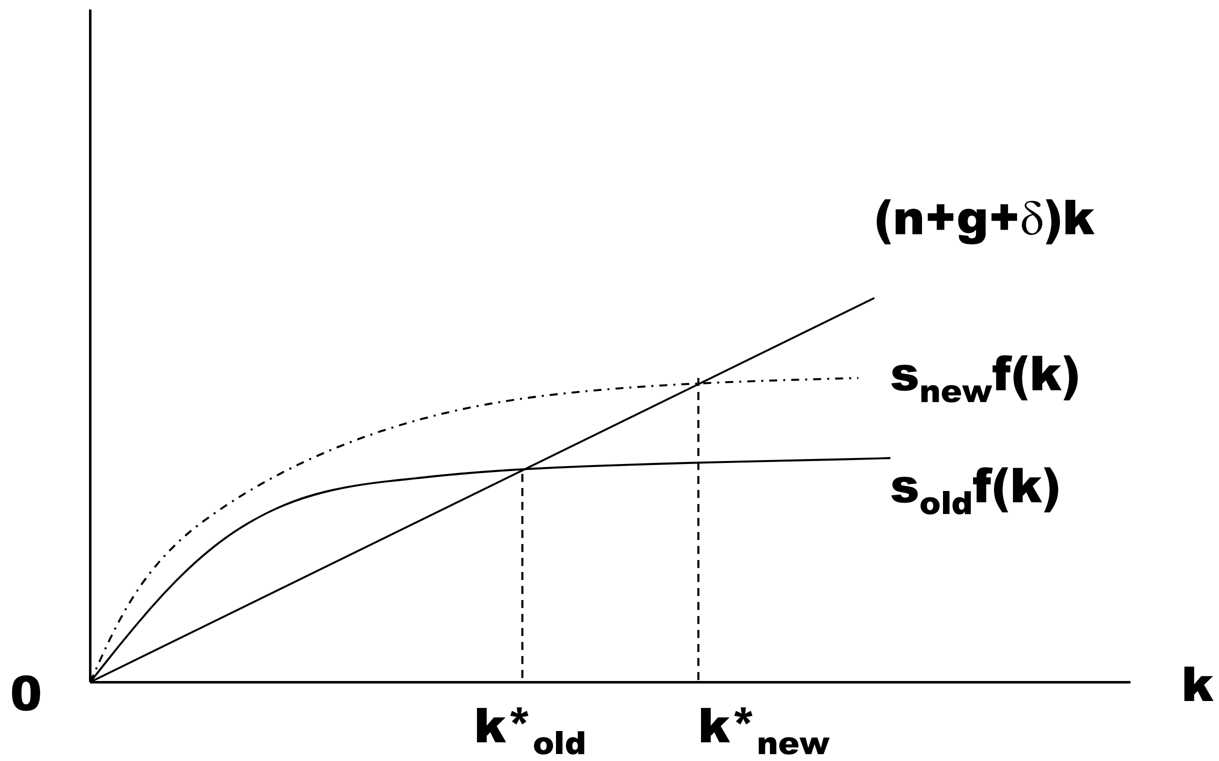
Macroeconomics

Lecture 10

Arayah Preechametta

The impact of a change in the saving rate on output



The impact of a change in the saving rate on output

- In sum, a change in the saving rate has a level effect but not a growth effect.
- Implication from Solow growth model, the change in the rate of technological progress (i.e., endogenous change in A(t)) may have growth effects.

The Solow Model and the Central Questions of Growth Theory

- Only growth in the effectiveness of labor can lead to permanent growth in output per worker.
- The impact of change in capital per worker on output per worker is modest.
- Hence, only differences in the effectiveness of labor can account for the vast differences in the wealth across time and space.

A model that resemble the Solow growth model

- **Infinite-horizon model (The Ramsey-Cass-Koopmans model): Competitive firms rent capital and hire labor to produce output, and a fixed number of households supply labor, hold capital, consume and save.**

The Ramsey-Cass-Koopmans Model

- There is a large number of identical firms.
- Each firm has access to the production function Y= F(K,AL).
- The firms hire workers and rent capital in a competitive factor markets, and sell their output in a competitive output market.
- Firms are owned by households, so any profits they earn accrue to the households.

The Ramsey-Cass-Koopmans Model

- There is a large number of households.
- The size of each household grows at rate n.
- Each member of household supplies one unit of labor at every point in time.
- The household rents capital its own to firms. It has initial capital holding of K(0)/H.

The behavior of firms

- Because the production function has constant returns and the economy is competitive, firms earn zero profits.
- Capital earns its marginal product. Assuming that there is no depreciation, the real rate of return to capital equals its earnings per unit time, or $r(t) = \partial F(K.AL)/\partial K = f'(k(t))$.
- The real wage per unit of effective labor is
- $w(t) = \partial F(K,AL)/\partial AL = f(k(t)) - k(t)f'(k(t))$

The behavior of households

- **The representative household takes the paths of r(t) and w(t) as given.**
- **Its objective function is to maximize**

$$U=\int_{t=0}^{\infty} e^{-\rho t}u\left(C\left(t\right)\right)\frac{L\left(t\right)}{H}dt$$

where $C(t)$ *is the consumption of each member of the household at time t.*

$u(.)$ *is the instantaneous utility function, and takes the form of a constant-relative-risk-aversion utility*

$$u\left(C\left(t\right)\right)=\frac{C\left(t\right)^{1-\theta}}{1-\theta},\qquad \theta>0,\ \rho-n-\left(1-\theta\right)g>0$$

$L(t)$ *is total population in the economy.*

The behavior of households

- **The household's budget constraint is**

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} A(t) w(t) \frac{L(t)}{H} dt \qquad (5)$$

where $R(t) = \int_{\tau=0}^{t} r(\tau) d\tau.$

One unit of capital good invested at time o yields

$e^{R(t)}$ *units of good at* t.

- **Let c(t) be consumption per effective labor, c(t)=[C(t)L(t)/H]/[A(t)L(t)/H].**

The behavior of households

- **Eq(5) can be rewritten as**

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) \frac{A(t)L(t)}{H} dt \le k(0) \frac{A(0)L(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w(t) \frac{A(t)L(t)}{H} dt \qquad (6)$$

$$Or \qquad \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} \left[w(t) - c(t) \right] \frac{A(t)L(t)}{H} dt \ge 0 \qquad (7)$$

$$\lim_{s\to\infty} \left[\frac{K(0)}{H} + \int_{t=0}^{s} e^{-R(t)} \left[w(t) - c(t) \right] \frac{A(t)L(t)}{H} dt \right] \ge 0 \qquad (8)$$

Note that the household's capital holdings at time s are

$$\frac{K(s)}{H} = \frac{K(0)}{H} e^{R(s)} + e^{R(s)} \int_{t=0}^{s} e^{-R(t)} \left[w(t) - c(t) \right] \frac{A(t)L(t)}{H} dt \qquad (9)$$

$\therefore Eq(9)$ *is* $e^{R(s)}$ *times the expression in the brackets in* $Eq(8)$, *hence*

$$\lim_{s\to\infty} e^{-R(s)} \frac{K(s)}{H} \ge 0 \qquad (10)$$

Since $A(t)L(t) = A(0)L(0)e^{(n+g)t}$, $Eq(6)$ *can be rewritten as*

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \le k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt \qquad (11)$$

$Eq(10)$ *and* (11) *together give*

$$\lim_{s\to\infty} e^{-R(s)} e^{(n+g)s} k(s) \ge 0 \qquad (12)$$

The behavior of households

- **Since C(t)=A(t)c(t) ,**

$$u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta} = \frac{[A(t)c(t)]^{1-\theta}}{1-\theta} = \frac{[A(0)e^{gt}]^{1-\theta}c(t)^{1-\theta}}{1-\theta}$$

$$= A(0)^{1-\theta}e^{(1-\theta)gt}\frac{c(t)^{1-\theta}}{1-\theta}$$

$$\therefore \qquad U = \int_{t=0}^{\infty} e^{-\rho t}\frac{C(t)^{1-\theta}}{1-\theta}\frac{L(t)}{H}dt = \int_{t=0}^{\infty} e^{-\rho t}\left[A(0)^{1-\theta}e^{(1-\theta)gt}\frac{c(t)^{1-\theta}}{1-\theta}\right]\frac{L(0)e^{nt}}{H}dt$$

$$= A(0)^{1-\theta}\frac{L(0)}{H}\int_{t=0}^{\infty} e^{-\rho t}e^{(1-\theta)gt}e^{nt}\frac{c(t)^{1-\theta}}{1-\theta}dt$$

$$= B\int_{t=0}^{\infty} e^{-\beta t}\frac{c(t)^{1-\theta}}{1-\theta}dt \qquad (13)$$

where $\qquad B \equiv A(0)^{1-\theta}\frac{L(0)}{H}, \quad and \quad \beta \equiv \rho - n - (1-\theta)g$

The behavior of households

The Lagrangian function

$$L = B\int_{t=0}^{\infty} e^{-\beta t}\frac{c(t)^{1-\theta}}{1-\theta}dt + \lambda\left[k(0) + \int_{t=0}^{\infty} e^{-R(t)}w(t)e^{(n+g)t}dt - \int_{t=0}^{\infty} e^{-R(t)}c(t)e^{(n+g)t}dt\right] \quad (14)$$

The first-order condition for an individual $c(t)$ *is*

$$Be^{-\beta t}c(t)^{-\theta} = \lambda e^{-R(t)}e^{(n+g)t} \quad (15)$$

Take ln *of both sides*

$$\ln B - \beta t - \theta \ln c(t) = \ln \lambda - R(t) + (n+g)t \quad (16)$$

The derivative of both sides of Eq(16), w.r.t. t, must be the same

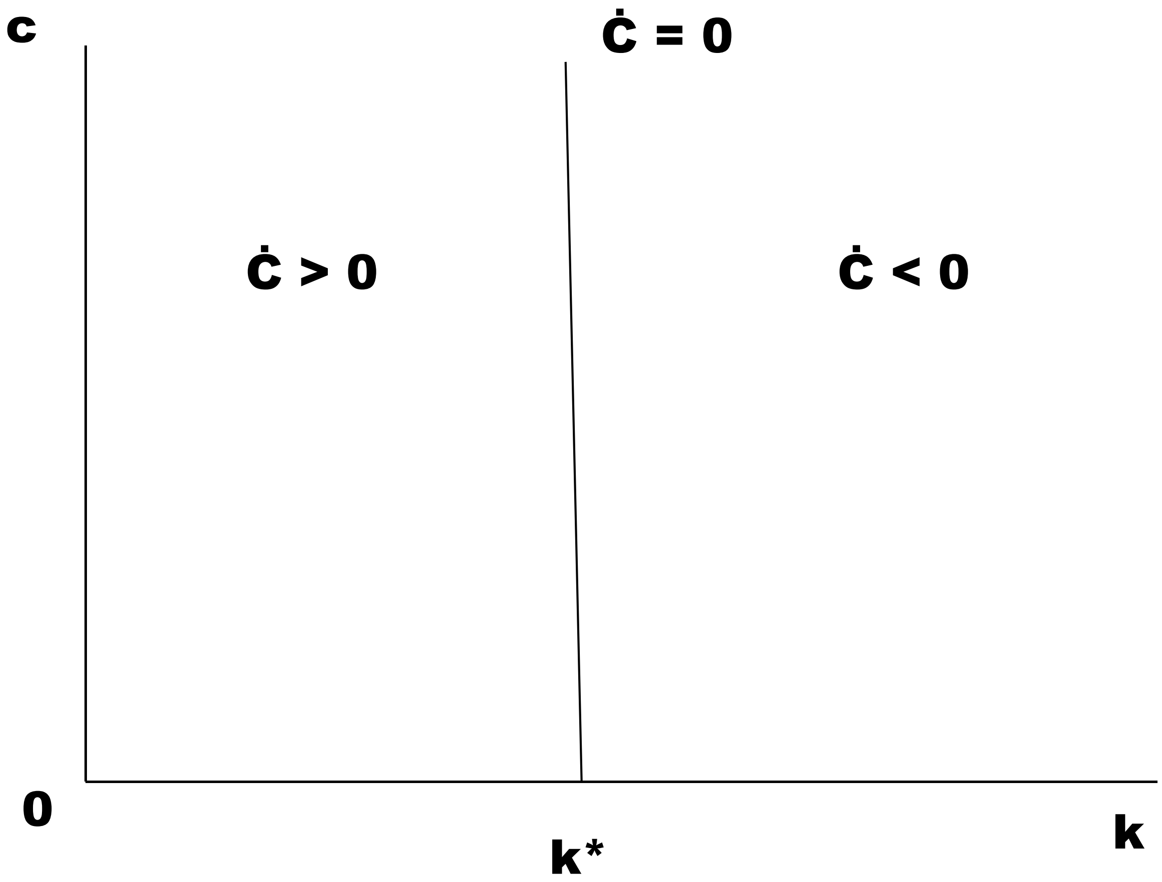
$$-\beta - \theta\frac{\dot{c}(t)}{c(t)} = -r(t) + (n+g)$$

$$\therefore \frac{\dot{c}(t)}{c(t)} = \frac{r(t) - n - g - \beta}{\theta} = \frac{r(t) - \rho - \theta g}{\theta} = \frac{f'(k(t)) - \rho - \theta g}{\theta} \quad (17)$$

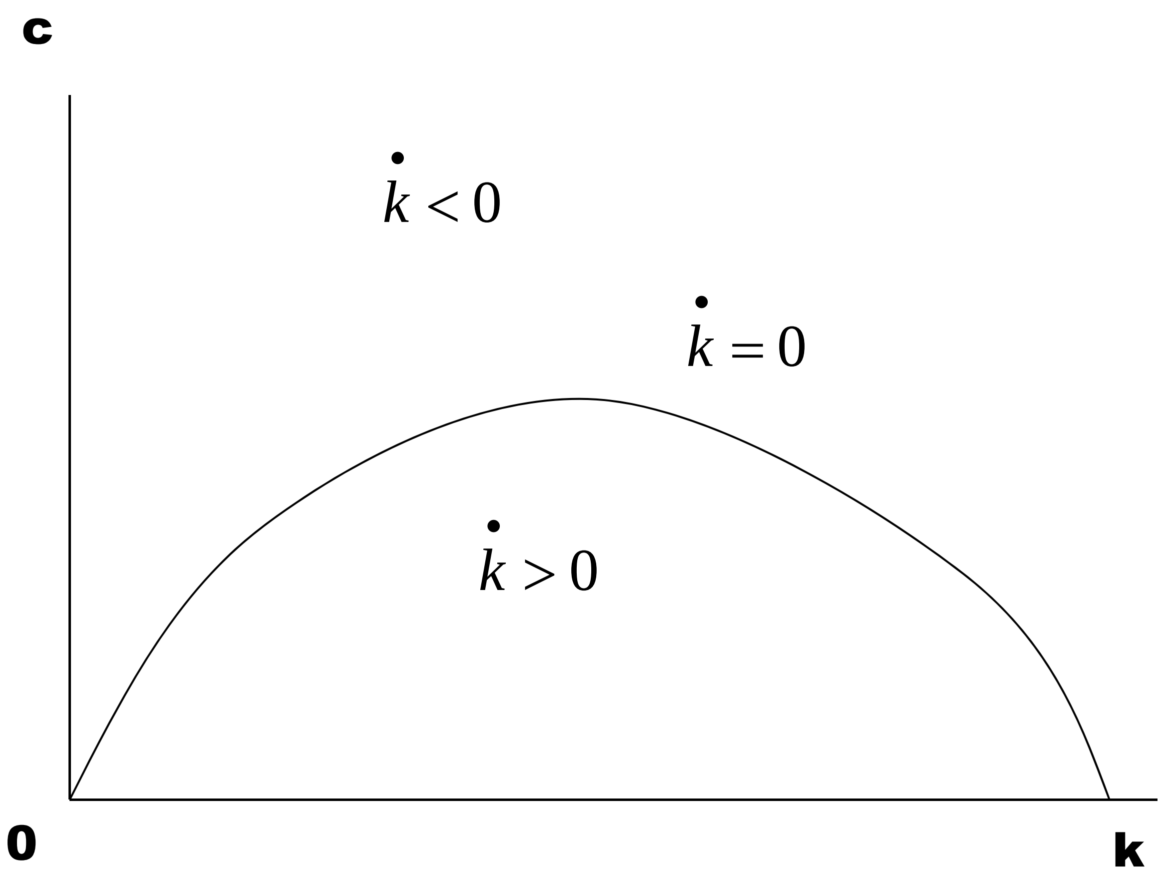
Since actual investment is output minus consumption, thus from Eq(4),

$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t) \quad (18)$$

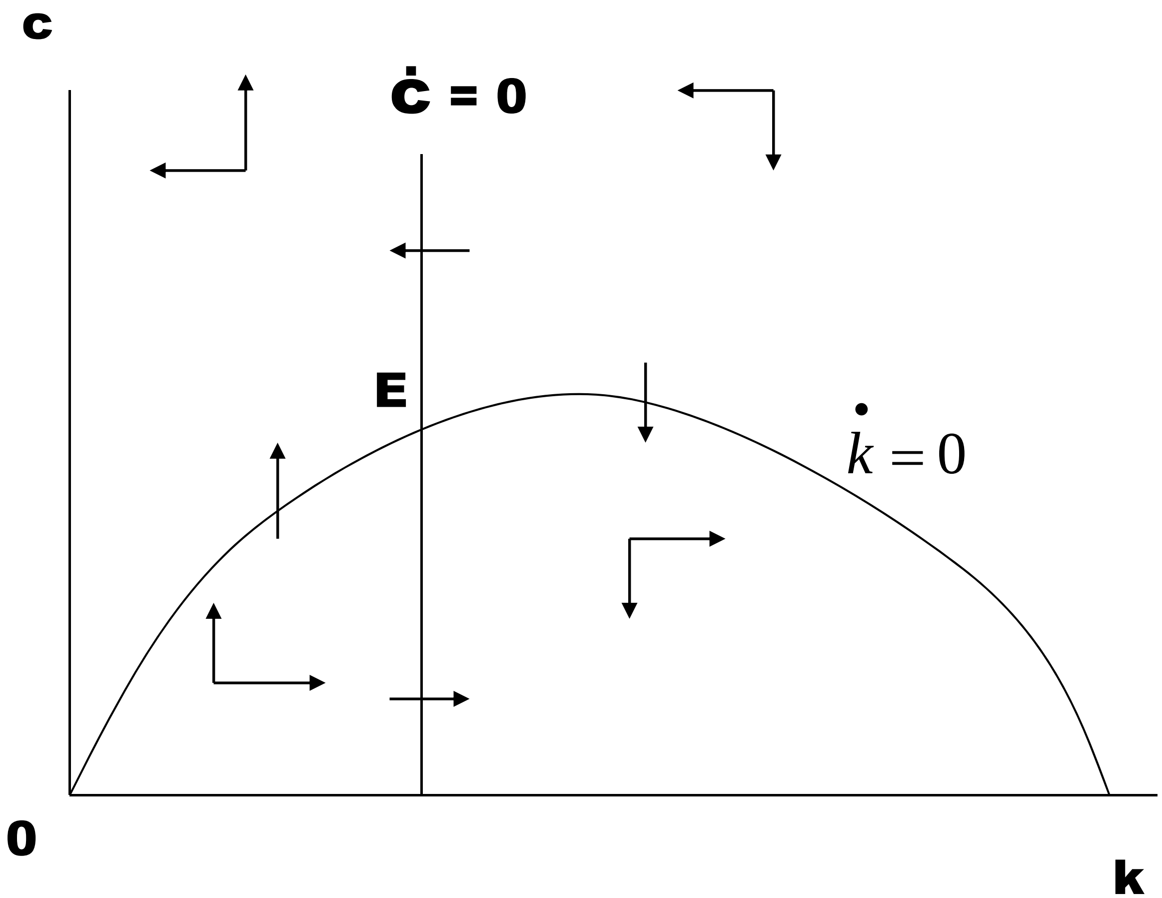
The Dynamic of c



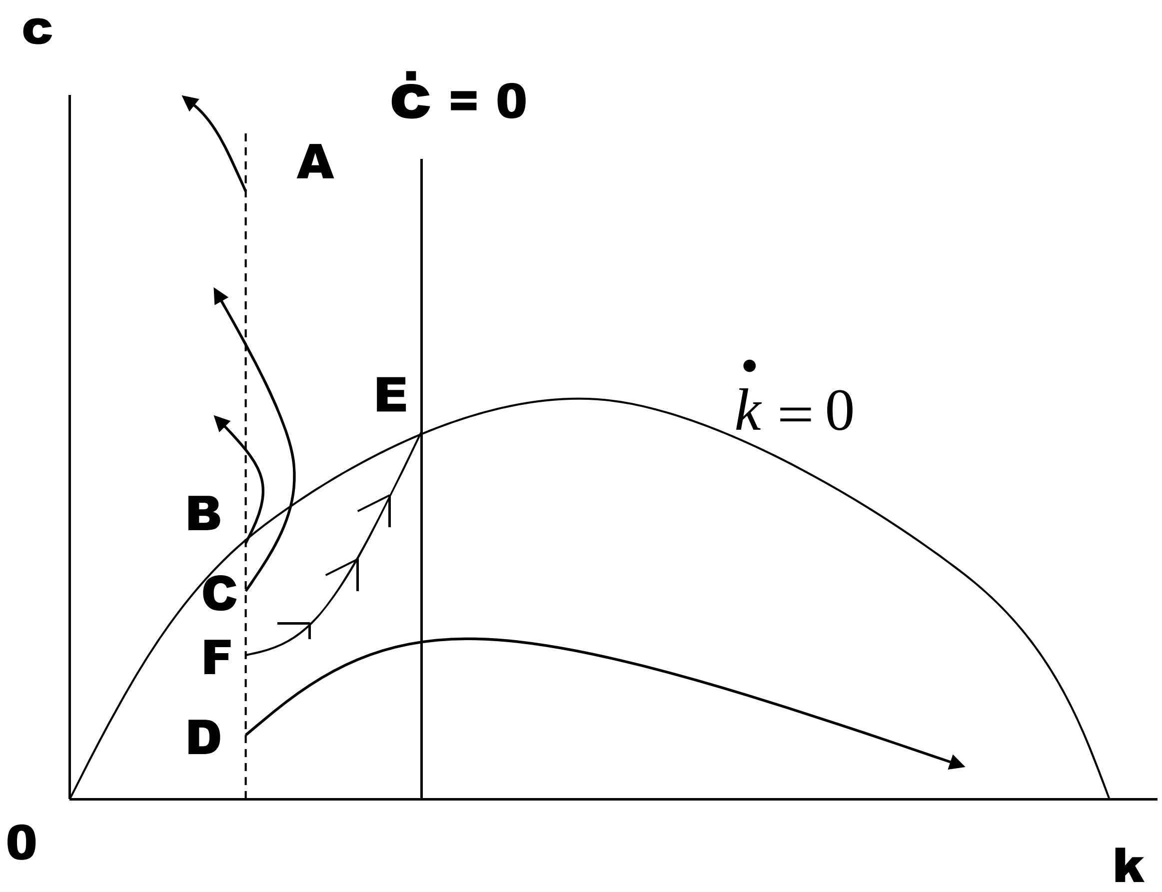
The Dynamic of k



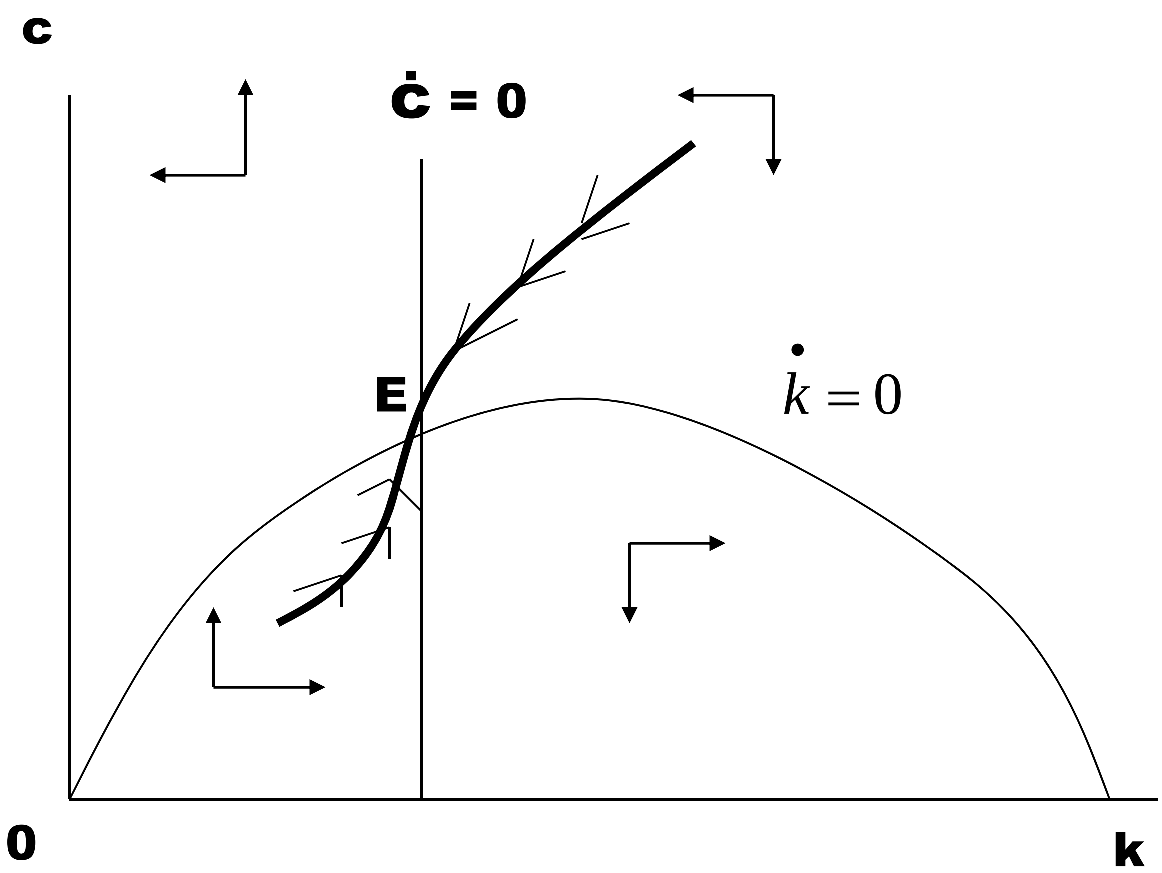
The Phase Diagram



The Initial Value of c



The Saddle Path



Welfare

- The First Welfare Theorem:
- If markets are competitive and there are no externalities, the decentralized equilibrium is Pareto-efficient. (It is impossible to make anyone better off without making someone else worse off.)
- Hence, the equilibrium in this model must be Pareto-efficient.