Assignment Solution
MLE

The study on bankruptcy firm employs the following regression model.

$$z_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i}$$

The log-likelihood function of this model is as follows:

$$\ln L = \begin{cases} \ln \Phi(z_i) \; if \;\; y_i = 1 \\ \ln \Phi(-z_i) \; if \;\; y_i = 0 \end{cases}$$

where: y_i is discrete binary variable, values either *0* or *1*.

x_{ki} are independent variables , *k=1, 2, 3.*

$\Phi(.)$ = Cumulative normal density function.

From the given data set (Assign02_MLE):

1. Estimate the above models using MLE with Newton-Ralphson algorithm and BHHH, then, evaluate the estimated results.

```
program ml_probit
        args lnf theta
        quietly replace `lnf'=ln(normal(`theta')) if $ML_y1==1
        quietly replace `lnf'=ln(normal(-`theta')) if $ML_y1==0
end
```

```
. ml model lf ml_probit (y=x*)

. ml maximize

initial:       log likelihood = -346.57359
alternative:   log likelihood = -299.06229
rescale:       log likelihood = -299.06229
Iteration 0:   log likelihood = -299.06229
Iteration 1:   log likelihood = -137.13306
Iteration 2:   log likelihood = -126.54535
Iteration 3:   log likelihood = -126.24974
Iteration 4:   log likelihood = -126.24956
Iteration 5:   log likelihood = -126.24956

                                                Number of obs   =        500
                                                Wald chi2(3)    =     138.82
Log likelihood = -126.24956                     Prob > chi2     =     0.0000

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .0176771   .0015548    11.37   0.000     .0146297    .0207244
          x2 |  -.0269468   .0038815    -6.94   0.000    -.0345544   -.0193393
          x3 |   .0178847    .003506     5.10   0.000     .0110131    .0247563
       _cons |   1.012678   .1126462     8.99   0.000     .7918951     1.23346
------------------------------------------------------------------------------

. ml model lf ml_probit (y=x*), tech(bhhh)

. ml maximize

initial:       log likelihood = -346.57359
alternative:   log likelihood = -299.06229
rescale:       log likelihood = -299.06229
Iteration 0:   log likelihood = -299.06229
Iteration 1:   log likelihood = -140.49244
```

```
...
Iteration 54:  log likelihood = -126.24956

                                                  Number of obs   =        500
                                                  Wald chi2(3)    =     134.45
Log likelihood = -126.24956                       Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .0176751   .0016539    10.69   0.000     .0144335    .0209167
          x2 |  -.0269433   .0037917    -7.11   0.000    -.0343749   -.0195116
          x3 |   .0178852   .0031771     5.63   0.000     .0116583    .0241121
       _cons |   1.012449   .1477093     6.85   0.000     .7229442    1.301954
------------------------------------------------------------------------------
```

2. Perform hypothesis testing whether $\beta_1 = \beta_2 = \beta_3 = 0$ using LR-test and Wald test.

```
. ml model lf ml_probit (y=x*)

. ml maximize

initial:       log likelihood = -346.57359
alternative:   log likelihood = -299.06229
rescale:       log likelihood = -299.06229
Iteration 0:   log likelihood = -299.06229
Iteration 1:   log likelihood = -137.13306
...
Iteration 5:   log likelihood = -126.24956

                                                  Number of obs   =        500
                                                  Wald chi2(3)    =     138.82
Log likelihood = -126.24956                       Prob > chi2     =     0.0000

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .0176771   .0015548    11.37   0.000     .0146297    .0207244
          x2 |  -.0269468   .0038815    -6.94   0.000    -.0345544   -.0193393
          x3 |   .0178847    .003506     5.10   0.000     .0110131    .0247563
       _cons |   1.012678   .1126462     8.99   0.000     .7918951     1.23346
------------------------------------------------------------------------------
```

Wald Test

```
. test (x1=0) (x2=0) (x3=0)

 ( 1)  [eq1]x1 = 0
 ( 2)  [eq1]x2 = 0
 ( 3)  [eq1]x3 = 0

           chi2(  3) =  138.82
         Prob > chi2 =    0.0000
```

LR Test

```
. est store ur

. ml model lf ml_probit (y=)

. ml maximize

initial:       log likelihood = -346.57359
alternative:   log likelihood = -299.06229
rescale:       log likelihood = -299.06229
Iteration 0:   log likelihood = -299.06229
Iteration 1:   log likelihood = -298.34586
Iteration 2:   log likelihood =  -298.3458
Iteration 3:   log likelihood =  -298.3458

                                                  Number of obs   =        500
                                                  Wald chi2(0)    =          .
Log likelihood =  -298.3458                       Prob > chi2     =          .

------------------------------------------------------------------------------
```

```
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .5709995   .0595003     9.60   0.000      .454381     .687618
------------------------------------------------------------------------------

. est store res

. lrtest ur res

Likelihood-ratio test                                 LR chi2(3)  =    344.19
(Assumption: res nested in ur)                        Prob > chi2 =    0.0000


. probit y x*

Iteration 0:   log likelihood =  -298.3458
...
Iteration 5:   log likelihood = -126.24956

Probit regression                                     Number of obs   =        500
                                                      LR chi2(3)      =     344.19
                                                      Prob > chi2     =     0.0000
Log likelihood = -126.24956                           Pseudo R2       =     0.5768

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .0176771   .0015548    11.37   0.000     .0146297    .0207244
          x2 |  -.0269468   .0038815    -6.94   0.000    -.0345544   -.0193393
          x3 |   .0178847    .003506     5.10   0.000     .0110131    .0247563
       _cons |   1.012678   .1126462     8.99   0.000     .7918951     1.23346
------------------------------------------------------------------------------
Note: 0 failures and 5 successes completely determined.
```