Oct 22 & 29 2024

EE211 Section 1

Costs

Costs in the Short Run

Term	Definition	Mathematical Description
Fixed costs	Costs that do not vary with the quantity of output produced	FC
Variable costs	Costs that vary with the quantity of output produced	VC
Total cost	The market value of all the inputs that a firm uses in production	$TC = FC + VC$
Average fixed cost	Fixed cost divided by the quantity of output	$AFC = FC/Q$
Average variable cost	Variable cost divided by the quantity of output	$AVC = VC/Q$
Average total cost	Total cost divided by the quantity of output	$ATC = TC/Q$
Marginal cost	The increase in total cost that arises from an extra unit of production	$MC = \Delta TC/\Delta Q$

Costs in the Long Run

Isocost line

- A set of input bundles each of which costs the same amount

Total cost C of producing any particular output is given by the sum of the firm's labor cost wL and its capital cost rK:

$$C = wL + rK$$

$$C = wL + rK$$

If we rewrite the total cost equation as an equation for a straight line, we get

$$K = \frac{C}{r} - \left(\frac{w}{r}\right)L$$

The isocost line has a slope of $\frac{\Delta K}{\Delta L} = -\frac{w}{r}$

It tells us that if the firm gave up a unit of labor (and recovered w dollars in cost) to buy $\frac{w}{r}$ units of capital at a cost of r dollars per unit, its total cost of production would remain the same.

E.g. if the wage rate were \$10 and the rental cost of capital \$5, the firm could replace one unit of labor with two units of capital with no change in total cost.

The isocost line has a slope of $\frac{\Delta K}{\Delta L} = -\frac{w}{r}$

Choosing the optimal input combination

Let us begin with the case of a firm that wants to maximize output from a given level of expenditure

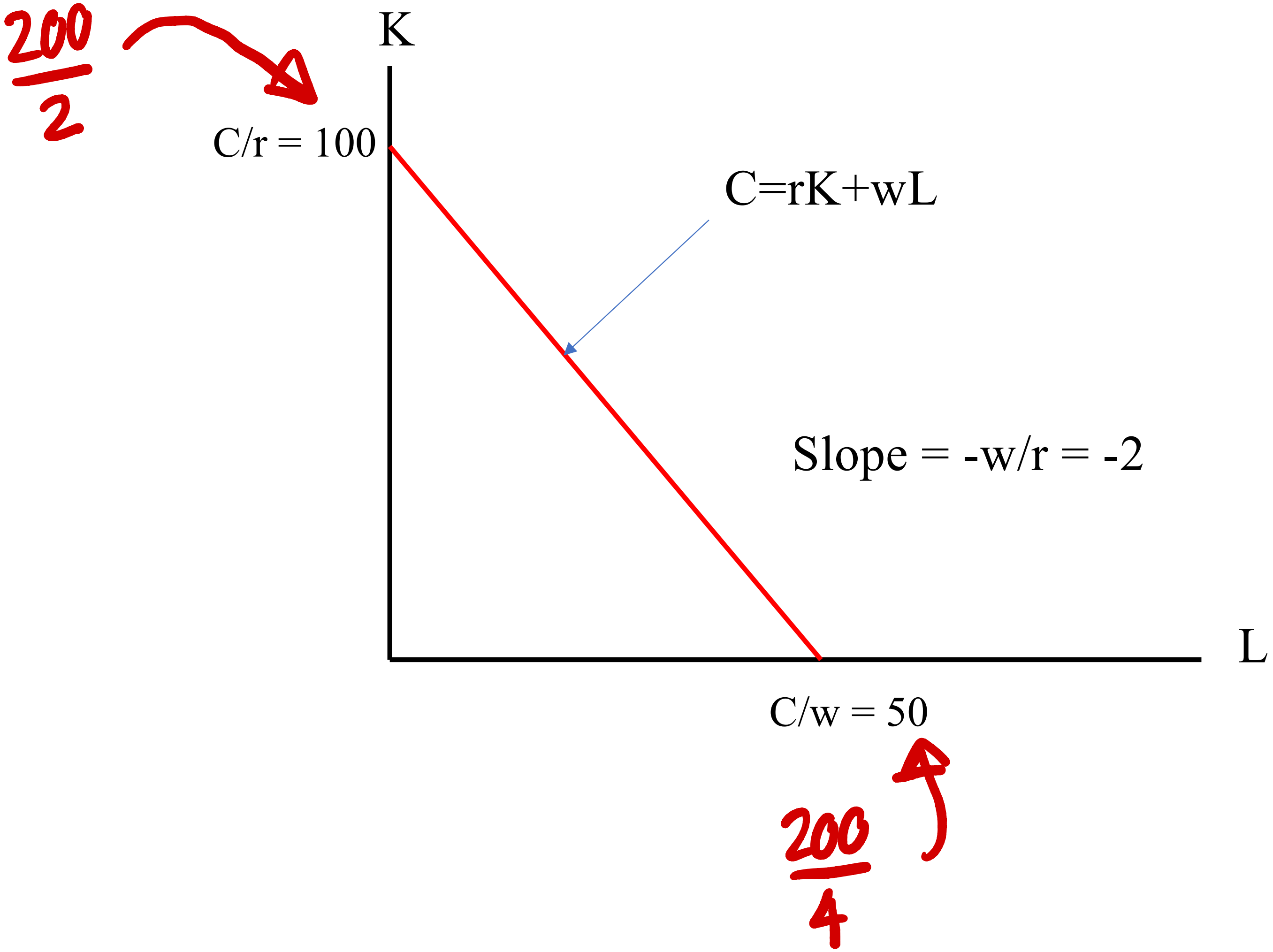
Suppose it uses only two inputs, capital (K) and labor (L), whose prices measured in dollars per unit of input per day, are $r=2$ and $w=4$, respectively.

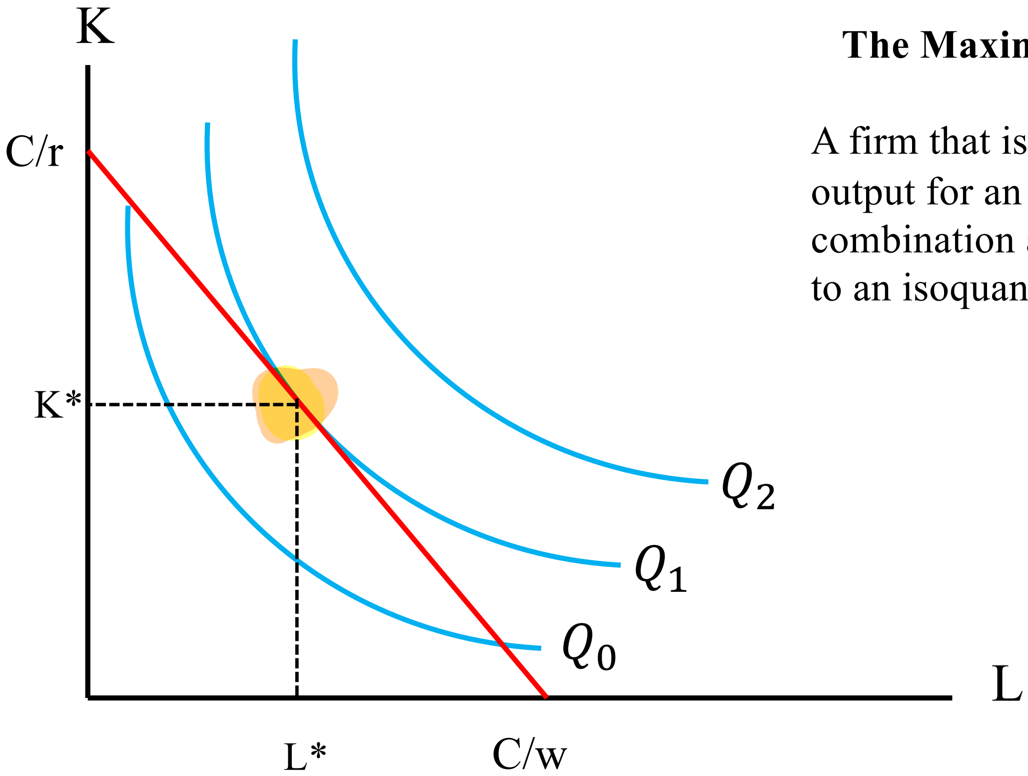
What different combinations of inputs can this firm purchase for a total expenditure of $C = \$200/$ day?

r=2

w=4

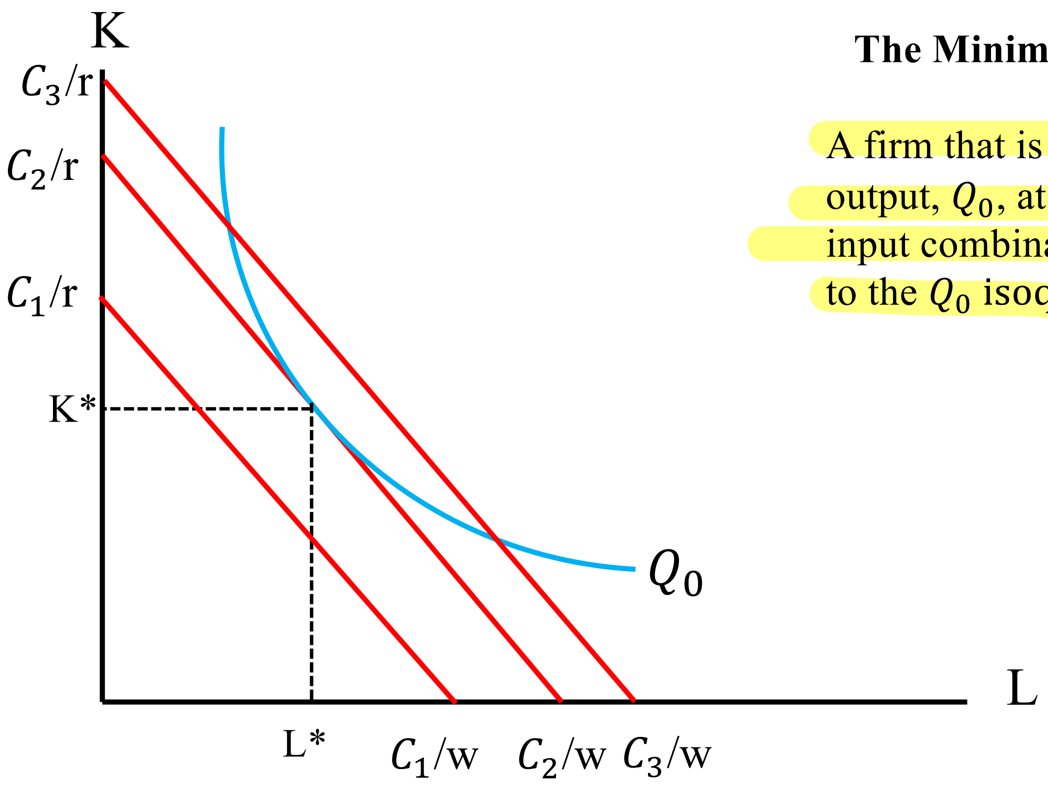
Total expenditure=200





The Maximum output for a Given Expenditure

A firm that is trying to produce the largest possible output for an expenditure of C will select the input combination at which the isocost line for C is tangent to an isoquant

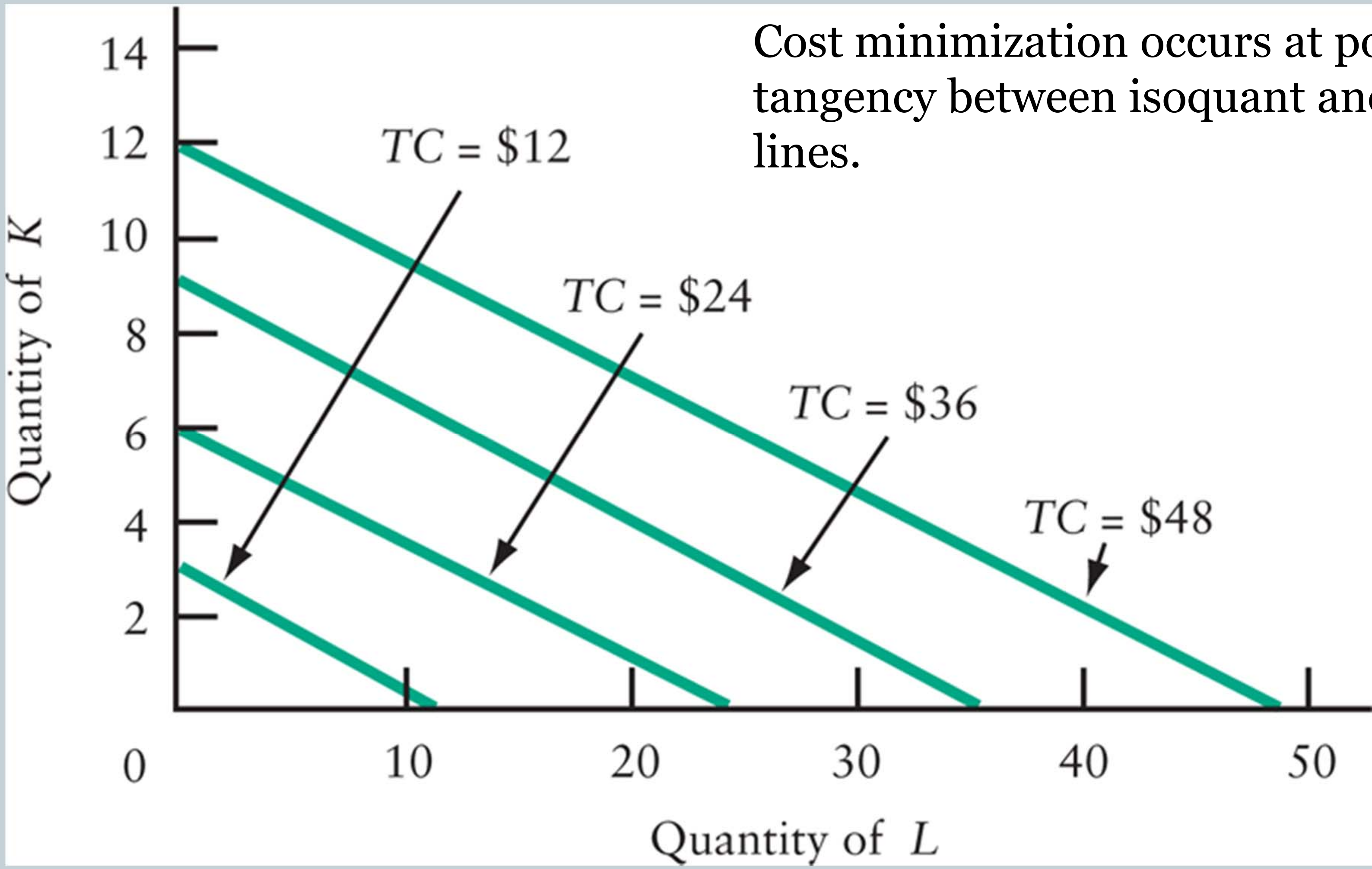


The Minimum cost for a Given Level of Output

A firm that is trying to produce a given level of output, Q_0, at the lowest possible cost will select the input combination at which an isocost line is tangent to the Q_0 isoquant.

Isocost Lines

Cost minimization occurs at points of tangency between isoquant and isocost lines.



- The slope of isoquant at any point is equal to $\frac{-MP_L}{MP_K}$, the negative of the ratio of the marginal product of labor to the marginal product of capital at that point. (The absolute value of this ratio is called the marginal rate of technical substitution)

- Combining with the result that minimum cost occurs at a point of tangency with the isocost line (whose slope is $-w/r$), it follows that

$$\frac{MP_{L*}}{MP_{K*}} = \frac{w}{r}$$

Where K* and L* again denote the minimum cost values of K and L. Cross-multiplying, we have

$$\frac{MP_{L*}}{w} = \frac{MP_{K*}}{r}$$

$$\frac{MP_L}{P_L} = \frac{MP_K}{P_K}$$

$$\frac{MP_{L*}}{w} = \frac{MP_{K*}}{r}$$

MP_{L*} is simply the extra output obtained from an extra unit of L at the cost-minimizing point.

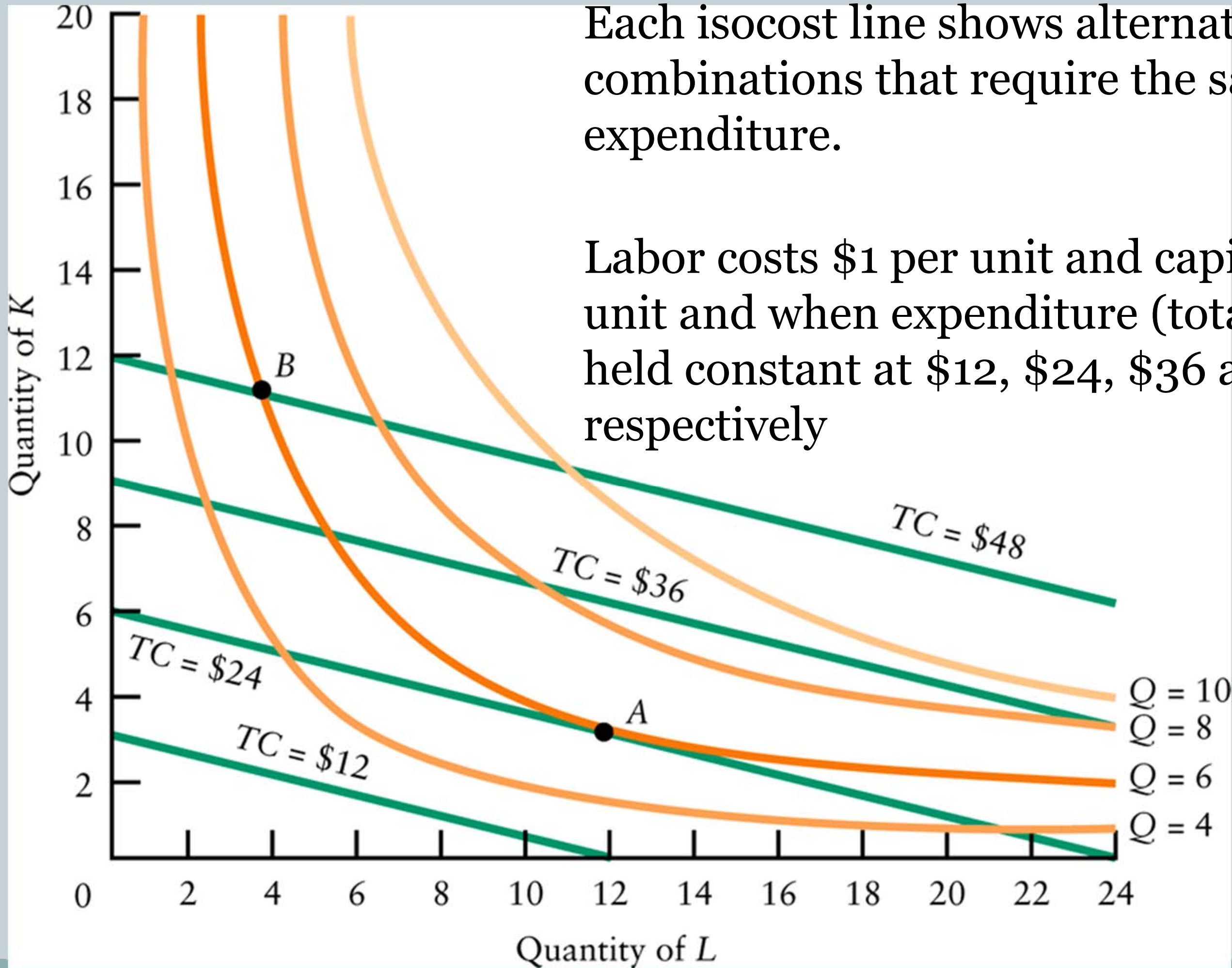
w is the cost, in dollars, of an extra unit of L.

The ratio $\frac{MP_{L*}}{w}$ is the extra output we get from the last dollar spent on L.

The ratio $\frac{MP_{K*}}{r}$ is the extra output we get from the last dollar spent on K.

When costs are at a minimum, the extra output we get from the last dollar spent on an input must be the same for all inputs.

Cost Minimization



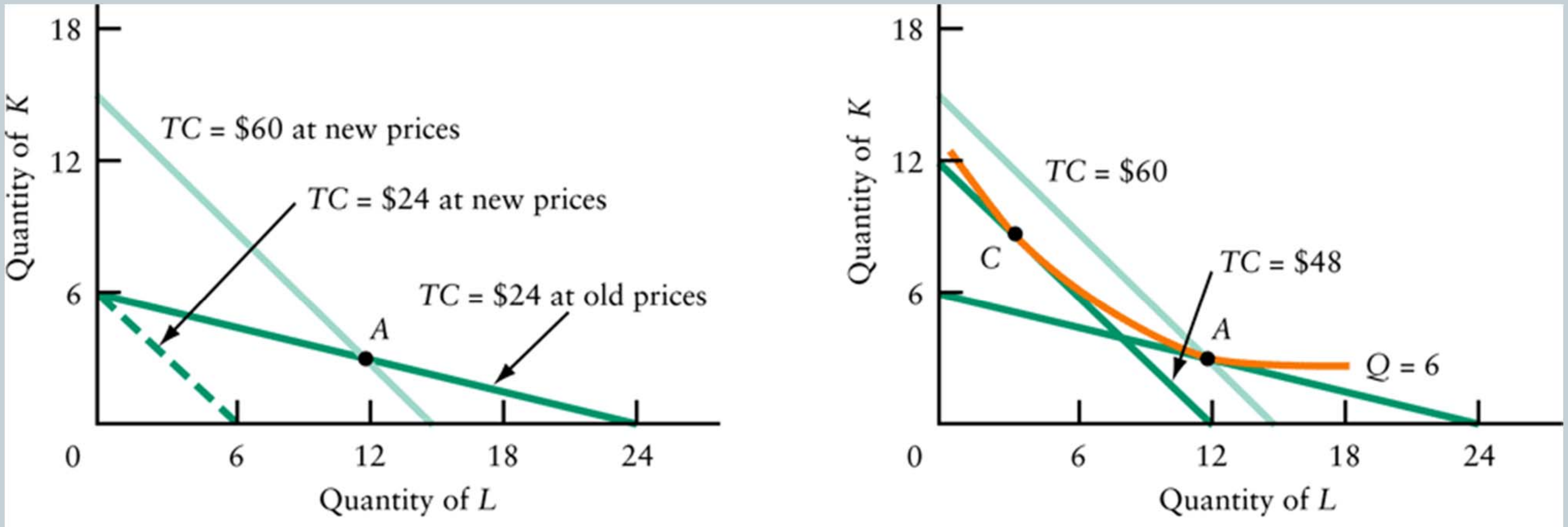
Each isocost line shows alternative factor combinations that require the same expenditure.

Labor costs $1 per unit and capital $4 per unit and when expenditure (total cost) is held constant at $12, $24, $36 and $48 respectively

The principle of substitution

Suppose that with technology unchanged, the price of one factor changes. In particular, suppose that with the price of capital unchanged at $4 per unit, the price of labor rises from $1 to $4 per unit.

The Effects of a Change in Factor Prices on Costs and Factor Proportions



(i) The effect on the isocost line of an increase in the price of labor

(ii) Substitution of capital for labor resulting from an increase in the price of labor

Changes in relative factor prices will cause a partial replacement of factors that have become relatively more expensive by factors that have become relatively cheaper.

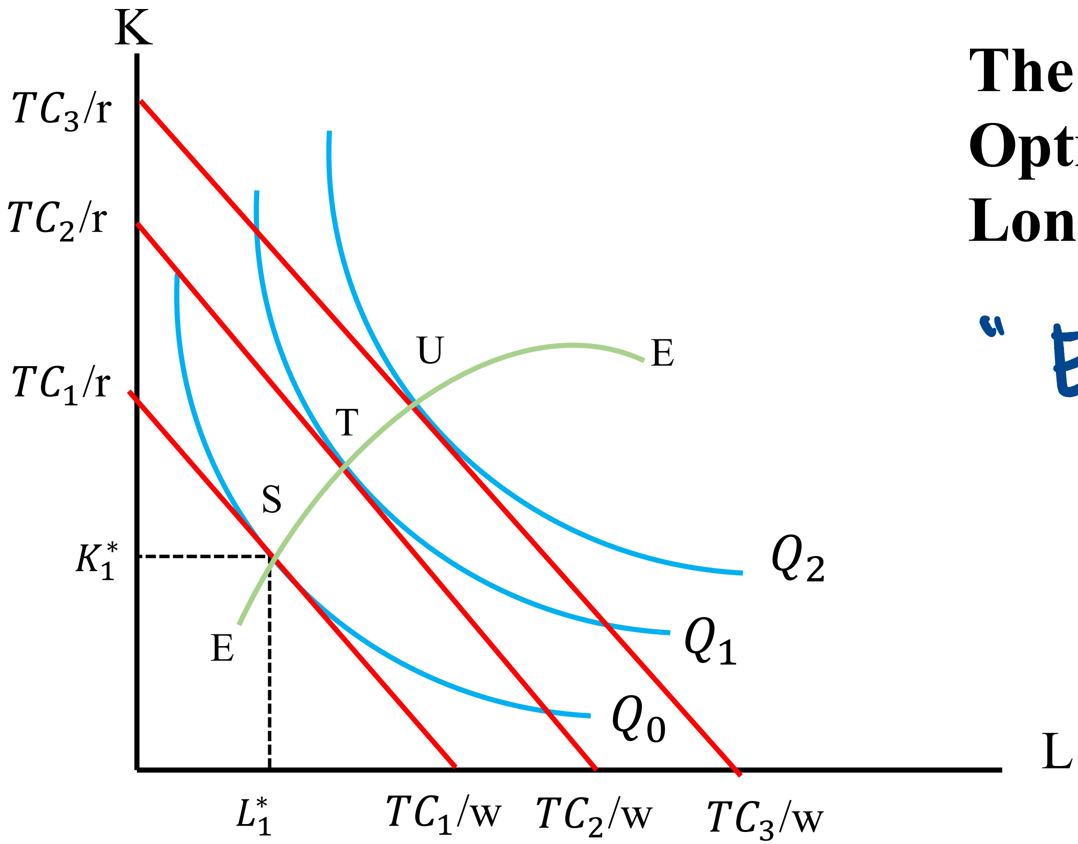
A rise in the price of one factor with all other factor prices held constant will

(1) Shift the cost curves of products that use that factor upward.

(2) Lead to a substitution of factors that are now relatively cheaper for the factor whose price has risen.

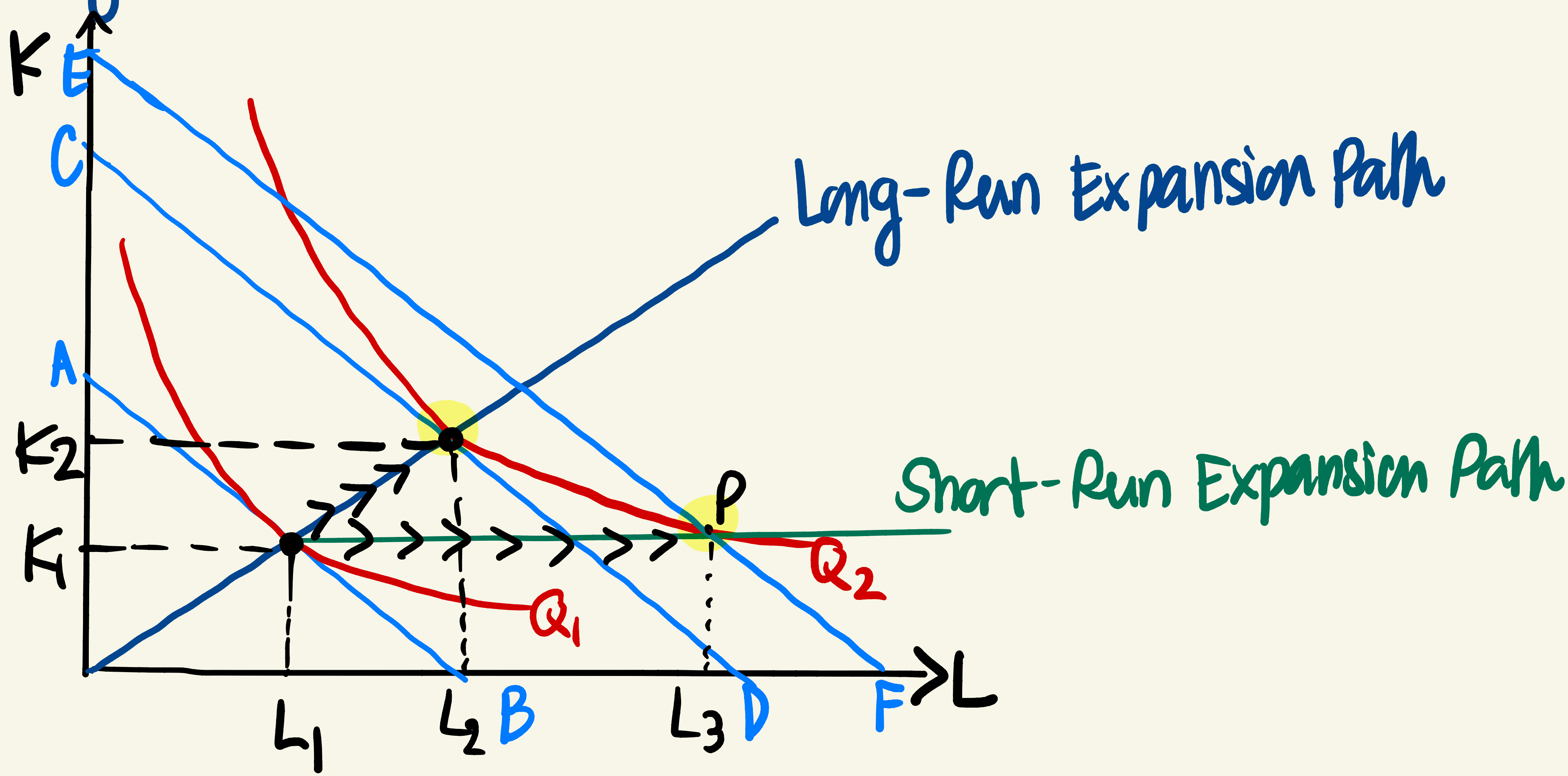
The relationship between Optimal Input Choice and Long-Run Costs

" Expansion path "



The firm can always buy the cost-minimizing input bundle that corresponds to any particular output level and relative input prices.
With fixed input prices r and w, bundles S,T,U, and others along the locus EE represent the least costly ways of producing the corresponding levels of output.

Long-Run versus Short-Run Cost Curves



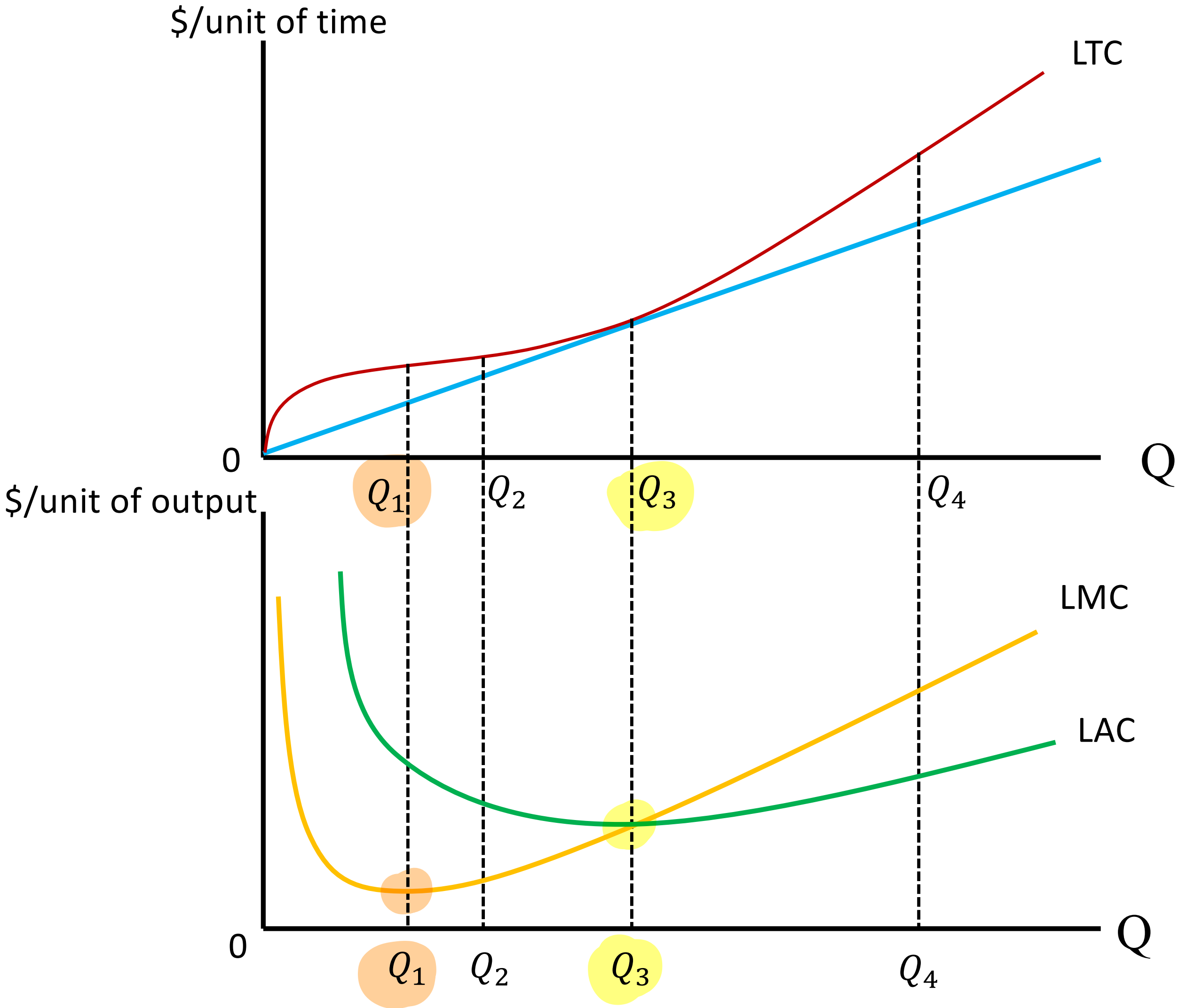
The inflexibility of Short-Run Production

When a firm operates in the short run, its cost of production may not be minimized because of inflexibility in the use of capital inputs.

Output is initially at level Q_1. In the short run, output Q_2 can be produced only by increasing labor from L_1 to L_3 because capital is fixed at K_1.

In the long run, the same output can be produced more cheaply by increasing labor from L_1 to L_2 and capital from K_1 to K_2.

The Long-Run Total, Average, and Marginal Cost Curves



In the long run, the firm always has the option of ceasing operations and ridding itself of all its inputs.

The long-run total cost curve will always pass through the origin.

The long-run average and long-run marginal cost curves are derived from the long-run total cost curve in a manner completely analogous to the short-run case.

The Long-Run Total, Average, and Marginal Cost Curves

- In the long run, the firm always has the option of ceasing operations and ridding itself of all its inputs.
- The long-run total cost curve will always pass through the origin because in the long run the firm can liquidate all of its inputs.
- If the firm elects to produce no output, it need not retain, or pay for, the services of any of its inputs.
- The long-run average and long-run marginal cost curves are derived from the long-run total cost curve in a manner completely analogous to the short-run case.

- Long run marginal cost (LMC) is the slope of the long-run total cost curve

$$LMC = \frac{\Delta LTC}{\Delta Q}$$

LMC is the cost to the firm, in the long run, of expanding its output by 1 unit.

- Long run average cost (LAC) is the ratio of the long-run total cost to output

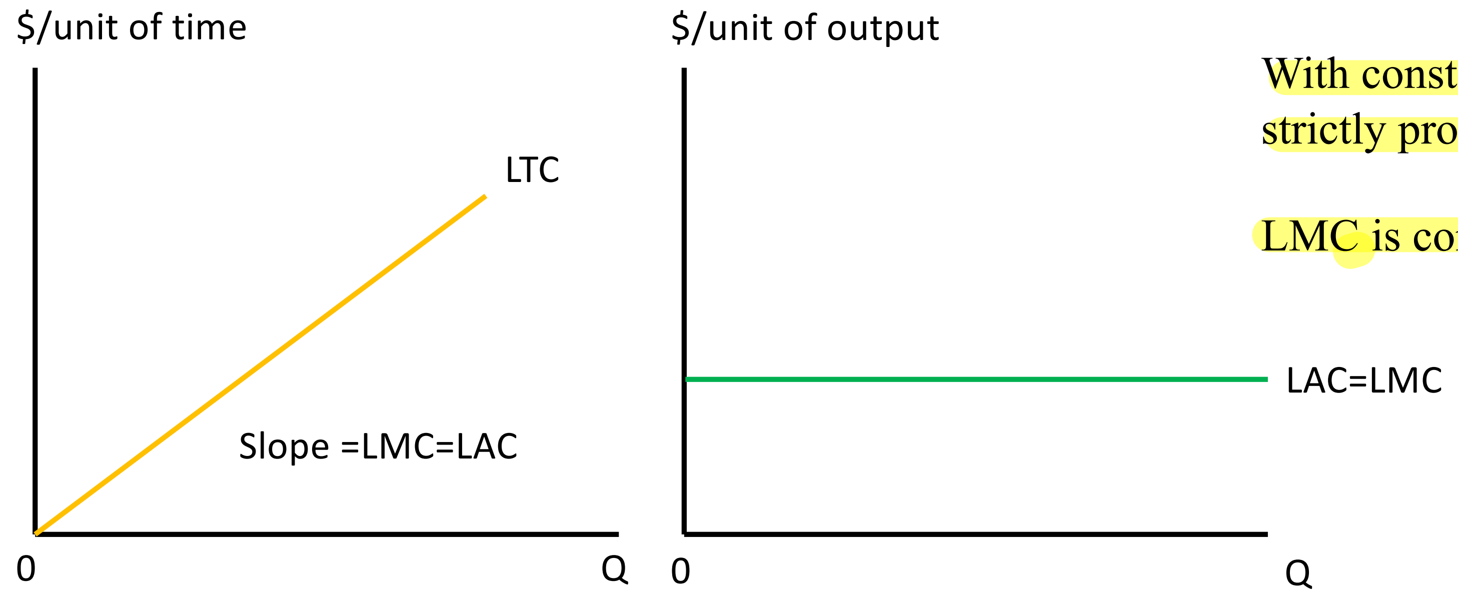
$$LAC = \frac{\Delta LTC}{Q} \quad \frac{LTC}{Q}$$

The slope of the LTC curve is diminishing up to the output level Q_1 and increasing thereafter, which means that the LMC curve takes its minimum value at Q_1.

The slope of LTC and the slope of the rate to LTC are the same at Q_3, which means that LAC and LMC intersect at that level of output.

The traditional average-marginal relationship holds: LAC is declinging whenever LMC lies below it, and rising whenever LMC lies above it

Figure A: The LTC, LMC, and LAC curves with Constant Returns to Scale

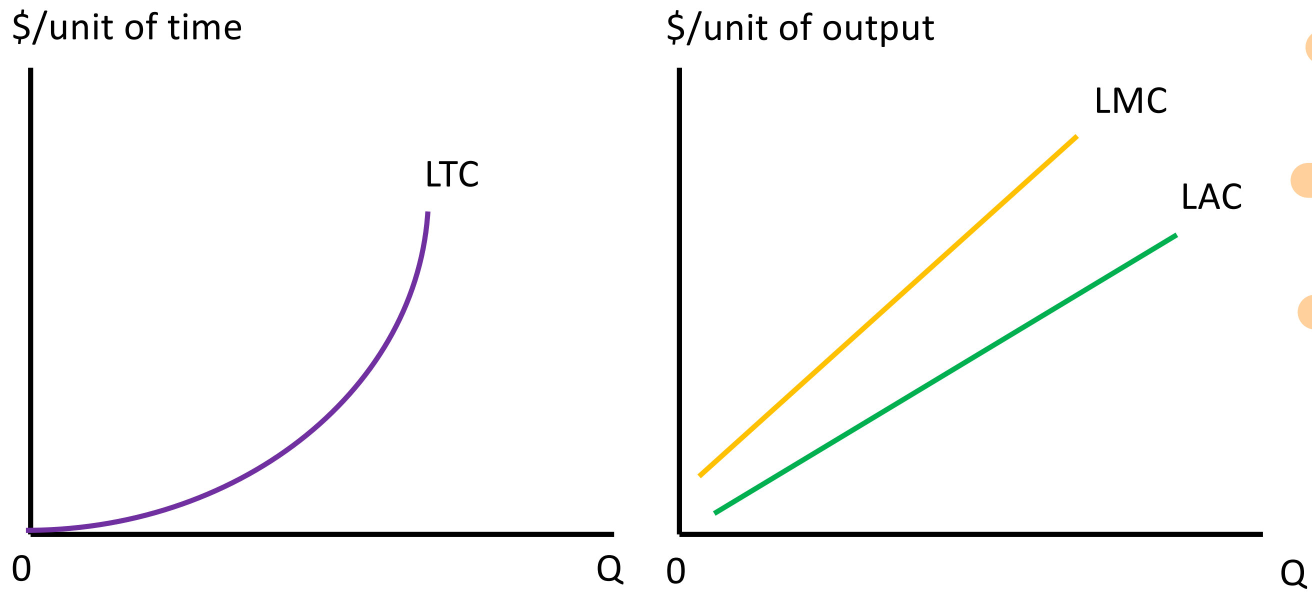


With constant returns, long-run total cost is strictly proportional to output.

LMC is constant and equal to LAC

For constant returns to scale production function, doubling output exactly doubles costs. The LTC curve for a production function with constant returns to scale is a straight line through the origin. Because the slope of LTC is constant, the associated LMC curve is a horizontal line, and is exactly the same as the LAC curve.

Figure B: The LTC, LMC, and LAC curves for a production process with Decreasing Returns to Scale



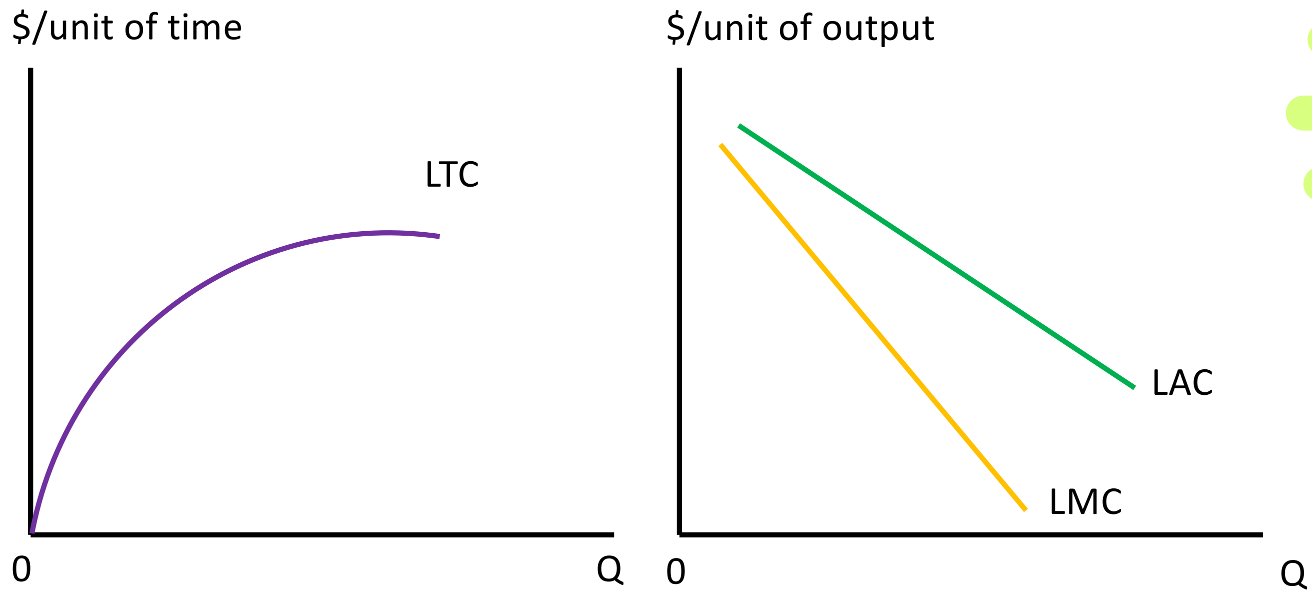
Under decreasing returns, output grows less than in proportion to the growth in inputs, which means that total cost grows more than in proportion to growth in output.

When the proportion function has decreasing returns to scale, a given proportional increase in output requires a greater proportional increase in all inputs and hence a greater proportional increase in costs.

The general property of the decreasing returns case is that it gives rise to an upward-sloping LTC curve and upward-sloping LAC and LMC curves.

LMC exceeds LAC ensures that LAC must rise with output.

Figure C: The LTC, LMC, and LAC curves for a production process with Increasing Returns to Scale



With increasing returns, the large-scale firm has lower average and marginal costs than the smaller-scale firm.

The case of increasing returns to scale- output grows more than in proportion to the increase in inputs. In consequence, long-run total cost rises less than in proportion to increases in output. The distinguishing feature of the LAC and LMC curves under increasing returns to scale is not the linear form shown in particular example, but in fact that they are downward sloping.

- The production processes whose long-run cost curves are pictured in figures A,B, and C are “pure cases,” exhibiting constant, decreasing, and increasing returns to scale, respectively, over their entire ranges of output.
- The degree of returns to scale of a production function need not be the same over the whole range of output.

Long-Run Costs and The Structure of Industry

The Relationship between Long-Run and Short-Run Cost Curves

Long-Run Cost Curves

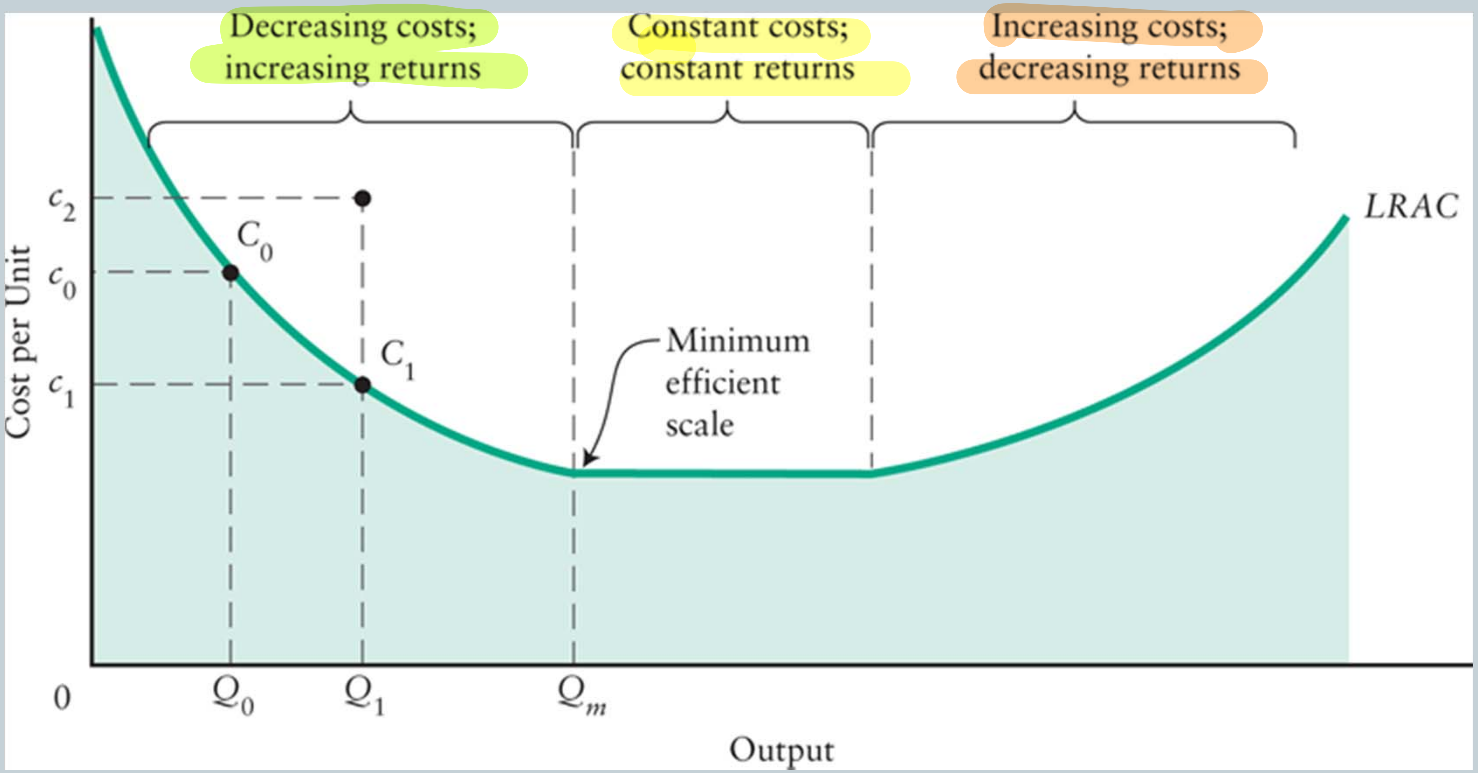
When all factors of production can be varied, consider the <u>least-cost</u> method of producing any level of output.

The long-run average cost (***LRAC***) curve shows the lowest possible cost of producing each level of output when all inputs can be varied.

The ***LRAC*** curve separates unattainable and attainable cost levels, given technology and factor prices.

The ***LRAC*** curve is usually U-shaped.

A "Saucer-Shaped" Long-Run Average Cost Curve



Decreasing costs

- Over the range of output from zero to Q_m
- The firm has falling long-run average costs- an expansion of output permits a reduction of average costs
- When long-run average costs fall as output rises, the firm is said to have **economies of scale**
- The decreasing-cost firm is often said to enjoy long-run **increasing return**
- **Increasing returns to scale- A situation in which output increases more than in proportion to inputs as the scale of a firm's production increases**

Constant Costs

- The firm's long-run average costs fall until output reaches Q_m

- **The firm's minimum efficient scale**

- LRAC reaches its minimum

- The firm would be encountering constant costs over the relevant range of output- the firm's long run average costs do not change as its output changes

- Factor prices are assumed to be fixed, the firm's output must be increasing exactly in proportion to the increase in inputs. (**constant returns**)

Increasing Costs

- When the LRAC curve is rising, a long-run expansion in production is accompanied by a rise in average costs

- If factor prices are constant, the firm's output must be increasing less than in proportion to the increase in inputs and this increasing cost firm is said to encounter long-run **decreasing returns**

- Decreasing returns imply that the firm suffers some **diseconomies of scale**

The Long-Run Average Cost Curve and Returns to Scale

Falling LRAC	=	Increasing returns to scale
Constant LRAC	=	Constant returns to scale
Rising LRAC	=	Decreasing returns to scale
Q_M	=	Minimum efficient scale

Returns To Scale

Increasing returns to scale – output increases more than in proportion to inputs as the scale of a firm's production increases.

Minimum efficient scale – the smallest output at which LRAC reaches its minimum.

Constant returns to scale – output increases in proportion to inputs as the scale of a firm's production increases.

Decreasing returns to scale – output increases less than in proportion to inputs as the scale of a firm's production increases.

(**)

Economies and Diseconomies of Scale

- **Economies of scale**
  - Long-run average total cost falls as the quantity of output increases
- **Constant returns to scale**
  - Long-run average total cost stays the same as the quantity of output changes
- **Diseconomies of scale**
  - Long-run average total cost rises as the quantity of output increases

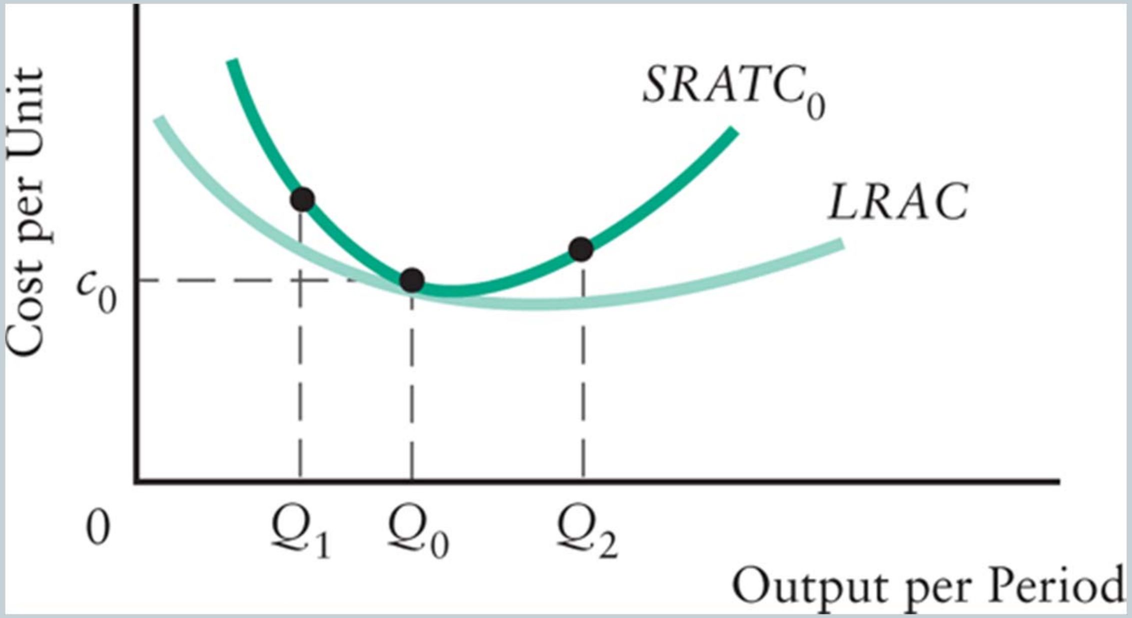
The relationship between LR and SR costs

- The LRAC curve shows the lowest cost of producing any output when all factors are variable

- Each SRATC (Short-Run Average Total Cost) curve shows the lowest cost of producing any output when one or more factors are fixed

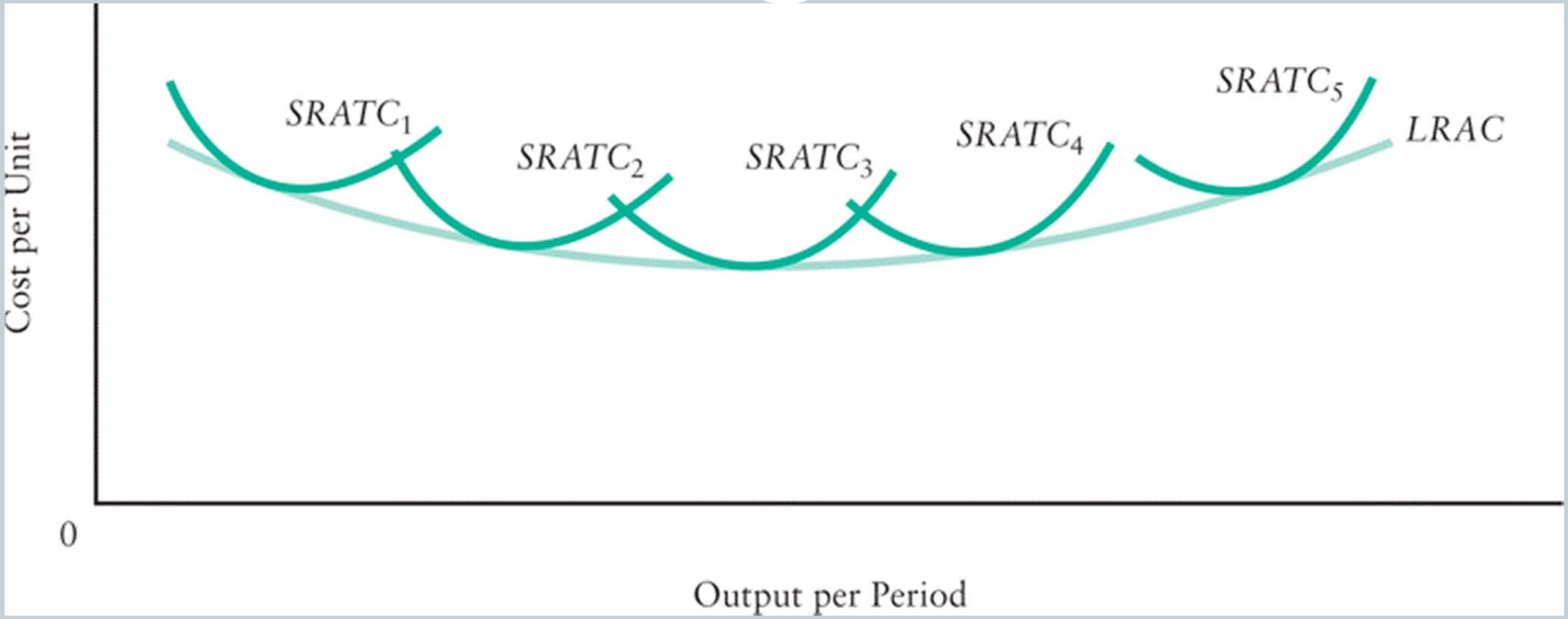
No SR cost curve can fall below the LR cost curve because the LRAC curve represents the lowest attainable cost for each possible output

LRAC and *SRATC* Curves



Each ***SRATC*** curve is tangent to the ***LRAC*** curve at the level of output for which the quantity of the fixed factor is optimal.

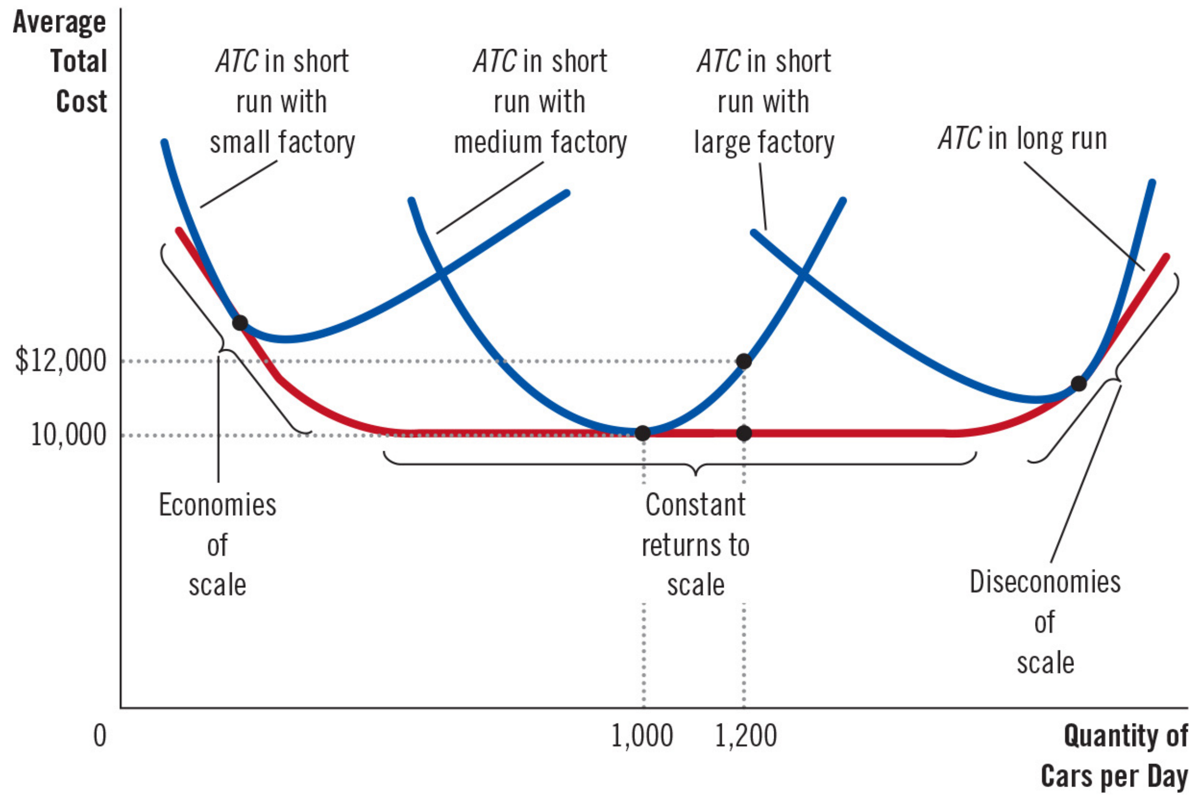
The relationship between the LRAC curve and the SRATC curves



To every point on the LRAC curve, there is an associated SRATC curve tangent at that point. Each short-run curve is drawn for a given plant size, shows how costs vary if output varies (holding constant the size of the plant). The level of output at the tangency between such SRATC curve and the LRAC curve shows the level of output for which the plant size is optimal

Average Total Cost in the Short and Long Runs

- Because fixed costs are variable in the long run, the average-total-cost curve in the short run differs from the average-total-cost curve in the long run.

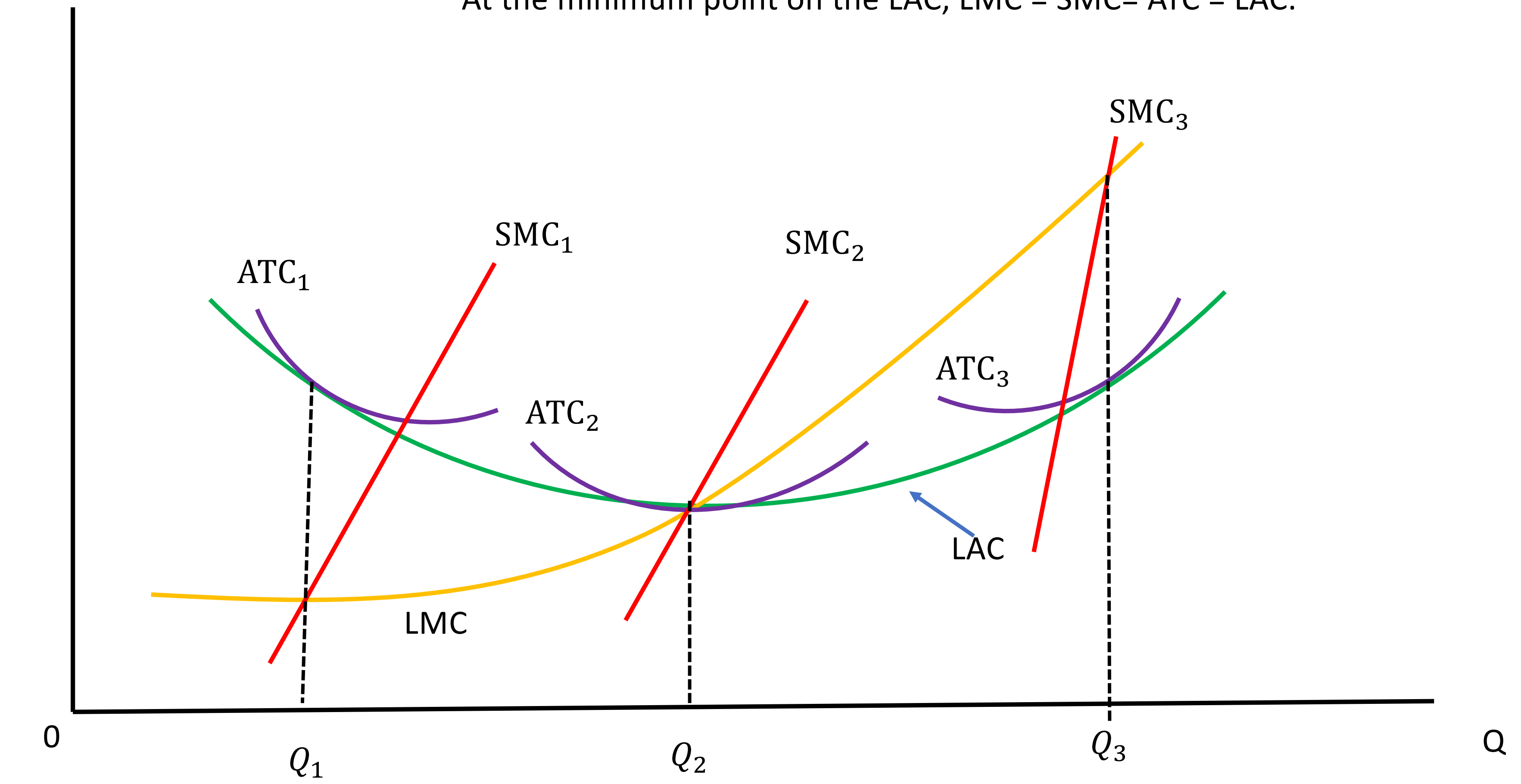


The Relationship between Long-Run and Short-Run Cost Curves

- One way of thinking of the LAC curve is as an envelope of all the short-run average total cost (ATC) curves.
- The output level at which a given ATC is tangent to the LAC, the long run marginal cost (LMC) of producing that level of output is the same as the short-run marginal cost (SMC). Thus $LMC\ (Q_1) = SMC\ (Q_1)$, $LMC\ (Q_2) = SMC\ (Q_2)$ and $LMC\ (Q_3) = SMC\ (Q_3)$.
- Each point along a given ATC curve, except for the tangency point, lies above the corresponding point on the LAC curve.
- At the minimum point on the LAC curve ($Q = Q_2$), the long run and short run marginal and average costs all take exactly the same value.

- Some intuition about ATC-LAC relationship for a given ATC curve is afforded by noting that to the left of the ATC-LAC tangency, the firm has “too much” capital, with the result that its fixed costs are higher than necessary;
- That to the right of the ATC-LAC tangency, the firm has “too little” capital, so that diminishing returns to labor drive its costs up.
- Only at the tangency point does the firm have the optimal quantities of both labor and capital for producing the correponding level of output.

The LAC curve is the outer envelope of the ATC curves.
LMC=SMC at the Q value for which the ATC is tangent to the LAC.
At the minimum point on the LAC, LMC = SMC= ATC = LAC.



Shifts in *LRAC* Curves

- Changes in technology and factor prices cause the long-run cost curve to shift.
- A rise in factor prices shifts the ***LRAC*** curve upward.
- A fall in factor prices or a technological improvement shifts the ***LRAC*** curve downward.

The Very Long Run: Changes in Technology

- In the very long run, there are changes in the available techniques and resources for firms. Such changes shifts the long-run cost curves.
- Technological change refers to all changes in the available techniques of production.
- Economists use the notion of productivity to measure the extent of technological change.

Technological Change

Three kinds of changes in the very long run:

1. New techniques — process innovation
2. Improved inputs
3. New products — product innovation

New Techniques

Also called process innovation, which was dramatic throughout the nineteenth and twentieth centuries.

Examples:

- Electricity replaced burning fossil fuels
- Gas combustion and wind-powered turbines replaced nuclear, hydro, or fossil fuel-burning generating stations.

Improved Inputs

Improvements in health and education raise the quality of labor services.

Improvements in material inputs are also constantly occurring.

New production techniques and new and better inputs are important aspects of technological improvement.

- They lead to reductions in firm costs and a downward shift in *LRAC* curves.

References

- Lipsey, Regan, and Storer (2008)
- Frank, R.H. (2010)
- Mankiw, N.G., (2023)