EE 320 Introductory Mathematical Economics

Homework 3

Due 19 March 2013

There are five questions in total, each worth equally.

1. Consider the following demand function of good A

$$Q_A = aP_A^{\alpha_1} P_B^{\alpha_2} Y^{\beta},$$

where Y is the income, and P_A and P_B are the prices of good A and good B, respectively.

a) Compute the three elasticities (own price, cross price, and income elasticities), and show that they are constants.
b) What are reasonable ranges for the parameters α_1 and α_2?

2. Given the production function

$$Q = AK^{\propto} L^{\beta}$$

where A is a positive constant, $0 < \alpha < 1$, and $0 < \beta < 1$.

a) Given that $\alpha + \beta = 1$, show that this production function exhibits constant returns to scale (that is, when all inputs are increased by a given proportion k, output increases by the same proportion).

b) Suppose that both capital and labor are functions of time (t): $K = K_0 - \delta t$ and $L = e^{\eta t}$, where K_0 is a positive constant, $0 < \delta < 1$, and $0 < \eta < 1$. Find the rate of change of output with respect to time.

3. Find the partial total derivatives $\left.\frac{dW}{du}\right|_{\Delta v=0}$ and $\left.\frac{dW}{dv}\right|_{\Delta u=0}$ if

a) $W = g(x_1, x_2, u, v)$ where $x_1 = 3u + v^2$ and $x_2 = uv$

b) $W = 5x_1x_2 - 8x_2^2$ where $x_1 = f(u, v)$ and $x_2 = h(u, v)$

4. Let the demand for a commodity be

$$Q_d = D(P, t_0), \qquad D_p < 0;\ D_{t0} > 0$$
$$Q_s = S(P, T_0), \qquad S_p > 0;\ S_{T0} < 0$$

where t_0 is consumers' taste for the commodity and T_0 is the tax on the commodity. All derivatives are continuous. Find $\frac{\partial P*}{\partial t_0}$ and $\frac{\partial P*}{\partial T_0}$, and discuss their economic implications.

5. Consider the constant elasticity of substitution (CES) production function

$$F(K, L) = A[\alpha K^{-\gamma} + \beta L^{-\gamma}]^{-1/\gamma},$$

where A, α, and β are positive constants, $\gamma > -1$, and $\gamma \neq 0$. Use the implicit function rule to find the marginal rate of technical substitution (MRTS) of the labor input (L) for the capital input (K).