EE320 Chapter 3
Static and Comparative Static Equilibrium Analysis

Panit Wattanakoon
Faculty of Economics, Thammasat University

1 Breakeven Analysis

A breakeven point is the quantity firm produces and yields itself zero profit. Before we analyze a breakeven, we need a profit function:

$$\begin{aligned}
\Pi &= \text{Total Revenue } - \text{Total Cost} \\
TC &= TFC + a \cdot Q \\
TR &= P \cdot Q
\end{aligned}$$

Given the above equations, what are exogeneous and endogeneous variables?

1.1 Static Equilibrium Analysis

Breakeven point is Q^* such that

Linear function

$$\begin{aligned} TR - TC &= 0 \\ TR &= TC \end{aligned}$$

$$Q^* > 0 \quad \text{if} \quad \underbrace{p - a > 0}_{\text{or p>a}} \quad (\text{TFC always} \geq 0)$$

Example: Find breakeven quantity, given that

$$\begin{aligned} TFC &= 2000 \\ P &= 5 \\ a &= 4 \end{aligned}$$

$$\because \Pi = TR - TC = 0$$

What will happen if P <a ?

1.2 Comparative Static Equilibrium Analysis

Example:

$$\begin{aligned} TFC &= 2000 \\ P &= 5 \\ a &= 4 \\ \text{If } P' &= 6 \\ \Delta P &= P' - P \end{aligned}$$

Find new breakeven quantity

$$\Pi = TR - TC = 0$$

Non-linear function

$$\begin{aligned} TR &= PQ \\ P &= \alpha - \beta Q \end{aligned}$$

$$\begin{aligned} \therefore TR &= \\ TC &= TFC + aQ \end{aligned}$$

Find breakeven quantity: $TR = TC$

Note on solution of $ax^2 + bx + c = 0$

2 Individual and Market Demand

Example: Demand for good x depends on a variety of factors

$$Q_x^d = f(P_x, P_y, P_z, \text{Income}, \text{Taste}, etc.)$$

$$Q_a^d = 100 - P_a + 2I - P_b + P_c$$

2.1 Individual Demand

$Q^d = f(P) = a - bP \quad ; b > 0$

2.2 Market Demand

Suppose that there are n number of potential buyers for goods x:

$$Q^d_M = Q^d_1 + Q^d_2 + Q^d_3 + \cdots + Q^d_n$$

Example:

$$\begin{array}{lrcl} \text{Consumer 1} & Q^d_1 & = & 50 - 2P \\ \text{Consumer 2} & Q^d_2 & = & 100 - 4P \\ & \therefore Q^d_M & = & \end{array}$$

But, we normally plot a demand function as $P = \alpha + \beta Q^d$. So we need to find the inverse version of this demand functions.

$$Q^d_1 = 50 - 2P \quad \Rightarrow$$

$$Q^d_2 = 100 - 4P \quad \Rightarrow$$

$$\therefore Q^d_M = 150 - 6P \quad \Rightarrow$$

Example: Suppose that

$$Q_x^d = 100 - P_x + 2I - P_B + P_C$$

and I=25, $P_B = 1$, and $P_C = 1$, what is the inverse of this demand function?

3 Individual and Market Supply

Similar to the demand function, the supply behaves as follows:

$$Q^s_x = f(P) \text{ i.e. } Q^s_x = c + dP_x$$

3.1 Individual Supply

$$Q^s_x = f(P) = c + dP; d > 0$$

3.2 Market Supply

Suppose that there are n number of firms producing goods x:

$$Q^s_M = Q^s_1 + Q^s_2 + Q^s_3 + \cdots + Q^s_n$$

Example:

$$\begin{aligned} Q^s_1 &= -50 + 2P \\ Q^s_2 &= -50 + 2P \\ \therefore Q^s_M &= -100 + 4P \end{aligned}$$

We can also find an inverse of supply functions:

$$\begin{aligned} Q^s_M &= -100 + 4P &\rightarrow\quad P_M &= 25 + \tfrac{1}{4}Q_M \\ Q^s_1 &= -50 + 2P &\rightarrow\quad P_1 &= 25 + \tfrac{1}{2}Q_1 \\ Q^s_2 &= \end{aligned}$$

4 Partial Market Equilibrium

Equilibrium is the situation that is there is no tendency to change except that there is an external force disturbing it. We will start off with a linear model and then move to non linear after that.

Linear Model

Let's consider a market equilibrium in goods x. Such goods are demanded and supplied following the below equations:

$$\begin{aligned} Q^d &= a - bP \\ Q^s &= -c + dP \end{aligned}$$

What are the equilibrium price and quantity of good x?

Example: Suppose that the demand for goods x is $Q^d = 20 - 5P$ and its supply is $Q^s = 5 + P$. What are the equilibrium price P^* and quantity Q^*

Non-linear Model

Consider a market equilibrium of goods x. Such goods are demanded and supplied following the below equations:

$$\begin{aligned} Q^d &= a - bP^2 \\ Q^s &= -c + dP \end{aligned}$$

What are the equilibrium price and quantity of good x?

Example: Suppose that the demand for good x is $Q^d = 4 - P^2$ and its supply is $Q^s = -1 + 4P$. What are the equilibrium price P^* and quantity Q^*

5 Excise Tax and Market Equilibrium

An excise tax is a tax charged on a particular goods and services. There are two types of it.

1. Specific tax - based on physical unit, in t $ per unit

$$P^t = P + t$$

2. Ad-valorem tax - based on monetary value, in t %

$$P^t = (1 + t)P$$

In this section, we are going to analyze the effect of tax on demand, supply and its equilibrium price and quantity.

Normal Demand and Supply Model:

Suppose that a market demand and market supply follow:

$$\begin{aligned} Q^d &= a - bP^d; b > 0 \\ Q^s &= -c + dP^s; d > 0 \end{aligned}$$

We can solve for its equilibrium price and quantity:

$P^* = \frac{a+c}{b+d}$ and $Q^* = \frac{ad-bc}{b+d}$

5.1 Tax on Producers

Given $Q^d = a - bP^d$ and $Q^s = -c + dP^s$, suppose that the government imposes an excise tax on producer. Each good sold will give lower revenue for producer. What will happen to market equilibrium? How much tax government received from consumer and producer?

First, we need to think how tax enters our equations.

$$P^s =$$

$$\begin{aligned} Q^d &= a - bP^d \\ Q^{s'} &= -c + dP^s \end{aligned}$$

5.2 Tax on Consumers

With excise tax charged from consumers, each good bought by consumer will be more expensive.

$$P^d =$$

$$\begin{aligned} Q^{d'} &= a - bP^d \\ Q^s &= -c + dP^s \end{aligned}$$

How about ad-valorem tax?

6 Elasticity Concept

6.1 Price Elasticity

Price elasticity tells us how sensitive a demand for goods is in response to a change in its price. We write out its mathematical formula as:

$$\varepsilon_p = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$$

$$\begin{aligned}
\varepsilon_1 &= \tfrac{\Delta Q}{\Delta P}\tfrac{P}{Q} \\
\varepsilon_2 &= \tfrac{\Delta Q}{\Delta P}\tfrac{P}{Q} \\
\varepsilon_3 &= \tfrac{\Delta Q}{\Delta P}\tfrac{P}{Q}
\end{aligned}$$

Elasticity is <u>not</u> always equal to slope

$$\text{slope} = \tfrac{\Delta Y}{\Delta X} = \tfrac{Y_2 - Y_1}{X_2 - X_1}$$

$$\begin{aligned}
\text{Example}: Y &= 10 + 2x \\
\tfrac{\Delta Y}{\Delta X} &= 2 \quad \rightarrow \quad \text{if x} \uparrow \text{by 1 unit, Y} \uparrow \text{by 2 units.}
\end{aligned}$$

For demand function $Q^d = f(P)$, consider the responsiveness of the change of quantity demanded due to the change in price

Example:

$$\left.\begin{array}{rlrl} P_x = & 100 & Q_x = & 10 \\ P_x = & 110 & Q_x = & 5 \end{array}\right\} \frac{\Delta Q_x}{\Delta P_x} = \frac{5-10}{110-100} = \frac{-5}{10} = -\frac{1}{2}$$

Example:

$$\left.\begin{array}{rlrl} P_y = & 10000 & Q_y = & 10 \\ P_y = & 10010 & Q_y = & 5 \end{array}\right\} \frac{\Delta Q_y}{\Delta P_y} = \frac{5-10}{10010-10000} = \frac{-5}{10} = -\frac{1}{2}$$

Both goods have the same slope, so same responsiveness. But, look at it closely $\rightarrow$ there is only a slight increase (0.1%) in P_y, halving the quantity, but 10% for P_x.

Slope is not enough to capture the sensitivity, percentage change is better.

Example:

$$\begin{aligned} \varepsilon_p &= \frac{\Delta Q}{\Delta P}\frac{P}{Q} \\ \varepsilon_p^x &= \frac{\%\Delta Q_x}{\%\Delta P_x} \\ \varepsilon_p^y &= \frac{\%\Delta Q_y}{\%\Delta P_y} \end{aligned}$$

Product Y is more responsive to Δ in price

Note that

$$\begin{array}{lcll} \mid \varepsilon_p \mid & > & 1 & \text{elastic} \\ & < & 1 & \text{inelastic} \\ & = & 1 & \text{unitary} \end{array}$$

6.2 Price Elasticity and Total Revenue

Recall that $TR = P \times Q$

$|\varepsilon_p| > 1 \quad \rightarrow \quad |\%\Delta Q| > |\%\Delta P|$

P $\uparrow \rightarrow Q \downarrow\downarrow \rightarrow TR \downarrow$ quantity dominates

$|\varepsilon_p| < 1 \quad \rightarrow \quad |\%\Delta Q| < |\%\Delta P|$

P$\uparrow\uparrow \rightarrow Q \downarrow \rightarrow TR \uparrow$ price dominates

$|\varepsilon_p| = 0 \quad \rightarrow \quad \%\Delta P$ leads to no $\%\Delta Q$

"perfectly inelastic"

$|\varepsilon_p| = \infty \quad \rightarrow \quad \%\Delta P$ leads to infinite $\%\Delta Q$

"perfectly elastic"

Application with tax incidence

Those parties have less of price elasticity will bear more burden.

- Consumer bear all (ε_d = perfectly inelastic)

- Producer bear all (ε_d = perfectly elastic)

6.3 Cross-Price Elasticity

$$\varepsilon = \frac{\%\Delta Q_x}{\%\Delta P_y} = \frac{\Delta Q_x}{\Delta P_y}\frac{P_y}{Q_x}\begin{cases} > 0 \text{ X and Y are substitute} \\ < 0 \text{ X and Y are compliment} \end{cases}$$

6.4 Income Elasticity

$$\varepsilon_I = \frac{\%\Delta Q_x}{\%\Delta I} = \frac{\Delta Q_x}{\Delta I}\frac{I}{Q_x}\begin{cases} > 0 \text{ normal goods} \\ < 0 \text{ inferior goods} \end{cases}$$

7 Simple Macroeconomics Model

$$\begin{aligned} \text{Closed economy}: C &= C_0 + cY_D \\ Y_D &= Y - T \\ T &= T_0 \\ I &= I_0 \\ G &= G_0 \end{aligned}$$

$$\begin{aligned} \text{@Equilibrium} \quad Y &= DAE \\ &= C + I + G \\ &= C_0 + c(Y - T_0) + I_0 + G_0 \\ \\ Y^* &= \tfrac{1}{1-c}(C_0 - cT_0 + I_0 + G_0) \end{aligned}$$

$$\left.\begin{aligned} \frac{\Delta Y}{\Delta C} = \frac{\Delta Y}{\Delta I} = \frac{\Delta Y}{\Delta G} &= \frac{1}{1-c} \\ \frac{\Delta Y}{\Delta T} &= \frac{-c}{1-c} \end{aligned}\right\}\text{multiplier}$$

Consider other cases:

1. income tax $(T = tY)$

2. induced investment $(I = iY)$

3. open economy $(DAE = C + I + G + X - M)$

Note that this analysis is only in goods market

8 IS-LM Model

2 endogenous variables $\rightarrow Y$ national income and r interest rate

Output Market

$$\begin{aligned} Y &= C + I + G \\ C &= C_0 + cY_D \quad , \quad 0 < c < 1 \\ I &= I_0 - jr \quad , \quad j > 0 \\ G &= G_0 \\ Y_D &= Y \end{aligned}$$

In equilibrium
$$\begin{aligned} Y &= C_0 + cY + I_0 - jr + G_0 \\ r^*_{IS} &= \end{aligned}$$

Money Market

$$\begin{aligned} M^d &= L_0 + l_1 Y - l_2 r \quad , \quad l_1 > 0 \\ M^s &= M_0 \quad , \quad l_2 > 0 \end{aligned}$$

In equilibrium
$$\begin{aligned} L_0 + l_1 Y - l_2 r &= M_0 \\ r^*_{LM} &= \end{aligned}$$

Endogeneous Variable $\rightarrow Y, r$

Exogeneous Variable $\rightarrow C_0,\ I_0,\ G_0,\ L_0,\ M_0$ shifts

parameters $\rightarrow c,\ j,\ l_1,\ l_2$ rotates

Equilibrium in both markets :

$$r^*_{IS} = r^*_{LM}$$

Reference

Aroonruengsawat, Anin. EE320 Lecture Handouts.

Chiang, A. C. and Wainwright, K. (2005) "Fundamental Methods of Mathematical Economics," 4th edition, McGraw-Hill, Inc., Singapore.

Dowling, E. T. (2001) "Schaums Outline of Theory and Problem of Introduction to Mathematical Economics", 3rd edition, The McGraw-Hill Companies, Inc.

Holden, K. and Pearson, A.W. (1992) "Introductory Mathematics for Economics and Business," Second edition, The Macmillan Press Ltd.

Sydsaeter, K. and P. Hammond. (2006) "Essential Mathematics for Economic Analysis," 2nd edition, Prentice Hall.