     0.000     .0251437     .0491318
      hml |    .0562866     .00609        9.24     0.000     .0443492     .068224
      _cons |   -.0073088     .0125928      0.58     0.562    -.0173752     .0319928

      test smb hml

      ( 1)  smb = 0
      ( 2)  hml = 0

      F( 2, 11955) =    61.20
      Prob > F =      0.0000
  
```

$$UR: \hat{r}_{jt} = \alpha_j + \beta_{j1} r_{mt} + \beta_{j2} smb_t + \beta_{j3} hml_t + \hat{u}_t$$

$$R: \hat{r}_{jt} = \alpha_j + \beta_{j1} r_{mt} + \hat{u}_t$$

$F(2, 11955) = 61.20 < 0.025 \rightarrow$ F-value falls into rejection region which means

$$\beta_{smb} \neq 0 \text{ or } \beta_{hml} \neq 0$$

$$H_0: \beta_{smb} = \beta_{hml} = 0$$

$$H_a: \beta_{smb} \neq 0 \text{ or } \beta_{hml} \neq 0$$

\therefore FF model is more appropriate than CAPM.

(6)

Source	SS	df	MS	Number of obs	=	11,959
Model	11683.8263	4	2920.95657	F(4, 11954)	=	1543.31
Residual	22624.8427	11,954	1.89265875	Prob > F	=	0.0000
				R-squared	=	0.3406
				Adj R-squared	=	0.3403
Total	34308.669	11,958	2.86909759	Root MSE	=	1.3757

rj	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
d1	.05393	.045781	1.18	0.239	-.0358082 .1436682
rm	1.005405	.0128275	78.38	0.000	.9802607 1.030549
smb	.0369291	.0061214	6.03	0.000	.0249302 .048928
hml	.0562495	.00609	9.24	0.000	.0443121 .0681868
_cons	.0028773	.0131425	0.22	0.827	-.0228842 .0286388

$$r_{jt} = \alpha_j + \gamma_j D_{1t} + \beta_{j1} r_{mt} + \beta_{j2} r_{smbt} + \beta_{j3} r_{hmlt} + \varepsilon_{jt}$$

$$\hat{r}_t = \hat{\alpha}_j + 0.0539 D_{1t} + 1.0054 r_{mt} + 0.0369 r_{smbt} + 0.0562 r_{hmlt} + \hat{u}_t$$

$$H_0: \gamma = 1$$

$$H_a: \gamma \neq 1$$

$$t = \frac{0.05393 - 1}{0.045781} = -20.4651$$

$P(|t| > | -20.4651 |) < 0.025 \rightarrow H_0$ is rejected which means $\gamma \neq 1$.

\therefore There is not significant January effects.

(7) Make interpretation of estimated result of model (3) (including (1) sign, (2) overall test, (3) R-square, and (4) individual test).

$$r_{jt} = \alpha_j + \gamma_j D_{1t} + \beta_{j1} r_{mt} + \beta_{j2} r_{smbt} + \beta_{j3} r_{hmlt} + \varepsilon_{jt}$$

$$\hat{r}_t = \hat{\alpha}_j + 0.0539 D_{1t} + 1.0054 r_{mt} + 0.0369 r_{smbt} + 0.0562 r_{hmlt} + \hat{u}_t$$

Source	SS	df	MS	Number of obs	=	11,959
Model	11683.8263	4	2920.95657	F(4, 11954)	=	1543.31
Residual	22624.8427	11,954	1.89265875	Prob > F	=	0.0000
				R-squared	=	0.3406
				Adj R-squared	=	0.3403
Total	34308.669	11,958	2.86909759	Root MSE	=	1.3757

rj	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
d1	.05393	.045781	1.18	0.239	-.0358082 .1436682
rm	1.005405	.0128275	78.38	0.000	.9802607 1.030549
smb	.0369291	.0061214	6.03	0.000	.0249302 .048928
hml	.0562495	.00609	9.24	0.000	.0443121 .0681868
_cons	.0028773	.0131425	0.22	0.827	-.0228842 .0286388

$$\hat{\beta}_{j1} > 1$$

This implies if excess return on market (r_m) increases by 1, excess return on stock j would increase by 1.0054. The positive sign of beta means there is on positive correlation between r_{mt} and r_{jt} . And it also implies that the stock is highly sensitive to the market.

$0 < \hat{\beta}_2 < 1$: This implies r_{smb} and r_{jt} are positively correlated which means if r_{smb} increases by 1 unit, r_{jt} would increase by less than 1 (0.0369).

$0 < \hat{\beta}_3 < 1$: Similar to the interpretation of β_2 , r_{hmlt} and r_t are positively correlated which means if r_{smb} increases by 1 unit, r_t would increase by less than 1 (0.0562).

$0 < \hat{\gamma} < 1$: This implies D_{1t} has positive impact on r_t . It means if $D_1 = 1$ (when the special event occurs), excess return on the stock would increase by less than 1 (0.0539).

Overall test:

$$H_0: \gamma = \beta_1 = \beta_2 = \beta_3 = 0$$

H_a : otherwise

$$F_{4, 1954} = 1543.31 \Rightarrow P(F > 1543.31) < 0.05$$

Since $P(F < 1543.31)$ does not exceed 0.05, F-value falls into rejection region.

Therefore, H_0 rejected which means one or more than one estimated coefficients $\neq 0$.

R-square

According to the result, $R^2 = 0.3406$ which means 34.06% of r_t could be explained by the model. Since $0.3 < R^2 < 0.7$, it implies that the model generally has low effect size.

Individual test

• Test if γ is significant

$$H_0: \gamma = 0 \quad t = 1.18 \Rightarrow P(t > 1.18) = 0.239 > 0.025$$

$$H_a: \gamma \neq 0 \quad \text{Since } P(t > 1.18) \text{ exceeds } 0.05, \text{ t-value does not fall into rejection region.}$$

Therefore, H_0 is not rejected meaning that γ is not significant.

• Test if β_1 is significant

$$H_0: \beta_1 = 0 \quad t = 73.38 \Rightarrow P(t > 73.38) < 0.05$$

$$H_a: \beta_1 \neq 0 \quad H_0 \text{ is rejected since } P(t > 73.38) < 0.05. \text{ Therefore, } \beta_1 \text{ is significant.}$$

• Test if β_2 is significant.

$$H_0: \beta_2 = 0 \quad t = 6.03 \Rightarrow P(t > 6.03) < 0.05$$

$$H_a: \beta_2 \neq 0 \quad H_0 \text{ is rejected since } P(t > 6.03) < 0.05. \text{ Therefore, } \beta_2 \text{ is significant.}$$

• Test if β_3 is significant

$$H_0: \beta_3 = 0 \quad t = 9.24 \Rightarrow P(t > 9.24) < 0.05$$

$$H_a: \beta_3 \neq 0 \quad H_0 \text{ is rejected since } P(t > 9.24) < 0.05. \text{ Therefore, } \beta_3 \text{ is significant.}$$

(8)

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. reg rj rm smb hml d1 d1rm dismb dihml
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Source	SS	df	MS	Number of obs	=	11,959
Model	11685.5157	7	1669.35938	F(7, 11951)	=	881.86
Residual	22623.1533	11,951	1.89299249	Prob > F	=	0.0000
				R-squared	=	0.3406
				Adj R-squared	=	0.3402
Total	34308.669	11,958	2.86909759	Root MSE	=	1.3759

rj	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
rm	1.008159	.0133675	75.42	0.000	.9819563 1.034361
smb	.0364768	.0064084	5.69	0.000	.0239153 .0490383
hml	.0553364	.0063695	8.69	0.000	.0428511 .0678216
d1	.0552912	.0461135	1.20	0.231	-.0350988 .1456811
d1rm	-.035594	.0475853	-0.75	0.454	-.1288689 .0576808
dismb	.0037628	.0217997	0.17	0.863	-.0389682 .0464937
dihml	.0106311	.0218876	0.49	0.627	-.0322721 .0535344
_cons	.0027652	.0131445	0.21	0.833	-.0230002 .0285307

$$\hat{r}_{jt} = 0.0028 + 1.0082 r_{mt} + 0.0365 r_{smbt} + 0.0553 r_{hmlt} + 0.0553 D_{1t} - 0.0356 D_{1t} r_{mt} + 0.0038 D_{1t} r_{smbt} + 0.0106 D_{1t} r_{hmlt} + \hat{u}_t$$

① All : $r_t = \lambda_0 + \lambda_1 r_{mt} + \lambda_2 r_{smbt} + \lambda_3 r_{hmlt} + u_t \rightarrow$ FF Model

② $D_1=0$: $r_t = \alpha_0 + \alpha_1 r_{mt} + \alpha_2 r_{smbt} + \alpha_3 r_{hmlt} + u_t$

③ $D_1=1$: $r_t = \beta_0 + \beta_1 r_{mt} + \beta_2 r_{smbt} + \beta_3 r_{hmlt} + u_t$

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. reg rj rm smb hml if d1==0
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Source	SS	df	MS	Number of obs	=	10,974
Model	10805.6192	3	3601.87308	F(3, 10970)	=	1887.21
Residual	20936.975	10,970	1.90856654	Prob > F	=	0.0000
				R-squared	=	0.3404
				Adj R-squared	=	0.3402
Total	31742.5942	10,973	2.89279087	Root MSE	=	1.3815

rj	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
rm	1.008159	.0134224	75.11	0.000	.9818484 1.034469
smb	.0364768	.0064347	5.67	0.000	.0238636 .04909
hml	.0553364	.0063956	8.65	0.000	.0427998 .0678729
_cons	.0027652	.0131985	0.21	0.834	-.0231062 .0286367

\rightarrow ②

```
. reg rj rm smb hml if d1==1
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Source	SS	df	MS	Number of obs	=	985
Model	872.032797	3	290.677599	F(3, 981)	=	169.11
Residual	1686.17832	981	1.71883621	Prob > F	=	0.0000
				R-squared	=	0.3409
				Adj R-squared	=	0.3389
Total	2558.21111	984	2.59980804	Root MSE	=	1.311

\rightarrow ③

$$RSS_1 = 22629.4691$$

$$RSS_2 = 20936.975$$

$$RSS_3 = 1686.17832$$

$$k = 3, n_1 = 10,974, n_2 = 985$$

Chow test

$$H_0: \alpha_0 = \beta_0 = \gamma_0$$
$$\text{and } \alpha_1 = \beta_1 = \gamma_1$$
$$\text{and } \alpha_2 = \beta_2 = \gamma_2$$
$$\text{and } \alpha_3 = \beta_3 = \gamma_3$$

H_a : otherwise

$$F = \frac{(RSS_1 - RSS_2 - RSS_3)/k}{(RSS_2 + RSS_3)/(n_1 + n_2 - 2k)}$$
$$= \frac{(22627.4691 - 20936.975 - 1686.17832)/3}{(20936.975 + 1686.17832)/(10,974 + 985 - 2(3))}$$
$$= 0.76008 \rightarrow p\text{-value} = 0.48 > 0.05$$

H_0 is not rejected since $p\text{-value} > 0.05$.

\therefore January and other month still share the same structure as FF model.

