

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^5 (a + bx_i) = 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$

b. $\sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

c. $\sum_{i=1}^{10} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$
 $= 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 = 385$

d. $\sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = [2(1)+(2)] + [2(1)+(3)] + [2(2)+(2)] + [2(2)+(3)]$
 $= 4 + 5 + 6 + 7 = 22$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$$\int_{-2}^4 f(x) = 1 \rightarrow 8b = 1 \rightarrow b = \frac{1}{8} = 0.125$$

b. Find the answer for $P(X \leq 2)$

$$1 - P(X > 2) = 1 - P(X=3) - P(X=4) = 1 - \frac{1}{16} - \frac{1}{32} = \frac{32-2-1}{32} = \frac{29}{32}$$

c. Find the answer for $P(-2 \leq X \leq 3)$

$$1 - P(X=4) = 1 - \frac{1}{32} = \frac{31}{32}$$

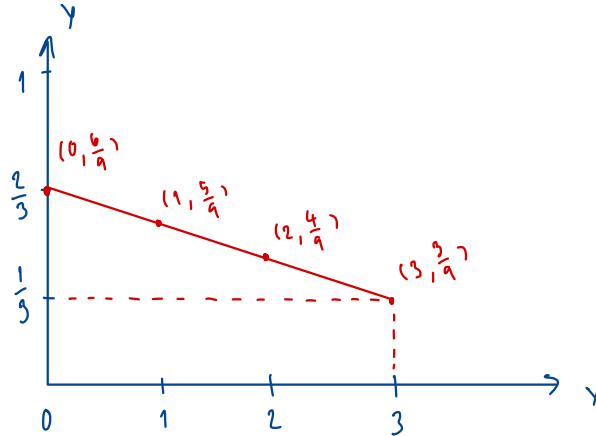
d. Find the answer for $P(X \geq 1)$

$$P(X=1) + P(X=2) + P(X=3) + P(X=4) = \frac{1}{4} + \frac{5}{16} + \frac{1}{16} + \frac{1}{32} = \frac{8+6+2+1}{32} = \frac{17}{32}$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\int_1^3 -\frac{1}{9}x + \frac{6}{9} dx$$

$$-\frac{1}{9} \frac{x^2}{2} + \frac{6}{9}x \Big|_1^3 \rightarrow \begin{matrix} 3 \rightarrow -\frac{1}{2} + 2 = \frac{3}{2} \\ 1 \rightarrow -\frac{1}{18} + \frac{6}{9} = \frac{11}{18} \end{matrix} \quad \Bigg/ \quad \frac{3}{2} - \frac{11}{18} = \frac{16}{18}$$

- c. Find the answer for $P(X \geq 2)$

$$\int_2^3 -\frac{1}{9}x + \frac{6}{9} dx$$

$$-\frac{1}{9} \frac{x^2}{2} + \frac{6}{9}x \Big|_2^3 \rightarrow \begin{matrix} 3 \rightarrow -\frac{1}{2} + 2 = \frac{3}{2} \\ 2 \rightarrow -\frac{2}{9} + \frac{12}{9} = \frac{10}{9} \end{matrix} \quad \Bigg/ \quad \frac{3}{2} - \frac{10}{9} = \frac{7}{18}$$

- d. Find the expected value of X

$$\int_0^3 x f(x) dx$$

$$\int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9}\right) dx$$

$$\int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x dx$$

$$-\frac{1}{9} \frac{x^3}{3} + \frac{6}{9} \frac{x^2}{2} \Big|_0^3 \rightarrow \begin{matrix} 3 \rightarrow -1 + 3 = 2 \\ 0 \rightarrow 0 \end{matrix} \quad \Bigg/ \quad 2 - 0 = 2$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

a. Construct the joint probability distribution function (PDF) table of X and Y

$Y \backslash X$	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

b. Find the marginal probability distribution function (PDF) of X

$$P(X = 1, 2, 3, 4, 5) = \frac{1}{6}$$

c. Find the marginal probability distribution function (PDF) of Y

$$P(Y = 0) = \frac{1}{2}$$

$$P(Y = 1) = \frac{1}{2}$$

d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X = x_i | Y = 1) = \frac{P(X = x_i, Y = 1)}{P(Y = 1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

e. Find the expected value of X given Y is equal to 1

$$E(X = x_i | Y = 1) = \sum_{i=1}^6 x_i P(X = x_i | Y = 1) = \sum_{i=1}^6 x_i \frac{P(X = x_i, Y = 1)}{P(Y = 1)} = \frac{1(\frac{1}{12}) + 2(\frac{1}{12}) + \dots + 6(\frac{1}{12})}{\frac{1}{2}} = \frac{\frac{1}{12} + \frac{2}{12} + \frac{3}{12} + \frac{4}{12} + \frac{5}{12} + \frac{6}{12}}{\frac{1}{2}}$$

f. Find the variance of X given Y is equal to 1

$$\text{Var}(X) = \sum_{i=1}^6 (x_i - E(X))^2 \cdot f(x_i | Y = 1)$$

$$= (1 - 3.5)^2 \frac{1}{6} + (2 - 3.5)^2 \frac{1}{6} + (3 - 3.5)^2 \frac{1}{6} + (4 - 3.5)^2 \frac{1}{6} + (5 - 3.5)^2 \frac{1}{6} + (6 - 3.5)^2 \frac{1}{6}$$

$$= 1.04167 + 0.375 + 0.04167 + 0.04167 + 0.775 + 1.04167$$

$$= 2.91666$$

$$= \frac{21}{12} \cdot \frac{1}{2} = \frac{21}{6} = 3.5$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

\bar{X} is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{X_1 + X_2 + X_3}{3}\right)$$

$$= \frac{1}{3} E(X_1 + X_2 + X_3)$$

$$= \frac{1}{3} (3\mu_X)$$

$$= \mu_X$$

$$\text{Var}(X) = \text{Var}\left(\frac{1}{3}(X_1 + X_2 + X_3)\right)$$

$$= \frac{1}{9} \left(\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) \right.$$

$$+ 2\text{Cov}(X_1, X_2) + 2\text{Cov}(X_2, X_3)$$

$$+ 2\text{Cov}(X_1, X_3) \left. \right)$$

$$= \frac{1}{9} \left(\sigma^2 + \sigma^2 + \sigma^2 + 2\left(\frac{1}{4}\sigma^2\right)(3) \right)$$

$$= \frac{1}{9} \left(3\sigma^2 + \frac{3}{2}\sigma^2 \right)$$

$$= \frac{1}{9} \left(\frac{9}{2}\sigma^2 \right)$$

$$= \frac{1}{2}\sigma^2$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . \bar{X} is estimator used to estimate

mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

- a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left(\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right) = \frac{1}{4} E(X_1 + X_2 + X_3 + X_4) = \frac{4\mu}{4} = \mu$$

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right) = \frac{1}{16} \text{Var}(X_1 + X_2 + X_3 + X_4) = \frac{4\sigma^2}{16} = \frac{\sigma^2}{4}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that \tilde{X} is an unbiased estimator of μ

$$\begin{aligned}
 E(\tilde{X}) &= E\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right) \\
 &= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4) \\
 &= \mu\left(\frac{1}{8} + \frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right) \\
 &= \mu\left(\frac{1}{8} + \frac{2}{8} + \frac{1}{8} + \frac{4}{8}\right) \\
 &= \mu
 \end{aligned}$$

So, \tilde{X} is an unbiased estimator of μ

- c. Between \bar{X} and \tilde{X} , which one is the better estimator for μ ? Why?

$$\begin{aligned}
 \text{Var}(\bar{X}) &= \frac{\sigma^2}{4} \\
 \text{Var}(\tilde{X}) &= \frac{1}{64}\text{Var}(X_1) + \frac{1}{16}\text{Var}(X_2) + \frac{1}{64}\text{Var}(X_3) + \frac{1}{4}\text{Var}(X_4) \\
 &= \sigma^2\left(\frac{1}{64} + \frac{1}{16} + \frac{1}{64} + \frac{1}{4}\right) = \sigma^2\left(\frac{1}{64} + \frac{4}{64} + \frac{1}{64} + \frac{16}{64}\right) = \frac{22}{64}\sigma^2
 \end{aligned}$$

The estimator with lower Variance is considered to be the better estimator

Therefore, \bar{X} is the better estimator