HW

1. firm 1 ; $\pi_1 = TR_1 - TC_1$

$$\pi_1 = (a - bq_1 - bq_2 - bq_3)q_1 - C_1$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 = 0$$

$$a - bq_2 - bq_3 = 2bq_1$$

$$q_1 = \frac{a - bq_2 - bq_3}{2b}$$

substitute ② $\longrightarrow$ $q_1 = \dfrac{a - \left(\frac{a - q_3 b}{3b}\right) b - q_3 b}{2b}$

$$q_1 = \frac{3a - a + q_3 b - 3q_3 b}{6b}$$

$$q_1 = \frac{a - q_3 b}{3b} \quad \text{— ①}$$

substitute ③ $\longrightarrow$ $q_1 = \dfrac{a - \left(\frac{a}{4b}\right) b}{3b}$

$$q_1 = \frac{4a - a}{12b} = \frac{a}{4b}$$

firm 2 ; $\pi_2 = TR_2 - TC_2$

$$\pi_2 = (a - bq_1 - bq_2 - bq_3)q_2 - C_2$$

$$\frac{\partial \pi_2}{\partial q_2} = a - bq_3 - 2bq_2 - bq_3 = 0$$

$$a - bq_1 - bq_3 = 2bq_2$$

substitute ① $\longrightarrow$ $2q_2 b = a - \left(\frac{a - q_2 b - q_3 b}{2b}\right) b - q_3 b$

$$2q_2 b = \frac{2a - a + q_2 b + q_3 b - 2q_3 b}{2}$$

$$4q_2 b = a + q_2 b - q_3 b$$

$$3q_2 b = a - q_3 b$$

$$q_2 = \frac{a - q_3 b}{3b} \quad \text{— ②}$$

substitute ③ $\longrightarrow$ $q_2 = \dfrac{a - \left(\frac{a}{4b}\right) b}{3b}$

$$q_2 = \frac{4a - a}{12b} = \frac{a}{4b}$$

firm 3 ; $\pi_3 = TR_3 - TC_3$

$$\pi_3 = (a - bq_1 - bq_2 - bq_3)q_3 - C_3$$

$$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 = 0$$

$$a - bq_1 - bq_2 = 2bq_3$$

$$q_3 = a - bq_1 - bq_2$$

substitute ① and ② $\longrightarrow$ $q_3 = \dfrac{a - b\left(\frac{a - q_3 b}{3b}\right) - b\left(\frac{a - q_3 b}{3b}\right)}{2b}$

$$q_3 = \frac{3a - a + q_3 b - a + q_3 b}{6b}$$

$$q_3 = \frac{a + 2q_3 b}{6b}$$

$$6bq_3 = a + 2q_3 b$$

$$4bq_3 = a$$

$$4bq_3 = a$$

$$q_3 = \frac{a}{4b}$$

Equilibrium price ; $P = a - bQ$

$$P = a - b(q_1 + q_2 + q_3)$$

$$P = a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$$

$$P = a - \left(\frac{3a}{4}\right)$$

$$P = \frac{a}{4} = 0.25a$$

firm 1; $\pi_1 = P \cdot Q - C_1$

$\pi_1 = 0.25a \cdot \frac{a}{4b} - C_1$

$$\boxed{\pi_1 = \frac{a^2}{16b} - C_1}$$

Firm 2; $\pi_2 = P \cdot q_2 - C_2$

$\pi_2 = 0.25a - \frac{a}{4b} - C_2$

$$\boxed{\pi_2 = \frac{a^2}{16b} - C_2}$$

Firm 3; $\pi_3 = P \cdot q_3 - C_3$

$\pi_3 = 0.25a \cdot \frac{a}{4b} - C_3$

$$\boxed{\pi_3 = \frac{a^2}{16b} - C_3}$$

2. We assume $q_1 + q_2 + q_3 + q_n = A$

$P = a - b(q_1 + q_2 + q_3 + \ldots + q_n)$

$P = a - bq_1 - bq_2 - \ldots - bq_n$

$\pi_1 = (a - bq_1 - bq_2 - bq_3 - \ldots - bq_n) \cdot q_1 - C_1$

$\vdots$

$\pi_n = (a - bq_1 - bq_2 - bq_3 - \ldots - bq_n) \cdot q_1 - C_n$

$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 - \ldots - bq_n = 0$

$$q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \ldots + q_n)$$

$$q_n = \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 + \ldots q_{n-1})$$

$\therefore q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 + \ldots + q_n)$

$0.5q_1 = \frac{a}{2b} - 0.5A$

$q_1 = \frac{a}{b} - A$ — (1)

$q_2 = \frac{a}{b} - A$

$q_3 = \frac{a}{b} - A$

$\vdots$

$q_n = \frac{a}{b} - A$

since $A = q_1 + q_2 + q_3 + \ldots q_n$, $A = n\left(\frac{a}{b} - A\right)$

$A = n\frac{a}{b} - nA$

$A + nA = n\left(\frac{a}{b}\right)$

$A(1+n) = n\left(\frac{a}{b}\right)$

$A = \frac{na}{(n+1)b} \longrightarrow$ substitute into (1); $q_1 = \frac{a}{(n+1)b}$

$\therefore q_i = \frac{a}{(n+1)b}$

Equilibrium price ; $P = a - b(A)$

$$P = a - b\left(\frac{na}{(n+1)b}\right)$$

$$P = a - \left(\frac{n}{n+1}\right)a$$

$$P = \frac{a(n+1) - na}{n+1}$$

$$P = \frac{na + a - na}{n+1}$$

$$P = \frac{a}{n+1}$$

$$\pi_i = P \cdot q_i - C_i$$

$$\pi_i = \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - C_i$$

$$\boxed{\pi_i = \frac{a^2}{(n+1)^2 b} - C_1}$$

3. if $n \rightarrow \alpha$, it will make $q_i = \frac{a}{(n+1)b} \longrightarrow$ nearly zero and each firm will sell at $q$ nearly zero unit #

; it will make $A = nq_i \longrightarrow$ nearly zero. Q of every firms combinded will be nearly $\rightarrow \alpha$ unit #

; it will make $P = \frac{a}{n+1} \longrightarrow$ nearly zero. When supply increase, price will decrease $\longrightarrow$ nearly zero #

; it will make $\pi_i = \frac{a^2}{(n+1)^2 b} - C_i \rightarrow -C_i$. Each firm will lose the profit #

if $n = 1$ ; it will make $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$. Since $Q = \frac{a}{2b} < Q = \frac{na}{(n-1)b}$, monopoly will sell less quantity #

; it will make $A = nq_i = Q$. Since $n = 1$, the firm will be a monopoly #

; it will make $P = \frac{a}{n+1} = \frac{a}{2}$. Since $P_M = \frac{a}{2} > P = \frac{a}{n+1}$, Monopoly will set higher price #

; it will make $\pi_i = \frac{a^2}{(n+1)^2 b} - C_i = \frac{a^2}{4b} - C_i$, which higher than $\pi_i = \frac{a^2}{(n+1)^2 b}$. As a result, the monopoly will get higher profit.#