Limited Dependent Variable Models

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Part 2

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Multinomial models

Probabilities and MEs

- The outcome, y_i, for individual i is one of m alternatives.
- Set $y_j = j$ if the outcome is the jth althernative, $j = 1, 2, ..., m$
- The probability that the outcome for individual i is alternative j, conditional on the regressors $\mathbf{x}_j$, is
  $P_{ij} = P(y_i = j) = F_j(x_i, \beta)$, j=1,...,m, i = 1,...,n
  - where functional form, $F_j(\cdot)$, corresponds to multinomial models, e.g. multinomial logit model
- Only m-1 of the probabilities can be freely specified because probabilities sum to 1. Multinomial models therefore require a normalization

Multinomial models

Probabilities and MEs

- The parameters of multinomial models are generally not directly interpretable.
- We need marginal effects (MEs). For individual i, the ME of a change in the kth regressor on the probability that alternative j is the outcome is
$ME_{ijk} = \frac{\partial P(y_i=j)}{\partial x_{ik}} = \frac{\partial F_j(x_i,\beta)}{\partial x_{ik}}$
- For each regressor, there will be *m* MEs corresponding to the m probabilities, and these *m* MEs sum to zero because probabilities sum to one.

Additive random-utility model

Unordered multinomial outcomes

- For individual i and alternative j, we suppose that utility U_{ij} is the sum of deterministic component, V_{ij}, that depends on regressors and unknown parameters, and an unobserved randome component ε_{ij}:
  $U_{ij} = V_{ij} + \varepsilon_{ij}$
- This is called an additive random-utility model (ARUM).
- We observed the outcome $y_i = j$ if alternative j has the highest untility of the alternatives.
  $P(y_i = j) = P(U_{ij} > U_{ik})$, for all k
  $= P(\varepsilon_{ik} - \varepsilon_{ij} \leq V_{ij} - V_{ik})$, for all k
- Different assumptions about the joint distribution of $\varepsilon_{i1}, \ldots, \varepsilon_{im}$ lead to different multinomial models with different specifications for $F_j(\mathbf{x}_i, \beta)$

Multinomial logit model

- Datasets have only case-specific variables.
- Multinomial logit model (MNL): $P_{ij} = \frac{exp(x_i\beta_j)}{\sum_{l=1}^{m} exp(x_i\beta_l)}$, j = 1,..., m
  - $\sum_{j=1}^{m} P_{ij} = 1$
  - To ensure model identification, β_j is set to zero for one of the categories, and coefficients are then interpreted with respect to the based category
- For simplicity, we set the base category to be the first category. Then, the MNL model:
  $P(y_i = j | y_i = 1) = \frac{exp(x_i\beta_j)}{1+\sum_{l=2}^{m} exp(x_i\beta_l)}$

Multinomial logit model

- Thus, $\hat{\beta}_j$ can be viewed as parameters of a binary logit model between alternative j and alternative 1.
- So, a positive coefficient means that as the regressor increases, we are more likely to choose alternative j than alternative 1
- We can transform to odds ratios or relative-risk ratio of choosing alternative j rather than alternative 1 is $\frac{P(y_i=j)}{P(y_i=1)} = exp(\mathbf{x}_i\beta_j)$
- So, $e^{\beta_{ir}}$ gives the proportionate change in the relative risk of choosing alternative j rather than alternative 1 when x_{ir} changes by one unit.

Multinomial logit model

- For the MNL model, the MEs can be shown to be $\frac{\partial P_{ij}}{\partial x_i} = P_{ij} \cdot (\beta_j - \bar{\beta}_i)$
  - where $\bar{\beta}_i = \sum_l P_{il}\beta_l$ is a probability weighted average of the β_l
- The MEs vary with the point of evaluation, x_i, because P_{ij} varies with x_i
- The signs of the β_js do not give the signs of the MEs.
- For a variable x, the ME is positive if $\beta_j > \bar{\beta}_i$

Ordered outcomes

- A latent variable crosses progressively higher thresholds.
- For an m-alternative ordered model, we define
  $y_i = j$ if $\alpha_{j-1} < y_i^* \leq \alpha_j$, $j = 1,\ldots,m$
  where $\alpha_0 = -\infty$ and $\alpha_m = \infty$
- Then, $P(y_i = j) = P(\alpha_{j-1} < x_i\beta + u_i \leq \alpha_j)$
  $= P(\alpha_{j-1} - x_i\beta < u_i \leq \alpha_j - x_i\beta)$
  $= F(\alpha_j - x_i\beta) - F(\alpha_{j-1} - x_i\beta)$
- The m-1 threshold parameters, $\alpha_1, \ldots, \alpha_{m-1}$, are obtained by maximizing the log likelihood with $P(y_i = j)$

Ordered outcomes

- Ordered logit model: u is logistically distributed with $F(z) = e^z/(1 + e^z)$
- Ordered probit model: u is standard normally distributed with $F(\cdot) = \Phi(\cdot)$, the standard normal cdf
- The sign of β can be interpreted as determining whether the latent variable increases with the regressor.
- If $\beta_j > 0$, then an increase in x_{ij} will decrease the probability of being in the lowest category and increase the probability of being in the highest category.
- ME: $\frac{\partial P(y_i=j)}{\partial x_{ri}} = [F'(\alpha_j - x_i\beta) - F'(\alpha_{j-1} - x_i\beta)]\beta_r$