

## MA217 Calculus for Social Science II QUIZ I 2014 Numerical Solutions

$$1. (a) \left. \frac{\partial x}{\partial n} \right|_{(-1,1,-1)} = 2, \quad \left. \frac{\partial y}{\partial n} \right|_{(-1,1,-1)} = 0,$$

$$\frac{\partial z}{\partial x} = \frac{-\left(\frac{1}{x+z} + 2xz\right)}{\left(\frac{1}{x+z} - e^y + x^2\right)},$$

$$\frac{\partial z}{\partial y} = \frac{ze^y}{\left(\frac{1}{x+z} - e^y + x^2\right)},$$

Since  $z = +\infty$  at  $(m, n, p) = (-1, 1, -1)$ ,  $\frac{\partial z}{\partial n}$  is undefined.

$$(b) \frac{\partial}{\partial n} \left( \frac{\partial y}{\partial n} \right) = -60n(m^2 - n^3)^9 + 810n^4(m^2 - n^3)^8$$

$$(c) \text{ using Young's theorem } \frac{\partial}{\partial n} \left( \frac{\partial y}{\partial m} \right) = \frac{\partial}{\partial m} \left( \frac{\partial y}{\partial n} \right) = -540mn^2(m^2 - n^3)^8$$

$$2. (a) \frac{\partial P}{\partial x} = \frac{b}{(a+b)^2}$$

$$(b) \frac{\partial P}{\partial y} = \frac{a}{(a+b)^2}$$

$$(c) \left( \frac{\partial P}{\partial x} + \frac{\partial P}{\partial y} \right)_{x=y} = \frac{1}{a+b}.$$

3.  $(-1, -1)$  is a saddle point.

4. The critical point is  $(0,0)$  and its classification is depending on the value of  $b$ .

$$f_{xx} = 2, \quad f_{yy} = 2, \quad f_{xy} = 2b$$

$$D = 4(1 - b^2)$$

For  $D > 0$  and  $f_{xx} = 0 > 2$ ,  $(0,0)$  is a relative minimum when  $b^2 < 1$  or  $-1 < b < 1$

For  $D < 0$ ,  $(0,0)$  is a saddle point when  $b^2 > 1$  or  $[b < -1 \text{ and } b > -1]$

For  $D = 0$ ,  $(0,0)$  has no conclusion when  $b = \pm 1$