Solution: Quiz 4

1. Find the solution set for
$$5|x| \ < \ \big|\ 3x+5-|3x|\ \big|.$$

Solution: Two approaches are given here.

Approach 1:

$$\begin{aligned} 5|x| &< \big|\ 3x+5-|3x|\ \big| \\ (5x)^2-\big(\ 3x+5-|3x|\ \big)^2 &< 0 \\ (5x-[3x+5-|3x|])(5x+[3x+5-|3x|]) &< 0 \\ (2x-5+|3x|)(8x+5-|3x|) &< 0 \end{aligned}$$

Consider 2 cases for $|x|=\begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$. Note: $|3x|=3|x|$.

Case1: $x<0$ which implies $|x|=-x$

$$(2x-5+|3x|)(8x+5-|3x|)=(2x-5-3x)(8x+5+3x)=(-x-5)(11x+5)<0$$

$\Rightarrow (x+5)(11x+5)>0$

	$x\in(-\infty,-5)$	$x\in(-5,-\frac{5}{11})$	$x\in(-5/11,\infty)$
$x+5$	$-$	$+$	$+$
$11x+5$	$-$	$-$	$+$
$(x+5)(11x+5)$	$\boxed{+}$	$-$	$\boxed{+}$

From the table, $x\in(-\infty,-5)\cup(-5/11,\infty)$ and the solution set in this case is
$\boxed{\{(-\infty,-5)\cup(-5/11,\infty)\}\cap(-\infty,0)=(-\infty,-5)\cup(-5/11,0)}$.

Case2: $x\geq 0$ which implies $|x|=x$

$$(2x-5+|3x|)(8x+5-|3x|)=(2x-5+3x)(8x+5-3x)=(5x-5)(5x+5)<0 \Rightarrow\ (x-1)(x+1)<0$$

$\Rightarrow \quad (x+1)(x-1)<0$

	$x\in(-\infty,-1)$	$x\in(-1,1)$	$x\in(1,\infty)$
$x+1$	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
$(x+1)(x-1)$	$+$	$\boxed{-}$	$+$

From the table, $x\in(-1,1)$ and the solution set in this case is $\boxed{(-1,1)\cap[0,\infty)=[0,1)}$.

From cases 1 and 2, the solution set is

$$\{(-\infty,-5)\cup(-5/11,0)\}\cup[0,1)=(-\infty,-5)\cup(-5/11,1)$$

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Approach 2:

Consider 2 cases for $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$.

Case1: $x < 0$ which implies $|x| = -x$. Note: $|3x| = 3|x|$.

$$\begin{aligned} 5|x| &< \left| \, 3x + 5 - |3x| \, \right| \\ -5x &< \left| \, 3x + 5 + 3x \, \right| \\ -5x &< \left| \, 6x + 5 \, \right| \end{aligned}$$

Consider 2 sub-cases:

1.1 $x < -5/6$ or $|6x + 5| = -(6x + 5)$

$$-5x < -(6x + 5) \quad \Leftrightarrow \quad x < -5 \Rightarrow \quad x \in (-\infty, -5) \cap (-\infty, -5/6) = (-\infty, -5)$$

1.2 $x \geq -5/6$ or $|6x + 5| = 6x + 5$

$$-5x < 6x+5 \quad \Leftrightarrow \; -5 < 11x \Rightarrow \; x > -5/11 \; \Rightarrow x \in (-5/11, \infty) \cap [-5/6, \infty) = (-5/11, \infty)$$

From subcases 1.1 and 1.2, case 1 has solution set:
$\boxed{\{(-\infty, -5) \cup (-5/11, \infty)\} \cap (-\infty, 0) = (-\infty, -5) \cup (-5/11, 0)}$

Case 2: $x \geq 0$ which implies $|x| = x$. Note: $|3x| = 3|x|$.

$$\begin{aligned} 5|x| &< \left| \, 3x + 5 - |3x| \, \right| \\ 5x &< \left| \, 3x + 5 - 3x \, \right| \\ 5x &< \left| \, 5 \, \right| = 5 \\ x &< 1 \end{aligned}$$

Hence, the solution set for case 2 is: $\boxed{(-\infty, 1) \cap [0, \infty) = [0, 1)}$

From cases 1 and 2, the solution set is:

$$\{(-\infty, -5) \cup (-5/11, 0)\} \cup [0, 1) = (-\infty, -5) \cup (-5/11, 1)$$

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