Solution: Quiz 4

1. Find the solution set for
$$3|x| \ < \ \big|\ |4x+9| - |x|\ \big|.$$

Solution:
The inequality $3|x| \ < \ \big|\ |4x+9| - |x|\ \big|$ implies

$$|4x+9| - |x| < -3|x| \qquad \text{or} \qquad |4x+9| - |x| > 3|x|$$

(i) For $|4x+9| - |x| < -3|x|$, we have $|4x+9| < -2|x|$ since $-2|x| < 0$ and so it implies in this case that $|4x+9| < 0$, which is impossible. Hence the solution set in this case is an empty set $\emptyset$.

(ii) For $|4x+9| - |x| > 3|x|$, we have

$$\begin{aligned} |4x+9| - |x| &> 3|x| \\ |4x+9| &> 4|x|. \end{aligned}$$

Since both sides of the inequality above are positive and $(4|x|)^2 = 4^2|x|^2 = 4^2x^2 = (4x)^2$, solving this inequality is equivalent to solving $(4x+9)^2 > (4x)^2$. That is,

$$\begin{aligned} (4x+9)^2 &> (4x)^2 \\ ((4x+9)-4x)((4x+9)+4x) &> 0 \\ 9(8x+9) &> 0 \\ 8x+9 &> 0 \\ x &> -\frac{9}{8}. \end{aligned}$$

That is, in this case (ii), the solution set is $(-9/8, \infty)$.

Since the given inequality is true when either (i) or (ii) is true, the solution set is $\emptyset \cup (-9/8, \infty) = (-9/8, \infty)$.

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